Massachusetts Institute of Technology OpenCourseWare

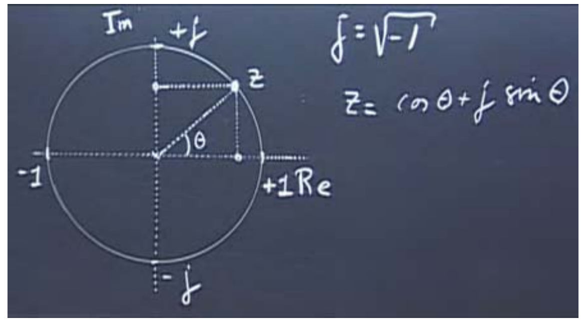
8.03SC **Fall 2012**

Notes for Lecture #1: Periodic Phenomena

The lecture begins by introducing the concept of periodic or oscillatory motion and demonstrating a wide variety of examples, including the quite complex motion of the Euler's disk (Note that the German language name *Euler* is pronounced "Oiler"). Frank Wilczek, mentioned in this section, is an MIT Nobel Prize winning physicist. This section ends with a demonstration of different sound frequencies. The unit of frequency is the hertz (Hz) and 1 Hz is one vibration per second. You should be able to hear many of the frequencies demonstrated: but it is not clear when Dr. Lewin really turned off the tone generator! The term "simple harmonic oscillation" (SHO) is introduced and the SHO equation of motion $x = x_0 \cos(\omega t + \phi)$ is shown (**15:00**). As an aside, the comment about "8.01" refers to an introductory mechanics course at MIT. The term x_0 is the amplitude, the largest value of the displacement. The argument of a trigonometric function is an angle, which we measure in radians. Thus the product ωt is an angle, with ω (omega) the *angular* frequency, measured in radians/s (different from Hz; there are 360°=2π radians in one turn, so ω is 2π times the frequency in Hz). Replacing ωt by $\omega(t+T)$ gives back the same value if $T = 2\pi/\omega$, so T is the period of oscillation. $f = 1/T$ is the frequency in Hz. The concept of projection of circular motion onto an axis to give SHO along that axis is discussed. The angular velocity is unfortunately also called ω. ϕ (called the phase angle) is the angle at time t=0.

A spring (**18:15**) with spring constant k attached to a mass m obeys Hooke's law $F_x = -kx$ giving $ma = m\ddot{x} = -kx$, which is a differential equation. If rewritten as $\ddot{x} + \frac{k}{m}x = 0$, the SHO equation $x = x_0 \cos(\omega t + \phi)$ is a solution to this equation, provided that $\omega^2 = \frac{k}{m}$. This means $\omega = \sqrt{\frac{k}{m}}$, or $T = 2\pi\sqrt{\frac{m}{k}}$. Note that this frequency does not depend on the amplitude of the motion (x_0). The same equation is found for a mass hanging on a vertical spring, in which case the equilibrium position has the spring stretched. The validity of the SHO equation for a mass on a vertical spring is verified in the demo, by comparing directly to a projection of circular motion (**24:20**). Without determining the value of k, the behavior of the frequency with a change in mass is explored (**25:45**). It is actually pretty easy to determine k for a spring using Hooke's Law, and therefore that the equation $w = \sqrt{k/m}$ does work, but this is not done in the video. Look at this equation intuitively: in the period form, it says that a long period arises from a large mass or from a weak spring. Think about why this makes sense. In the actual demo done (**26:20**), pay attention to the use of errors, and the use of multiple oscillations to increase the timing accuracy. The fact that results are obtained which do NOT agree with initial expectations (**32:15**) are a good demonstration of the scientific method, and improvement of an initially inadequate theory. Accounting for the mass of the spring[1] (**34:45**), a more accurate formula is $T = 2\pi\sqrt{\frac{m+\frac{M}{3}}{k}}$ where the inclusion of the mass of the spring (M) corrects the initially observed deviation.

Complex notation can be very useful in discussing periodic motion (**39:15**). Defining $j = \sqrt{-1}$, a set of Cartesian axes can be laid out, labeled as Re and Im (Real and Imaginary, the equivalents of x and y coordinates, respectively) with the y coordinate being multiplied by j. It easily follows that a complex number on a unit circle (**41:15**) is $z = \cos(\theta) + j\sin(\theta)$.



