Electricity and Magnetism

- Today
  - More on waves
    - Energy Density
    - Power
    - Poynting vector
    - Intensity
  - MW experiment
  - Polarization

Maxwell's Equations in Vacuum

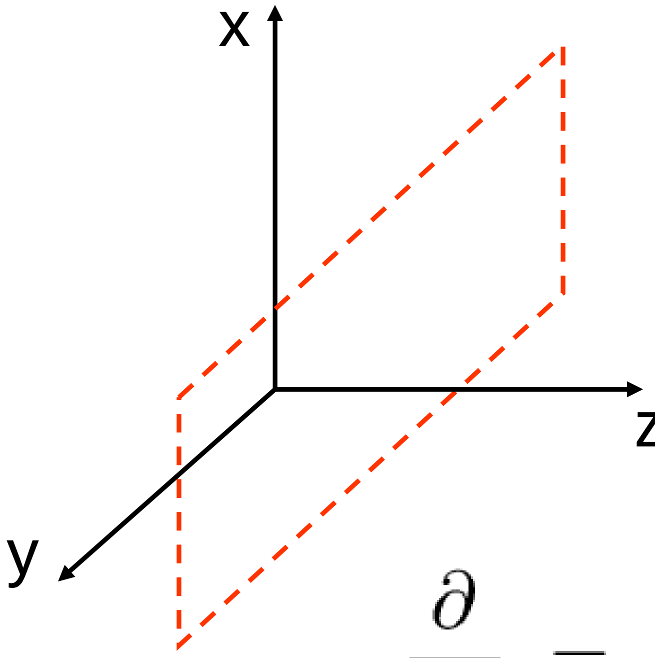
I. $\vec{\nabla} \cdot \vec{E} = 0$

II. $\vec{\nabla} \cdot \vec{B} = 0$

III. $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

VI. $\vec{\nabla} \times \vec{B} = \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$

Solve for a simple geometry



Allow variations only in z-direction:

$$\frac{\partial}{\partial x} = 0$$

$$\frac{\partial}{\partial y} = 0$$

Electromagnetic Waves

- We found wave equations:

$$\frac{\partial^2 B_y}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 B_y}{\partial t^2}$$
$$\frac{\partial^2 E_y}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2}$$

same for E_x, B_x

$v = c$

E and B are oscillating!

Plane waves

- Example solution: Plane waves

$$
\begin{aligned}
E_y &= E_0 \cos(kz - \omega t) \\
B_x &= B_0 \cos(kz - \omega t) \\
\text{with } k &= \frac{2\pi}{\lambda}, \omega = 2\pi f \text{ and } f\lambda = c.
\end{aligned}
$$

E.M. Wave Summary

- $\vec{E} \perp \vec{B}$ and perpendicular to direction of propagation
- Transverse waves
- Speed of propagation $v = c = \lambda f$
- $|\vec{E}|/|\vec{B}| = c$
- E.M. waves travel without medium

Reminder on Waves

- For a travelling wave (sound, water)

  Q: What is actually propagating?
- **-> Energy!**
- Speed of propagation: $v = \lambda f$

E.M. Waves and You

- Transport of information
  - Visible light
  - Communication (Radio, TV, phone,…)
  - MW experiment
- Transport of energy
  - E.M. Energy radiated from Sun supports life on earth
  - Source of *your* Energy via Photosynthesis

Energy in E.M. Waves

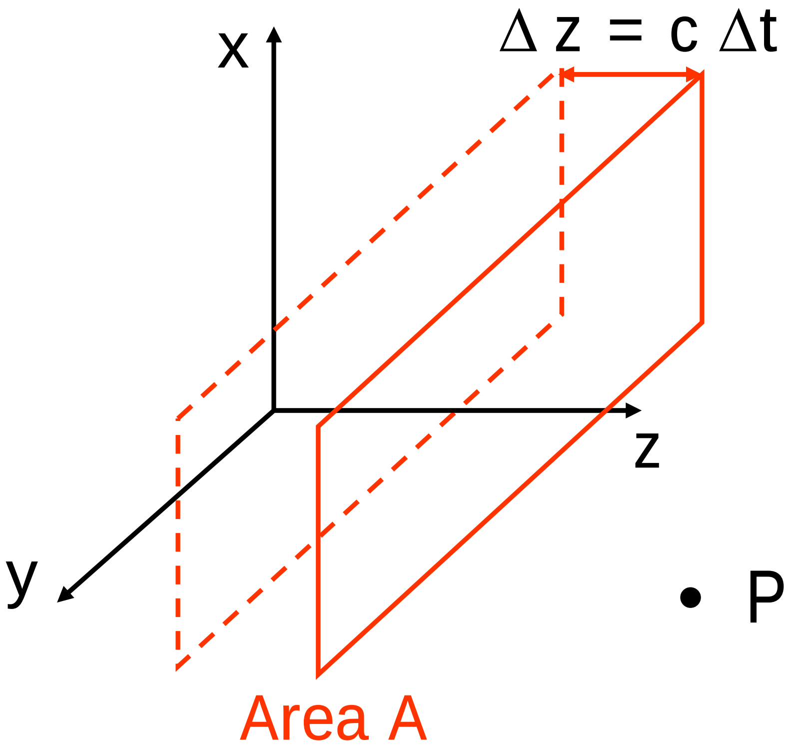
- Remember:
  - Energy/Volume given by $\frac{1}{2} \varepsilon_0 E^2$ and $\frac{1}{2} B^2/\mu_0$

