**8.03SC Physics III: Vibrations and Waves, Fall 2012**
**Transcript – Lecture 3: Forced Oscillations with Damping**

PROFESSOR: We have discussed free oscillations of harmonic oscillators without damping. And then we introduced damping. But in each of those cases, we let the simple oscillator do its own thing. We did not interfere with it.

Today, that's going to change. Today we are going to impose our will onto the simple harmonic oscillator. And we can impose our will on to it by driving it with a force and then see what the net result is. And let's Start with a simple example that I have here.

A spring with spring constant, k and an object mass, m. Let this be the equilibrium position, x equals 0. There will be damping. I will introduce again b/m=gamma and omega 0 squared equals k/m. We've seen this before, just a shorthand notation.

So now in addition to the fact that when the object is away from equilibrium that there is here a spring force, I am not going to apply on that object a force. It may not be easy to do. We'll get back to that, how we do that.

But I can apply a force on that, maybe through magnetic fields, maybe through electric fields. And this force is now going to have the character F0 times cosine omega t. I impose on that system now that frequency omega. And I can choose that anything I want to.

So now I can write down the differential equation of motion, Newton's Second Law, m x double dot equals minus kx, nothing new that's the spring force. Minus bx dot, nothing new. That's the damping. But now comes this external force F0 cosine omega t.

What I'm going to do now, I'm going to move this to the complex plane. Not that it is absolutely necessary, but I'm so used to that. So I'm going to write this now in terms of Z.

And then we take the real part of Z later that gives us back to x. So we're going to write these now in terms of Z double dot. I divide m out. We get plus gamma times Z dot plus omega zero squared times z. And that now becomes F0 divided by m.

Remember I divided m out. And then we get cosine omega t, for which I will write e to the power j omega t because I work now in the complex plane. And through Euler, I can always convert that back to cosine. My trial function for Z, which is a complex notation, is some amplitude times e to the power j omega t minus delta.

Now crucial is that you understand why this omega and this omega are the same. This is the omega of my driving this system. That is my will that I impose on that system. Clearly given enough time in the beginning the system may be unhappy and it may do all kinds of nasty things, which we will discuss next lecture.

But ultimately I will come out to be the winner and ultimately that system is bound to start oscillating with the frequency that I impose on it. If I start shaking you, in the beginning you may not like that and you may oppose to that. But ultimately I will be the winner and I'll make your shake with that frequency omega. So clearly the ultimate solution must have the same omega as the driver.

And what is the meaning of this delta? Well, it is not at all obvious that the object will be in the same face as the driver. It is possible that when the force is pointing in this direction, that the object maybe going in the other direction.

And you will see that indeed can happen. And so this delta is a face angle, which takes into account the possibility that the driver and the object in their motion are not exactly in phase.

We call this solution a steady state solution. Steady state, that you must wait long enough for the system not to fight you any longer. That will be part of my next lecture, the fighting issue. This is when I ultimately come out to be the winner and when the system follows my will.

So now I'm going to take the second derivative. So I get minus omega squared. j omega comes out twice, so I get minus omega squared. Then I get plus gamma times j omega.

Then I get plus omega 0 squared. And that whole thing is multiplied by A to the power e j omega t minus delta. That now equals F0 divided by m times e to the power j omega t. So the whole thing is now in the complex plane.

And you see that e to the power j omega t cancels on both sides. So I lose my e to the power j omega t. And I'm going to multiply both sides by e to the power j plus delta.

So I lose my delta here. But it appears then here. And so if I make that simple algebraic change, we're going to get minus omega squared plus gamma j omega plus omega 0 squared.

That multiplied by A must now be equal F0 divided by n times e to the power j delta. Omega t is gone. And I've moved the delta to the right side.

Can we live with that? And this can be written F0 divided by m times the cosine of delta plus j times the sine of delta. I'm still in the complex plane but that's Euler.

Now let's compare the apples with apples and oranges with oranges. This is an apple. And this is an apple. That means it's real. And that means this is an apple. But there are also oranges. This is an orange, has a j. And this is an orange that has a j.

And so for this equation to always hold at all moments in time, the apples must be equal to the apples on this side. And the oranges on this side must be equal to the oranges on that side. So it looks like one equation, but it really is two equations.

So we now get that minus omega squared plus omega 0 squared times A must be equal to F0 divided by m times the cosine of delta. Apples on both sides are equal. And now we get the oranges on both sides.

Gamma omega times A, I still have this A here. So gamma omega times A equals F0 divided by m times the sine of delta. Two equations with two unknowns, A as unknown and delta as unknown.

Now they're easy to solve. If you square them, then you get sine squared delta and cosine squared delta. You add them up, that's one.

