8.03SC Physics III: Vibrations and Waves, Fall 2012
Transcript – Lecture 13: Deriving Electromagnetic Waves

PROFESSOR: Today's lecture is going to be about electromagnetic radiation which is one of the highlights in 8.03 and one of the great victories of 19th century physics. So far we've dealt with electromagnetic waves, well excuse me, with mechanical waves. And we've dealt with sound waves. But today we're going to enter the domain of electromagnetic waves-- radio waves, radar, infrared, visible lights, ultraviolet, gamma rays, x-rays, electromagnetic radiation.

An electric field which is associated with a magnetic field, one cannot be thought of without the other, propagates through space even in empty space, even in a vacuum. And I will start with Maxwell's equations and then you will see a little bit more math which I do to brush up on what you may have forgotten.

So let's start with Maxwell's equations. The divergence of E equals rho divided by epsilon 0-- Gauss's law. In vacuum there is no charge density so this is 0.

Then we have the divergence of B. That is 0. If one of you discovers ever a magnetic monopole you would not only get the Nobel Prize but then the divergence of B would not be 0. And then we have the curl of E equals minus dB dt-- Faraday's law. And then we have the curl of B equals mu 0 times J. J being current density, but in vacuum. There is no current. So this term is 0. This was Ampere's law in the old days of 8.02. And then it was Maxwell himself who added this term which is generally referred to as the displacement current.

The del vector is an operator. d dx, x roof plus d dy y roof plus d dz z roof. If you apply the del vector on a function, phi, This is called the gradient of phi. That's a vector that is d phi dx x roof plus d phi dy y roof plus d phi dz z roof.

And then we have the divergence of a vector A. That itself is a scalar. And that is dAx dx dAy dy dAz dz.

The curl of A-- I will raise this again later. I want you to see this. The curl of A, the way I remember it, and you probably do the same, x y, z, d dx, d dy, d dz, and then you get your Ax, Ay, Az. And that then becomes dAz dy minus dAy dz. And that is in the x direction.

And then you get dAx dz minus dAz dx. That is in the y direction. And then you would get dAy dx minus dAx dy. And that is in the z direction. We'll have to check that because it's so easy to make a little slip. And that is very awkward for me and for you also later. That is fine. And there we get dAy dx, minus dAx dy. That is fine.

There is one and only one vector manipulation with the del vector that I want you to know. I don't want you to remember it. I certainly don't remember it, but we need it today. And that is the curl of the curl. The curl of the curl of A. And I will show you the result without proof, is the gradient of the divergence of A minus del dot del vector A.

And this we often write as simply del squared. So for this we write, del square of A. And this is also called the Laplacian. And so the del squared over vector is d2 A dx squared the whole vector, plus d2 A dy squared plus d2 A dz squared.

And this has nine terms. Because this has three terms, this has three terms, and this has three terms. I will do the first one and then you have to do the others. So I will only do the component in the x direction. Which then becomes d2 Ax dx squared. d2 Ax dy squared plus d2 Ax dz squared. And that is then the component in the x direction. And there is also one in the y direction and there's one in the z direction.

This first term comes from this one. The second term comes from this one. And the third term comes from this one.

So now we are in a good position to start to use this on Maxwell's equations. So I will raise this again and I will lower this so you can see both. And I will start working on the center board.

So what I'm going to do now I'm going to take the curl of the curl of E. And I know that the curl of the curl of E is minus d dt because the curl of E is minus dB dt. And so it is minus d dt times the curl of B.

Now I use the only thing that we should remember, and that is this identity, that the curl of the curl of A is the gradient of the divergence of A. But the divergence of E is 0 in empty space. You see that there. So this term, this first term is 0. So I only have this term, which is then minus the Laplacian operator on the vector A. So this is also minus del squared E.

Now the curl of B is epsilon 0 mu 0 dE dt. So when I take the time derivative I get minus epsilon 0 mu 0 times d2 B-- excuse me d2 E, I have an E there-- d2 E dt squared. So that is this part.

And now I get on the right side I get minus d2 E dx squared plus d2 E dy squared plus d2 E dz squared. This minus sign and this minus sign eats each other up.

And this equation that you're looking now at is a milestone in the history of mankind. This equation changed our whole way of thinking about the world and even about the universe. This was the great victory of Maxwell. This is a wave equation in an electric field in vacuum.

So this tells you that you must be able to create electric fields, this is a vector, which move with a speed v for which we always write c, which is 1 over the square root of epsilon 0 mu 0. Because, remember, this is a wave equation and what you see here is always 1 over v squared. And it was Maxwell, of course, who was the first to recognize that because this was only possible because he added this term to Ampere's equation. He was a genius.