- Energy density for E.M. wave:

$$u = \varepsilon_0 E^2$$

- What about power?

Energy in E.M. Waves



- Power/Unit Area (instantaneous)

$$P/A = 1/\mu_0 \, E \, B$$

Poynting Vector

- Not a typo: John Henry Poynting (1852-1914)
- Wave: Direction + Magnitude
- Summarize using vector: Poynting Vector

$$\vec{S} = 1/\mu_0 \, \vec{E} \times \vec{B}$$

Direction: $\vec{S} \perp \vec{B}$, $\vec{S} \perp \vec{E}$

Magnitude: $|S| = 1/\mu_0 \, EB$
Power/Unit Area

Intensity

- Frequency of waves is very high:

  $\lambda \sim 500$ nm (5×10^{-7} m) -> $f = c/\lambda \sim 6 \times 10^{14}$ Hz

- Look at time average
- Def. Intensity: $I = \langle S \rangle = 1/(2\mu_0)\, E_0 B_0$
- [I] = Power/Area

Intensity

- Example:
  - Radio station
  - Power ~ 10kW, R ~ 20km

    -> Surface A at R: $4\pi R^2 \sim 5 \times 10^9$ m^2

    -> $I = P/A = 10\text{kW}/5 \times 10^9$ m^2 ~ 2 μW/m^2

That's why you need a receiver/amplifier to listen!

Intensity

- Example:
  - Sun
  - I on earth: 100 W/m^2 , R ~ $1.5x10^{11}$ m

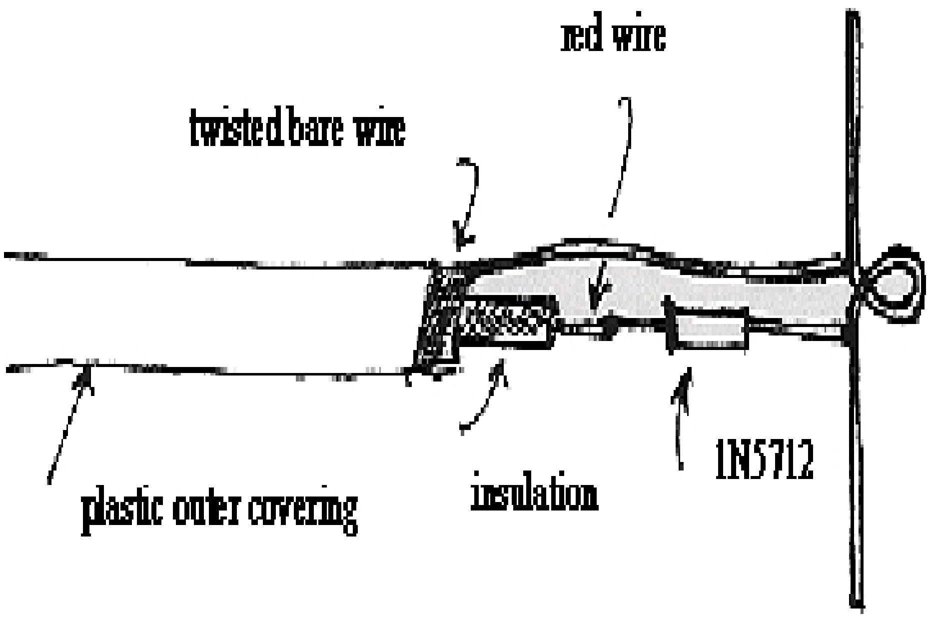
  -> P = I x A ~ 10^2 W/m^2 x $3x10^{23}$ m^2 ~ **3×10^{25} W**