The Euler formula $\cos(\theta) + j\sin(\theta) = e^{j\theta}$ is given without proof (**41:50**), although it is mentioned that a Taylor's expansion is the tool needed. This result dates from 1748! It means that complex exponents correspond to moving through an angle on the complex unit circle. The periodicity of this function is discussed. For a general point in the complex plane, $z = Ae^{j\theta}$. The SHO equation can be written with the real part of $Ae^{j(\omega t+\phi)}$. Clearly, $j = e^{j(\pi/2\pm 2\pi n)}$ (where n is any integer) from the geometry and periodicity. Before the break, an example of an oscillation caused by heat is shown (**49:15**).

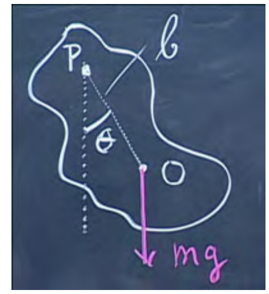
For a pendulum, $T = 2\pi\sqrt{\frac{l}{g}}$ (**51:00**), so a long pendulum should have a longer period. This is again in line with intuition, and readily demonstrated (**52:15**). The small angle approximation is discussed, meaning that in radians, $\sin(\theta) \approx \theta$. Another approximation, that the string is of negligible mass, is in fact very accurate (**55:20**). It's very difficult to make a spring strong but very light, so you typically need to correct for the mass, but it is easier to make a very light, strong string. Since g is known, and l easy to measure, it is easy to compare periods of pendulums to what is predicted. Data testing the small angle approximation $\sin(\theta) \approx \theta$ is shown (**56:15**) with careful attention to experimental errors. A clip from the Physics 8.01 lectures is included (**58:20**), but don't try this at home! Since mass is not in the formula, we do not expect any dependence on mass, and this is well demonstrated by adding Prof. Lewin's mass to a pendulum.

The case of the physical pendulum, in which the mass is not necessarily concentrated at one point, is next considered, using angular motion equations (**1:01:00**). The angular motion analogue of $\vec{F} = m\vec{a}$ is $\vec{\tau} = I\vec{\alpha}$, with $\vec{\tau}$ the torque, I the moment of inertia, and $\vec{\alpha}$ the angular acceleration.

Note that the angular acceleration and torque are vectors (much like $\vec{F}$ and $\vec{a}$ in Newton's law), and are perpendicular to the plane in which the force is applied. The torque is $\vec{\tau} = \vec{r}\times\vec{F}$ (**1:04:00**),

[1]Pages in "French", mentioned from time to time in the lectures, refer to pages in the textbook used at that time for 8.03: A.P. French *Vibrations and Waves (1971) ISBN: 9780393099362.*

where the $\times$ denotes not ordinary multiplication, but rather the vector cross product.



The choice of origin is important in angular motion problems, and in this case a clever choice is made to get rid of the torque from the force that must be present at the point of suspension. By defining this as the origin, $\vec{r}$ to that point is zero, and there is no torque due to the suspension force! For the force applied a distance b from this origin, which is the mg of the centre of mass, the magnitude of the cross product is $bmg\sin(\theta)$, and this is, from the angular equivalent of Newton's law, $I\vec{\alpha}$.

Taking into account the direction of the torque, and that $\vec{\alpha}$ is the second time derivative of the angle θ (actually also a vector, a detail we will ignore), we get $-bmg\sin\theta = I_P\ddot{\theta}$ (**1:06:20**), and in the small angle approximation $\sin(\theta) \approx \theta$, we get the differential equation $\ddot{\theta} + \frac{bmg\theta}{I_P} = 0$ (**1:07:00**). This is SHO with solution $\theta = \theta_0\cos(\omega t + \phi)$. Note the discussion of angular frequency versus angular velocity, which unfortunately have the same symbol ω. We get $T = 2\pi\sqrt{I_P/bmg}$, which is tested for a hoop of mass m and radius R. Nobody in the audience seems to recall how to calculate the moment of inertia (**1:12:00**); it is mR^2 if about the center. By the parallel axis theorem, for a point on the outer edge of the hoop, $I_P = mR^2 + mR^2 = 2mR^2$, and then $T = 2\pi\sqrt{\frac{2mR^2}{Rmg}} = 2\pi\sqrt{\frac{2R}{g}}$. Note that, as with a simple pendulum, m does not appear (**1:13:30**). This is the same as the period of a pendulum with all the mass at the bottom of the hoop. The formula is tested and works to (just) within experimental error.

8.03SC Physics III: Vibrations and Waves
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These viewing notes were written by Prof. Martin Connors in collaboration with Dr. George S.F. Stephans.