And so that immediately gives you then what A is. So A is going to be F0 divided by m upstairs. And downstairs you're going to get omega 0 squared minus omega squared squared plus omega gamma squared.

So this is the amplitude of the object. A real massage that we'll talk about this for at least the next ten minutes. It's a very complicated function. We want to see through that equation what that actually means.

And the tangent of delta is easy to find because you to define this equation by that one. You get immediately the tangent of delta. As disappear and F0 over m disappear. And so you'll get the tangent of that angle delta is omega gamma divided by omega 0 squared minus omega squared.

We can now return to the real world. And if we return to the real world, we have to change Z back into x. And so our final solution, which I will put in color, would then be that x as a function of t is this amplitude A times the cosine of omega t minus delta. And that is the omega. That is my will that I impose on the system.

Notice that there are no two adjustable constants, which we were so used to in the past. In the past we said, well you can start the system at t equals 0. You can define the position. You can give it a certain velocity. So you always expect that in your solution there are two adjustables in order to meet the initial conditions.

There are none here. And the reason for that is that this is a steady state solution, which means the system doesn't even remember any more what the situation was at t equals 0. It had lost all its memory.

And so A, which is the amplitude of that object, is not something that you may choose. A follows immediately from this equation, which is a complex function, omega, omega 0, F0, and so on. And delta is also non negotiable. Delta has nothing to do with tour initial conditions. Delta follows from gamma, omega, and omega 0.

So now we're going to look at A and try to understand the complexity of that amplitude. For one thing, it is pleasing that F0, is upstairs. It is intuitive pleasing that if the force that you apply becomes larger that the amplitude will become larger. That's reasonable.

It is also pleasing to see that there is a gamma here downstairs. That means if there is a huge amount of damping, you don't expect A to be very large. So that's also pleasing, that you see a gamma downstairs.

Now, I want to evaluate in detail what is hidden in this very difficult equation. And let me try out your intuition, common sense. Without looking at my solutions, without looking at differential equations, without looking at the equation A, just common sense now. Suppose I apply a force here on this object with a frequency which is near 0, so it takes 100 million years for it to reach a maximum.

And then it takes another 100 million years for the force to go to 0 and so on. When that force has a value F0, what do you think will be the position of that object? If you know that position, that may tell you what A is. What the amplitude is of that object without any differential equations.

Any one of you able to immediately say, of course A has to be this? And maybe that is a little tougher. I see some hands there, no. Oh, you were just doing your hair. Yeah?

AUDIENCE: [INAUDIBLE] use the string constant and [INAUDIBLE] how far it's [INAUDIBLE]?

PROFESSOR: Of course. If that force goes so slowly that at all moments in time there must be equilibrium between the spring force, which is, of course, kx, and the force that you apply, which is uf. And so, if you do it extremely slowly, the two must always cancel each other.

And so I make the prediction now that when omega goes to 0, that A should become F0 divided by k. That is that x. Now let's look at this equation. Let's see whether that is true.

We make omega 0. We make omega 0. So this equation tells us that it is F 0 divided by m. And then downstairs we have omega 0 squared. But omega 0 squared is k over m. And you see indeed that that's exactly what you get.

Now, without looking at equations can you guess what the phase difference is between the driver and the follower? If it takes 100 million years for that force to slowly reach its maximum and 100 million years to go back again, what do you think will be their phase difference between the two? It will be 0, of course. There's plenty of time for that object to follow. So you expect the delta becomes 0.

Well, if omega become 0, this 0 is omega squared. So this goes away. Here's the 0 upstairs. So you see the tangent of delta is 0, and that indeed is what you see. So to follow each other. It's extremely boring, the whole thing, to watch. And the amplitude is exactly what you predict.

Let's now do something more interesting and let us drive it at what we call the resonance frequency. We give it that word. That is the frequency that the system really would love to oscillate in the absence of any damping and in the absence of my doing the silly thing by driving it.

So now we are what we call at resonance, so this term goes away. And this term now becomes omega 0 gamma. So you now get that A becomes F0 divided by m. And then downstairs you have F omega 0 gamma.

Well, if you remember that we introduced quality factor omega 0 divided by gamma, which is a dimensionless number, then you can also write this as F0 divided by k times Q. But that's nice to remember. That at resonance, if you define this as resonance, the amplitude of the object is Q times higher than what it would be at extremely low frequency. Interesting to remember.

So this the amplitude at very low frequency and when you drive it at resonance, it is Q times higher. And then, I will put that here, when omega 2 goes to infinity, everything goes so fast that the object has no time to follow the driver. The object goes nuts because of this high frequency. It can't do anything.