- Very large amount of power
- Took until ~1940 to understand process of “nuclear fusion” (Hans Bethe)
- Weak Interaction
- Hydrogen fused into helium: Mass -> Energy

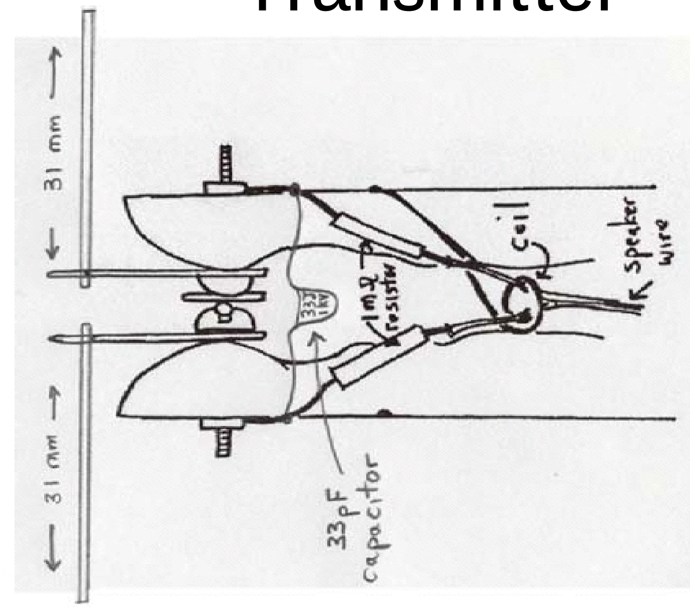
Experiment MW

- Generate and detect E.M. waves

Receiver



Transmitter

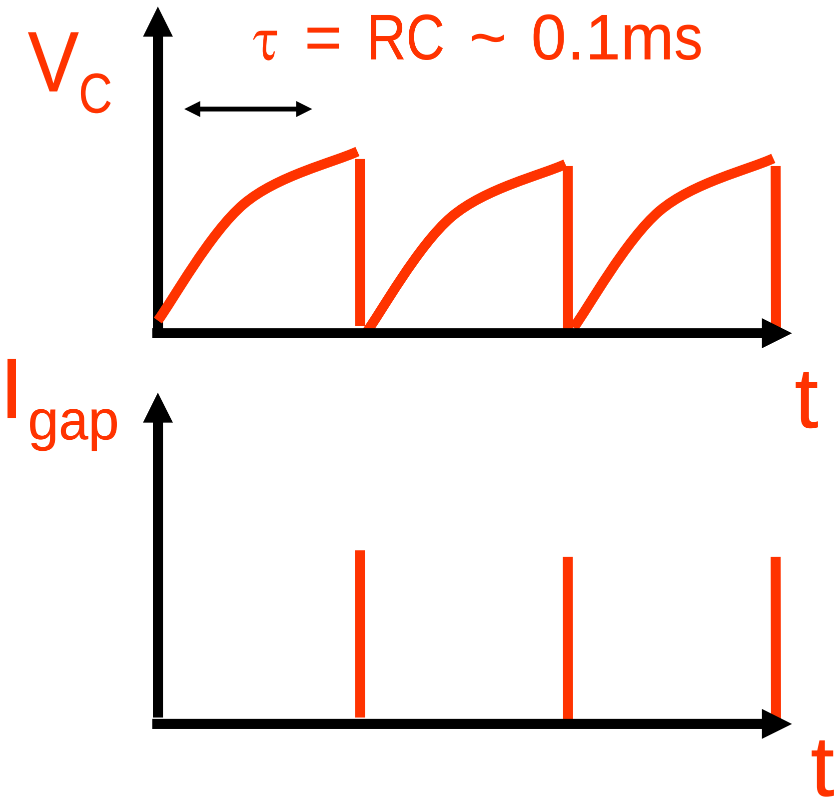


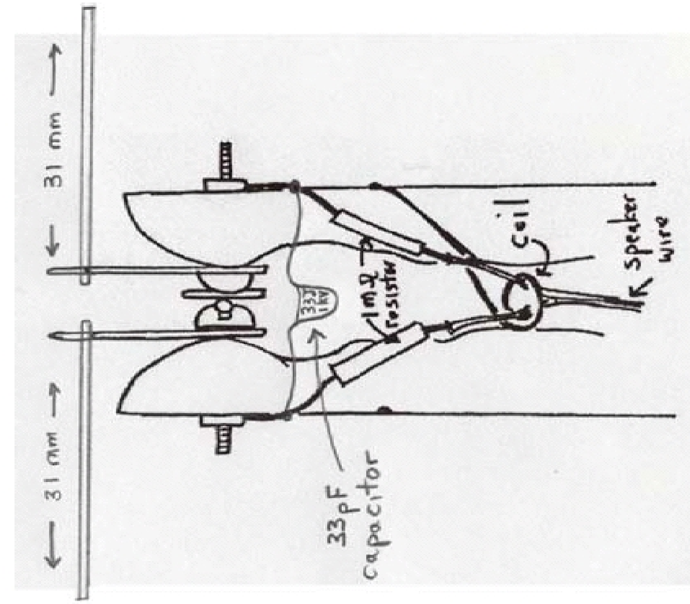
Exp AMP + diode + Antenna

Antenna + Exp EB

Experiment MW

- What determines wavelength?





