Massachusetts Institute of Technology OpenCourseWare

8.03SC **Fall 2012**

Problem Set #1

Problem 1.1 – Manipulation of complex vectors

a) Find the magnitude and direction of the vector $(4 - \sqrt{5}j)^3$.

b) What are the real and imaginary parts of $\dfrac{Ae^{j(\omega t+\pi/2)}}{4+5j}$ assuming that A and ω are real?

c) Write the following complex vectors Z in terms of $a + jb$ (a and b are real). Notice that there may be more than one solution. $Z_1 = (j)^j$ $\qquad Z_2 = (j)^{8.03}$

Problem 1.2 (French 1-10)[1] – Simple harmonic motion of y as a function of x

Verify that the differential equation $d^2y/dx^2 = -k^2y$ has as its solution $y = A\ cos(kx) + B\ sin(kx)$ where A and B are arbitrary constants. Show also that this solution can be written in the form

$$y = C\ cos(kx + \alpha) = C\ \mathrm{Re}[e^{j(kx+\alpha)}] = \mathrm{Re}[(Ce^{j\alpha})e^{jkx}]$$

and express C and α as functions of A and B.

Problem 1.3 (French 1-11) – Oscillating springs

A mass on the end of a spring oscillates with an amplitude of 5 cm at a frequency of 1 Hz (cycles per second). At $t = 0$, the mass is at its equilibrium position ($x = 0$).

a) Find the possible equations describing the position of the mass as a function of time, in the form $x = A\ cos(\omega t + \alpha)$. What are the numerical values of A, ω, and α?

b) What are the values of x, dx/dt, and d^2x/dt^2 at $t = \frac{8}{3}$ sec?

Problem 1.4 (French 3-4) – Floating cylinder

A cylinder of diameter d floats with l of its length submerged. The total height is L. Assume no damping. At time $t = 0$ the cylinder is pushed down a distance B and released.

a) What is the frequency of oscillation?

b) Draw a graph of velocity versus time from $t = 0$ to $t =$ one period. The correct amplitude and phase should be included.

Problem 1.5 (French 3-14) – A damped oscillating spring

An object of mass 0.2 kg is hung from a spring whose spring constant is 80 N/m. The object is subject to a resistive force given by $-b\,v$, where v is its velocity in meters per second.

a) Set up the differential equation of motion for free oscillations of the system.

b) If the damped frequency is 0.995 of the undamped one, what is the value of the constant b?

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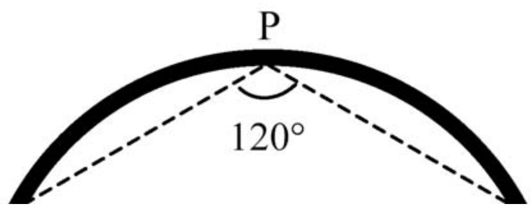
[1] The notation "French" indicates where this problem is located in one of the textbooks used for 8.03 in 2004: French, A. P. *Vibrations and Waves. The M.I.T. Introductory Physics Series.* Cambridge, MA: Massachusetts Institute of Technology, 1971. ISBN-10: 0393099369; ISBN-13: 9780393099362.

c) What is the Q of the system, and by what factor is the amplitude of the oscillation reduced after 4 complete cycles?

d) Which fraction of the original energy is left after 4 oscillations?

Problem 1.6 – A physical pendulum

A uniform rod of mass m is bent in a circular arc with radius R. It is suspended in the middle and can freely swing about point P. The length of the arc is $\frac{2}{3}\pi R$.



a) What is the period of small angle oscillations about P?

b) Compare your result with the period derived (and demonstrated) in lectures for a hoop with mass m and radius R.

Problem 1.7 – Damped oscillator and initial conditions

The displacement from equilibrium, $s(t)$, of the pen of a chart recorder can be modeled as a damped harmonic oscillator satisfying the homogeneous differential equation $\ddot{s}(t) + \gamma\dot{s}(t) + \omega_0^2 s(t) = 0$

a) Find the time evolution of the displacement if the pen is critically damped and subject to the initial conditions $s(t = 0) = 0$ and $\dot{s}(t = 0) = v_0$. Does $s(t)$ change sign before it settles to its equilibrium position at $s = 0$?

b) Find the response of an overdamped pen subject to the initial conditions $s(t = 0) = s_0$ and $\dot{s}(t = 0) = 0$.

c) Use your favorite mathematical tool[2] to plot your solution for $s(t)$ in part (b) as a function of time. Use $\omega_0 = 3/7 \times \pi$, $\gamma = 3$ and $s_0 = 1$ for the plot. Let time run from 0 to 10 seconds. For your own curiosity, once you have your code written, you can vary γ to see the effect of the damping on the response.

[2]Examples include Matlab, Mathematica and Maple.