And so A would then go to 0. And let's check that. If omega goes to infinity, you see the downstairs here goes to infinity, so A goes d2 0, so you have no amplitude at all.

What is not so obvious, that delta here is pi. And what is not so obvious here either, that delta goes to pi over 2 in case of resonance. In other words, at resonance the driver and the follower are 90 degrees out of phase. The follower is 90 degrees behind.

Very hard to imagine what that is like but I will demonstrate it. You will be able to see it. What it means here that at very high frequencies, the amplitude of the object goes to 0 but what I will be able to show you, that if the driver goes in this direction, that the object goes in this direction. So they are 180 degrees out of phase, and that I can show you. That's what it means when delta equals pi.

So now we can make a graph, a plot of A as a function of omega. So here is omega and here is A. And let this be the resonance frequency omega 0. I'll make it a little straighter.

So you start at a very low frequency. This is 0. You start here with F 0 divided by k. We all agreed that that was obvious.

And then the amplitude will build up, go through a maximum, goes down, and ultimately goes to 0. And at this value, omega 0, this value is Q times F0 divided by k.

Now for those of you who look very carefully, you may have noticed that the maximum here that I have drawn is not at omega equals omega 0, which may go against your instinct. This maximum occurs at a frequency, which we will call omega max, which is always a little bit below omega 0. But for high Q systems, as I will show you shortly, it is effectively the same. I will come back to this.

The delta, the phase as a function of omega, this is pi and this is pi over 2. And if this is omega 0, then that delta will change in the following way. It is way harder to imagine than that what A is doing, you are in phase at very low frequencies. At resonance, precisely at omega 0, you hit the pi over 2, 90 degrees out of phase, and at very high frequencies you will see that the two are out of phase. And I will be able to demonstrate that to you.

Coming back to this mysterious maximum. It's not so mysterious actually. Where is this, at what frequency, do we have really the maximum amplitude? Well, to calculate that you would have to take the derivative of that monstrous equation.

You will have to take the ad omega, and you ask that be 0. So that's when the maximum occurs. And I will leave you with that exercise. It may take you a few minutes to do that.

And you will find then that omega max-- so where the real maximum is located. The maximum in terms of amplitude, is omega 0 squared minus gamma squared over 2, not 4 but 2 to the power of 1/2. Not so intuitive that it is there.

And if you'd like to write that in terms of Q, which is often done, then omega max. So that is the frequency at which the amplitude reaches a maximum, is omega 0 times 1 minus 1 over 2Q squared. And then the square root of that whole thing. And the reason why this is nice, if you know Q, you can immediately evaluate what the difference is, percentage wise, between omega max and omega 0.

If you want to know what the maximum amplitude itself is, so what A max is, so that's really this value. It must be very close to Q times F0 over q, but it's a little higher. Then you can write that in the following form, and that's just a matter of algebraic manipulation. And you get a Q year, which you expect. And then downstairs you get something like this, 1 minus 1 over 4. And then you get a Q Squared to the power of 1/2.

And so now let's put in some numbers so that you get some feeling for the answers that we have. Suppose we have an example of Q equals 5. That's a modest value for Q. Most pendulums that we have, the Q is way higher than 5. So I take a modest number for Q.

If I go to this equation here, Q squared is 25. 2 times 25 is 50. That's 2%.

But I have to take the square root, so it's only 1% off. So omega max divided by omega 0, is 0.99. It's only 1% lower. It's only 1% below omega 0.

And then if you want to know now what A max is, so you would think that A max is very close to Q times F0 over k. But it is not. Q times, it is a little larger.

And so if we divide A max divided by A0, A0 now is meant to be the amplitude when omega equals 0, that is a shorthand notation. This number is not Q, a little higher. It is now 5.03.

And you can see it. If you Q is higher, then of course these numbers become even closer than omega max becomes even closer to omega zero. And then this maximum A becomes even closer to Q times F0 over k. Rarely ever will we be bothered too much with the fact that the resonance frequency, which we call omega 0, is not exactly the frequency whereby the response of the object is a maximum. Very rarely ever will that become an issue.

I want to show you now a transparency from your own book. So don't take notes. This is from French. You see here the function A. But it is divided by A0, which is that F0 divided by k. So that is the amplitude for zero frequency.

So when you start off it's 1. That ratio is 1 by definition, right? Because it's A omega divided by A0. And horizontally, you see omega divided by omega 0. So by definition, right here at the 1 sign, is that point that I put omega 0 there.

And you see here these various curves for different values of Q. And the one that I made green is Q equals 10. And no surprise that the height is at plus 10 because we predicted that it is Q times higher than the amplitude when we have low frequency.