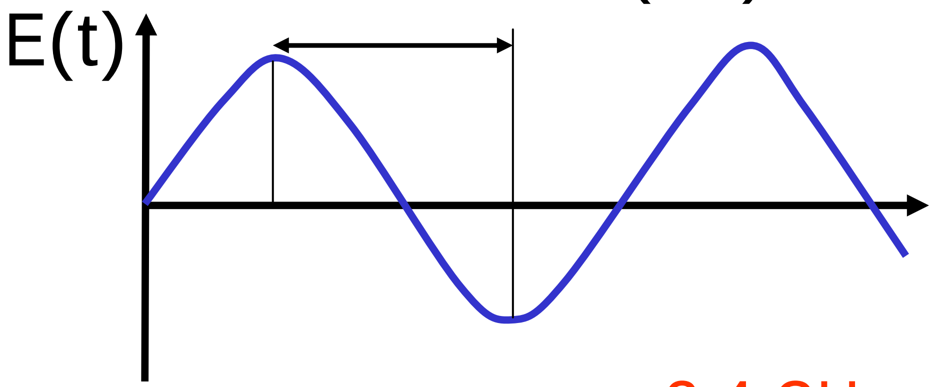
Sharp ‘ping’ at every discharge

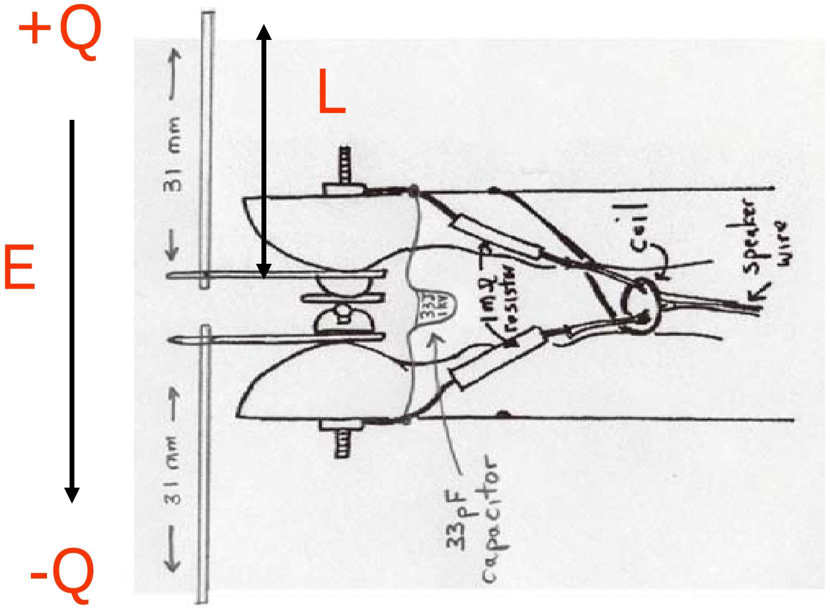
Experiment MW

- In-Class Demo: Tuning fork
- 'Ping' contains all frequencies
- Fork (Antenna) pick out *one* frequency -> Resonance

Experiment MW

$T/2 = 1/(2\ f)$





2.4 GHz

Speed $c = 2L/(T/2) = 4\ L\ f$

Wavelength λ ← 12.4 cm

Experiment MW

- Antenna determines wavelength in MW experiment
- Antenna alse determines direction of E (and therefore B)

-> Polarization