And you see indeed that the red one is very close to 10. If you look at the one that has a mark Q equals 3, which is this one. If you look very carefully you may see that the maximum A shifted slightly below the value 1, which is omega 0.

But even for equals 3, the differences is insignificantly small. And then at the bottom you see the delta function, the phase delay. The object follows the driver at very low frequency.

Precisely, delta is 0. At resonance it is precisely pi over 290 degrees, very hard to imagine. But I will try to show it to you. And it's very high frequencies. They go like this. They are 180 degrees out of phase.

So now comes the question, how do you apply a force on a system? It's nice to say there is a force but you have to think of a way that you can actually do that. And I will just discuss one case with you and then I will try to demonstrate it also.

If I have a pendulum and I want a force on this object, then I can do that, as you will see, in an indirect way by starting to move my hand here. You will see how that translates into a force on this object, by moving my hand. I am now the driver. My displacement now is in inches, not of force, but is in inches.

Here is the pendulum, length, L, mass, m. And this is the equilibrium position of my hand, and here is the object. But I'm going to move my hand in a way eta equals eta 0 times cosine omega t.

That is the frequency that I decide. I impose that frequency on the top of that pendulum. And the amplitude of my hands in terms of inches or miles or light years, that is linear scale at a 0. It's not a force. It is not a force. It's a displacement.

Well, I take a picture at one moment in time and what do I see? I see that this is what the pendulum looks like. This angle is theta. The pendulum is displaced over a distance x from equilibrium. And the top is displaced over a distance eta. This is Walter Lewin. I am doing that. I am there with my hands, I can't help it. This is where I am. And this is where the object is.

So now I want to put in all the forces that are at work. I will move it up a little bit because I want to have a little bit of room for my forces. So make the legs a little shorter. So here is the object and here is the object.

There are only two forces on this object. And that is gravity, which is mg. And that is the tangent, there is nothing else. I call this x equals 0. And I call this displacement x away from equilibrium.

For small angles I want to argue that T is very close to mg. For one thing, if you hold them vertically and you do nothing and there is no motion, it's obvious that T is mg. The two forces have to cancel each other out. That's clear.

But I can show you that even if the angle are modest that that should also be the case. Suppose I decomposed T in two directions. Vertical direction, so this is T times the cosine of theta. And in horizontal direction, so this is T times the sine of theta.

If the angles are very small, the object is hardly moving at all in this direction. The motion is almost exclusively in this direction. So there is no acceleration in the y direction. Or I should say the acceleration in the y direction is negligibly small. So that means to a high degree of accuracy, T cosine theta is always the same as mg, high degree of accuracy. But for small angles, cosine theta itself is 1. Therefore, T equals mg.

And so the force that is driving this object back to equilibrium is T sine theta. And so that force is mg sine theta to a high degree of accuracy. I'm going to introduce, again, that gamma is b/m, so there is damping.

And I'm going to introduce that omega squared equals g/l. Omega 0 squared is g/l. The square root of g/l being the resonance frequency of a pendulum length l independent of the mass of the object as we have seen before.

So now I'm going to write down Newton's Second Law. So I get mx double dot. And then I get minus bx dot. That is the damping, minus bx dot.

And now comes this force, which is the only one that wants to drive it back to equilibrium. It's the restoring force. And so that force, if you accept my T being mg, that is mg times the sine of theta. That's the differential equation that I now have to solve.

That is a driven system. Now, here I had a driven system. And boy, I saw force here. I don't see anything like that there. Where on earth does Walter Lewin come into this picture? Who is doing something? Have I overlooked myself, perhaps?

AUDIENCE: The angle?

PROFESSOR: Excuse me?

AUDIENCE: Did you change the angle [INAUDIBLE]?

PROFESSOR: I changed nothing. Would I change anything? I changed nothing but I don't see myself anymore. So what's wrong? Is there anything wrong with this? Where do I show up in this equation? Yeah?

AUDIENCE: [INAUDIBLE]?

PROFESSOR: So where in that equation do I show up? What is the sine of theta? What is the sine of theta? What is the sine of this angle?

AUDIENCE: [INAUDIBLE]?

PROFESSOR: x minus eta, that's Walter Lewin. x minus eta divided by l. There I am. And so I'm going to substitute that in here and I'm going to divide by m. No, let's not divide by m yet.

I'll just say x double dot minus bx dot. And then we get minus mg times x over l. And now we bring Walter Lewin to the other side. And so we get plus mg times eta divided by l.

And eta is eta 0 times cosine omega t. Because I am moving my hand, that eta is a function of time. Let me write down an mg in here and then we'll check this.

So mg times sine theta has two terms. It as an mg times x over l. But it has also and mg times eta over l. And I bring that eta over l on this side, but I know that eta is changing in time.

And so you see now Walter Lewin is right there. And now I divide by m. And I substitute omega 0 squared in here. Oh, I had an m here too. You should have screamed. That was an m there. I decided not to divide by m, remember?

Now I'm going to divide by m. So I get x double dot. There's an equal sign here. You should not be sleeping. You're not supposed to sleep.

This is an equal sign minus bx dot, right? Equals minus b, we're in business now. And so I'm going to-- yeah this with also, you're all sleeping. My all of you are sleeping, my goodness. mx double dot minus bx dot minus mg k x/l. And then the minus and the minus becomes plus. Alright? Try not to sleep.

So x double dot. Now we get plus gamma times x dot. And now we get plus omega 0 squared times-- because g/l is omega 0 squared-- times x. So I divide the m out. And now I get equals omega zero squared times eta 0 times cosine omega t. I will move this l up a teeny little bit.

And I'm going to look now at that equation at the bottom. And I am now overjoyed, happiness, because this one looks almost like a carbon copy of the one that I had here was an F0 cosine omega t. And now instead of an F0 divided by m cosine omega t, I now have this.

So this takes the place of my earlier F0 divided by m. F0 divided by m is an acceleration, by the way. It better be an acceleration because this is an acceleration and apples have to be apples. So this is an acceleration. This is an acceleration. And this is also an acceleration. Multiply omega squared by distance, then you get distance divided by time squared.

So you see now how the connection between the two go. Where originally I got an F0 over m times cosine omega t, now because of Walter Lewin's motion, I'm going to get an omega 0 squared times eta 0. And so you see now how these motion of my hand indeed translates into a force on the object.

Well, I have the solution. I don't have to do anything. All I have to do is change this by omega 0 squared times eta 0 and I'm done.

Differential equations are identical. I don't even have to change the tangent of delta. Nothing changes. This is the only thing that changes. So we're done.

We can now make some predictions. The prediction is that if I'm going to shake this pendulum, and I'm going to do that very slowly, taking one how to go to the left and taking one hour to the right. And if my amplitude is eta 0, what do you think that the amplitude A, the solution or my differential equation will be?

In other words, I'm going to shake very slowly, what do you think A will be without looking at the differential equations? So I just go with my hand like this. Amplitude eta 0, amplitude eta 0, and I do it very slowly. What will be the amplitude of A?

AUDIENCE: [INAUDIBLE]?

PROFESSOR: Eta 0. So we expect that this goes to eta 0. Well, if you don't believe it, go to this equation, substitute in here 0, in here 0, you get omega 0 squared, each of this omega 0 squared, until you see eta 0. That's exactly what that equation predicts. But your common sense says the same thing.

Now what do you think delta is if I'm going to move this pendulum very slowly to the left and to the right? Of course. Of course, the object will follow me. It will be ridiculous if I take one week to go from here to here, that the object will be there, right? Obviously, the object is always here. So we also predict that delta is 0.

And so now we can make a quick prediction that at resonance you probably get Q times eta zero. And then delta would become pi over 2. And when you go to very high frequency, then A will go to 0. And then delta will go to pi. And this is what I want to demonstrate to you.

So the final solution of this pendulum, which I will write down in red, is going to be that x equals A times the cosine of omega t minus delta, just as we had before. A is non negotiable, has nothing to do with the initial conditions. Delta eyes non negotiable, has nothing to do with the initial conditions. This is the steady state solution.

Alright. Let me take my shoes off because then you can see it better. Alright here's a pendulum. It's going to be very exciting. I'm going to tell you. I'm going to move this with omega very close to 0. Very exciting. I'm doing it right now. Aren't you thrilled?

No you're not thrilled? But I'm moving. And now I'm going to go back. Do we agree that A, the amplitude of that object, is exactly the same as the amplitude of my hand? Do we agree? Do you see that? That is why that A is eta 0. And that follows from that rather complicated equation.

Did you see that delta was 0? Did you see that we went hand in hand, so to speak, no pun implied. We're going hand in hand, right? That one follows exactly my hand. So delta is 0.

Let's now go to high frequency, very high frequency, way above resonance. And what you see now is that the object is not moving very much, but if you look very carefully you will see when my hand is here, the object tends to go there. And my hand is here, the object tends to go there. That is that pi. Ready?

You see that there's almost no motion. A is near 0. But can you really see that the phase difference is pi? Can you see the 180 degrees? You see if A is exactly 0, the of course, then you cannot tell. So I try not to go infinity fast. I go a little slower than infinity fast. Can you see it?

OK, now comes resonance. And now it will be very difficult to see this pi over 2. That's almost impossible. But that's not my objective. But my objective is to show you that an enormously small, very small eta zero here will give an amplitude there which is Q times higher, so you get a huge swing when my hand is hardly moving at all. That's the power of Q.

So there we go. I'll first get it into it. There it is. Now, this is resonance. Would you agree? This is resonance. Now look at my hand. My hand is moving probably no more than with an amplitude of 3 millimeters. No more.

And yet, I see an amplitude there of 60 centimeters. That would mean that very roughly, this pendulum has a Q of 200. Namely, 60 centimeters divided by 3 millimeters. So this is even a way to make an extremely rough guess, admittedly very rough, of the Q value. You can not even see my hand move. Be honest. You can't even see my hand move but I know I am moving it a little.

AUDIENCE: A little bit.

PROFESSOR: Oh, you're lying. Oh, you are not. No, you will not. OK. So you see all the goodies that we have calculated actually can be demonstrated and show up quite dramatically.

Suppose I have a spring system like this and I want to force on that object here. Well, what I can do is just shake it here in a way extremely similar to what I did there. And when I shake it there, we can make certain predictions. We can make predictions now based on the knowledge that we have.

Suppose I shake it with an amplitude eta zero. No differential equations, no nothing for now. But I know that somehow it will come out in terms of a force at the object.

So I know that when I write down the differential equation, of course it shows up exactly this way. I get an omega 0 squared times eta zero, except the omega 0 squared is now k/m. So what do you think if I shake it at omega equals 0. What is then the amplitude that this object will have relative to my motion, eta 0?

If I move my hands, eta 0, infinitely long, what will this object do?

AUDIENCE: [INAUDIBLE].

PROFESSOR: It will just follow me. So you get this answer. What will be the delta? It will be 0. When I hit resonance, what will be the amplitude of that object hanging from the spring? It will be Q times higher than my eta 0.

What will be the phase difference? 90 degrees. When I shake like crazy, A will go to 0.

So with the spring, if you shake it like this, which is part of your problem set, you will see exactly the same results that we have there for a pendulum. And this now, I want to independently demonstrate for you.

I have here an air track. I can blow out air so that the object here starts floating. So we can make the damping very small by making it float. But if we lower the air flow, the damping becomes a little higher.

I have a spring here with spring constant k. And I have another spring here with spring constant k. They both have spring constant k.

And now I'm going to drive this here at extremely low frequency over a distance eta 0 at maximum. Eta 0 cosine omega t. What do you think the amplitude of this object will be at that very low-- Yeah?

AUDIENCE: Half of the amplitude [INAUDIBLE].

PROFESSOR: Very, very good. Very good. Not eta 0. But why is it half?

AUDIENCE: Because [INAUDIBLE].

PROFESSOR: Because we have two springs. So effectively this spring constant is twice that. Exactly. So if I go very slowly, you will see that this displacement here will be twice as high as this displacement. But what I really want to show you is their in phase. This one will go to the right when this one goes to the right.

Now comes the catch. I showed you earlier there's a steady state solution. In the beginning the system doesn't like me. It hates me. It fights me. It doesn't like that omega. It wants to do something different, which is part of next week's lecture.

And you will see that in the beginning. And so we have to be a little patient before my will survives. You ready for that?

So I'm going to start now to drive the system at a frequency which is below resonance. I want you to see two things, that they go hand in hand. And I want you to see that the-- see you're going to very low frequency.

Here, this is twice the amplitude of the driver. Now it is here, the spring. And now the spring is here. So there's only this much to eta 0. So eta 0 is no more than 3/4 of an inch.

And now we're going to let that object be exposed to this driver. And we'll give it a little bit of time to recognize me. It takes a little bit of time to reach the steady state solution.

And next time we will learn how much time it actually takes. So if you want to be a little bit patient, then you will see. If we give it too much damping, too little air, then of course it starts to get stuck.

Yeah, we're close. We are close. For me, close enough. Now look at it. They're going both to the left for me and both to the right for me. For you they're going now both to the right and they're going both to the left. They're going both to the right and both to the left.

Now, this was the amplitude-- twice the amplitude of the driver. And when you look carefully here, it's less. It is that eta 0 over 2, that this gentleman immediately noticed, because we have two springs. So you see here, apart from the factor of 2, you see the delta 0. And you see that the amplitude indeed is half of the amplitude of the driver because of the two springs.

Now we're going to resonance omega 0. And now nasty things may happen. It may break. We have to give it time. You see what funny things it's doing?

It's not in steady state yet. We have to wait. Just a little patience. Give it more time.

Notice also that the-- remember, this is only moving this much. Look how much this is moving. I may not even be exactly at resonance, you know. We can only do the best we can hear. It may not be exactly at resonance.

Oh boy, I'm close to resonance now. Oh yeah. Oh, man. Look at that, ooh! Am I at resonance. I think I got it there. You see?

They're neither in phase nor out of phase. Now you see the 90 degrees. And look at this teeny weeny little displacement here.

And look what this man is doing. That is resonance. Remarkable. Markos, where's Markos? We hit it. Right on!

Now I will oscillate it way over resonance. Not way, but over resonance. The system must first calm down.

And now I will change the frequency above resonance so that now you will see the phenomenon that I discussed earlier, that the amplitude is very small. Again, we have to wait a little. Look how fast it's going. And that then will go 180 degrees out of phase.

Now, look this is going this much back and forth. And this one is not doing very much. Now you can't see it. I can. It looks like this. Can you see it? That's what 180 degrees out of phase is. Very clear.

Five minute break. See you back here in exactly five minutes.

Now so we have discussed today some simple systems. Pendulum, one object. Springs, one object, one resonance frequency. But soon in 803, we will discuss systems with more than one object.

For instance, if I put three cars on here with four springs, three resonance frequencies. If I have a triple pendulum, which I will demonstrate next week, one pendulum, below the other, below the other, three resonant frequencies. Five cars on there, five resonance frequencies. So simple objects like a dinner plate or just a regular glass has an enormous number of resonance frequencies. It can oscillate in many, many different ways.

If you drive your car, your wheels turnaround, that's oscillation, a certain period underlying. And you may notice at certain speeds that something in your car begins to rattle, very annoying. All you have to do is go a little slower or go little faster and it stops. You go off resonance for that object. Now, you may go on resonance for another object, of course. And some cars rattle at any speed.

You have a radiator in your room, which rotates. That is also and underlying oscillation in a period. That may start to cause resonance in the frame. You may hear some awful noise sometimes. Unfortunately, these fans, you cannot change the speed so easily. But you can go from state three, to two, to one and then this terrible noise will go away.

You take a washing machine or dryer-- I used to remember, a friend of mine in the Netherlands had a dryer. And when you started the dryer, at a very early phase, when it was at a certain frequency, the whole dryer would start to walk through room. It would. And then at higher frequencies, of course, it would stop.

Resonances are everywhere and they often occur when you don't expect them. You open a faucet. You think it's a steady stream of water, which I'm sure it is. But sometimes you're hear, errr! An unbelievable sound. It drives you almost nuts. I'm sure all of you have heard that sometimes.

If it isn't in your dormitory, maybe at hotels or at home. All you have to do is open the faucet a little more or a little less and it goes away. And it's really an extremely loud and annoying resonance.

If you take something as simple as a wine glass, which has a tremendous number of resonances, then I can make you listen to a well known resonance, which is by rubbing the rim of the glass. When I rub the rim of the glass, I'm not exciting it at one particular frequency. And certainly not at a resonance frequency.

I'm exciting it and lots and lots off frequencies. I dump on it the whole spectrum of frequencies. But the class is mean. It just picks out the one which is its resonance. That's where it builds up a large value for A. It ignores all the others. And that's why I can make it resonate at that particular frequency. Listen to it.

This is not one frequency, what I'm doing. It has nothing to do with the time that is for me to go around. It's very high pitched. But it's about 420 hertz.

So the rubbing is like dumping a spectrum of frequencies on it. And it selects what it likes the most. When I was a student, I remember that we often had after dinner speaker.

We have dinners at the fraternity and we had an after dinner speaker. And more often than not, we didn't like the after dinner speaker. We didn't like the speech. And so we made that very clear.

And the way we did that was, all our wine glasses. An enormous sound in that dining hall. And the speaker very quickly got the message, of course. That is an enormous sound that you generate. And most of these wine glasses were roughly the same. So it was always a tone that was loud and clear and almost one frequency.

You've seen footage lately of the storms. Three storms in a row. And you must remember sometimes that you saw traffic signs. Your pole, and then a traffic sign, and then even though there's some kind of a crazy wind going, the traffic sign goes like this. Shhh. Shhh. All resonant frequencies. And can even break.

Even though the wind appears to be relatively steady, the wind then generates, in a way, a whole spectrum of frequencies. And this traffic sign picks out the one that it likes the most. And then it goes nuts at a frequency that is a resonance frequency. And resonances can become destructive, of course. If these amplitudes are too high, then things can break down. You may have noticed I was worried here when the amplitude became so large that the spring might even break or the car might jump off that track.

And of course a classic example of this destructive resonance is the destruction of a very famous bridge in this country in 1940. The Tacoma Narrows Bridge, Washington state, was destroyed by winds. And this bridge had many different resonant frequencies. One like this. And some like this. And depending on the wind strengths, different resonances were excited at different moments in time. But that ultimately led to the destruction of the bridge.

Now most of you have seen this movie. But just for the few who haven't, I really want you to see this movie. You cannot go through MIT and not have seen the destruction of the Tacoma Bridge Movie.

There are countries where soldiers are not allowed to cross a bridge when they are marching in step. That's the case in the Netherlands, my home country. It's also the case in many European countries. The story has it that in England-- I think more than 100 years ago-- when soldiers went over the bridge in step that we bridge collapsed. Whether that was the result of the soldiers, we will never know. But in any case, from those days came the order that soldiers had to go out of step before they crossed the bridge.

There is a rumor, that most of you have heard, that there are women who are capable of singing with such a loud voice that it can break a wine glass. They have tune exactly at the resonance frequency of the wine glass. And they come out, huge volume, and the glass bricks. I don't believe it but it's a rumor.

There was a commercial many years ago, some of you may never have seen it, for Memorex. Memorex was a tape, a special tape for audio tape recorders. This guy is going to a concert and there's this woman singing. Loud voice, beautiful frequency, bingo, glass breaks.

He comes home and he tells his wife the story. Well, his wife was smart enough, of course, not to believe it. And he says, well it just so happens that I recorded it on my Memorex tape. So you plays the tape at home and at the moment that the woman's voice goes--

[CRACKING SOUND[

What happens? The glasses break at home in his cabinet.

So much for the physics of Memorex because you can imagine that the resonance frequency of the glasses at home were very different than the glasses in the orchestra. So it's all a swindle. But that's, of course, what commercials are all about. What is now the bottom line? The bottom line was that if you buy Memorex, these tapes, then the reproduction is so perfect that you can even take it home and you see the glasses break.

This brings up now the $64 million dollar question. And that is can it be done or can it not be done? If there are any women in my audience who want to give it a try, that would be wonderful.

We've asked ourselves that question. Can this be done or can this not be done by a person? And I think we came to the conclusion that a person alone without all kinds of electronic equipment could not do that. And I still believe that today.

But professor Felts, Michael Felts here at MIT with one of his graduate students many years ago, developed some powerful equipment which you see here, which was designed to make an attempt to break a wine glass. It doesn't always work. But it works often.

The idea being then that here is the wine glass. It's almost a carbon copy of this one. When they finally made this to work-- it's near 440 hertz. They went to Crate & Barrel and they asked, how many of these wine glasses do you have? They said we have 5,000. And they bought them all, 5,000. Because it's not obvious if you have to change the glasses that it will ever work again.

And so here is one of those glasses. Here is a loudspeaker. The sound comes from this side. And what's nice about this arrangement is that we can make you see the distortion of the glass in this resonance mode.

The glass oscillates like this. It goes from oval to circular, t oval to circular. And the way we can make you see that is by strobing the glass at a frequency which is a little bit different from the frequency of the sound.

Now think about it. Two frequencies almost the same give you a big phenomenon. So what that comes down to is you see the motion of the glass very slowly. And then, if we hit that resonance frequency, we will increase the volume. And then maybe it will break.

Now, I have to warn you. The sound will be unbearably high. And so for some of you here in the front row you may even want to cover your ears, or you may want to move back. It's up to you. But be careful. This is really an enormously strong signal that you're going to hear. So be careful. You can move back if you want to.

I have hearing aids, as you have noticed. And I have the option that I can turn them off. But in spite of that I'm not deaf without them. I will still cover my hears.

So let me first give you the light setting that we want. So we make it a little dark. And then I will show you the glass.

Oh, I have to change the setting here. There's the glass. There's no sound. It's not oscillating. And now I turn on the speaker.

So I'm now going to turn my hearing aids off and put this on. And I'm going to increase the volume. And if it doesn't want to break, I will change the sound frequency a little bit to sweep over that resonance curve so that I get on to the maximum. Because the Q is so high over this system, that if I'm a little bit off in frequency, then the amplitude will be low.

So let's first see what happens when I increase the volume. Did I do something wrong? Yeah, I changed the frequency.

It's very close.

Getting very close.

[APPLAUSE]

I think I have convinced you that a woman cannot do this. I have some last words of wisdom for you and that is falling in love is also a form of resonance. And it too can be destructive because it can break your heart. So try to remember that next time. Have a good weekend.