Now you can go through a similar reasoning instead of taking the curl of the curl of E, you can take the curl of the curl of B and then you will find that there must be an associated magnetic field for which you get that the del squared B equals mu 0 , epsilon 0 times d2 B dt squared.

One cannot exist without the other. To put it even more bluntly, one is the other. You cannot think of them as being separate. One cannot exist without the other.

So we're going to have three dimensional wave equations, and the electromagnetic waves are then characterized by two interdependent oscillations, one in the E field and one in B fields. And the speed with which they propagate in a vacuum is this. That follows immediately from Maxwell's equations. Untouched by human hands, so to speak. And if you use the value for epsilon 0 and mu 0 you will find that this is extremely close to 3.00 times 10 to the 8 meters per second.

And the amazing thing is that epsilon 0 can be measured without any time variability. Epsilon 0 follows from Coulomb's law, as static as you can have it. And mu 0 can be measured without any time variability. Mu 0 follows from Ampere's Law. And so with these two, what I call static quantities, Maxwell was able to demonstrate that they dictate the speed of light in vacuum.

Mu 0 is called the permeability of free space. You can look up what it is, it's 4 pi times 10 to the minus 7. And epsilon 0 is called the permittivity of free space, 8.8 times 10 to the minus 12.

Maxwell knew the speed of light. It was known way before Maxwell. In 1676 Romer, in a brilliant way, using the eclipsing times of the moons of Jupiter, derived the speed of light in 1676. And he came up with 214,000 kilometers per second. And the only reason why he was on the low side is that it wasn't well known in those days what the distances were between planets.

And then 1728 James Bradley used another brilliant technique. I will not go into the details, but it's called stellar aberration and he used that technique to determine that the speed of light was 301,000 kilometers per second.

And then in the late 19th century, in fact in 1849, both Foucalt and Fizeau measured the speed of light in their laboratory in France. One used a rotating mirror. The other used a rotating disk. And so they were even able to do it in the laboratory, and they found values which were within 5% of 300,000 kilometers per second.

So Maxwell knew the answer. And so when he saw this, he immediately realized, he postulated that light is an electromagnetic phenomenon. And in 1865, he laid the foundation of the very famous electromagnetic theory of light which changed the way that we look at the world.

So now comes the question, what are the solutions to this wave equation? Well, I will start with the simple one. And the simple one actually holds all the key information that you need, but then we'll build it up and make it a little bit more complicated. I start with an xyz coordinate system-- x, y, z.

And whenever in physics you deal with cross products, always make your coordinate system right handed. Don't even think of left handed coordinate systems. And a right handed coordinate system is a coordinate system whereby x roof crossed with y roof equals z roof. Excuse me. Do you mind turning that off?

So I assume a simple case that the E vector only exists in the x direction. So it has a certain amplitude. I call it E0x. And I make it a traveling wave, of course. Cosine omega t minus kz. So Ey is 0 and Ez is 0. The only E vector in the x direction.

The complicated three dimensional wave equation collapses now to a one dimensional wave equation. Of the nine terms that you have here only one term survives and that is d2Ex dz squared. And that then becomes mu0 times epsilon0. And where you take the Edt squared, d2E dt squared, only one term survives. Because there is no Ey. There is no Ez. And so this now becomes d2Ex dt squared. So this is now a one dimensional wave equation. I started easy.

Now remember that the curl of E is minus dB dt. So the curl of E, in this case, and you can check that because you know what the curl is. Here's the curl, the curl of E, there's only one term that survives. I'll write down here the curl of E. There's only one term that survives and that is dEx dz in the y direction. And that now must be equal to minus dB dt. Because that is Maxwell.

I will lower this and later I will raise it, so that you can see this. But it's important that you can see this above my head.

So my question now is, what is B? So this is the traveling wave in the z direction, which has an E vector only in the x direction as of now. And I want to know what is the associated B field. Well, for one thing, you can already see that the B field is going to be in the y direction. If we take dEx dz, so this is dEx, and we take the derivative again, this is z, we get a minus k, then you get the E0x and then the cosine becomes minus sine. So this becomes a plus times the sine of omega t minus kz. And that now equals minus dB dt.

So we're almost there. All we have to do now is do an integration in time. So we bring the dt-- oh, it's important that we get the y direction. That is very important. Very important. Remember, this y, very important. That's going to be the direction of the B field. Don't forget that y.

So now you bring the d2 here and you do an integration in time. So now you get the B field that is going to be associated with that traveling wave electric fields. So if you do an integral, the omega pops out below here. So you get kE0x divided by omega. The sine becomes minus the cosine, but I also have a minus sign here so those two minus signs cancel. And so I get the cosine of omega t minus kz.

But omega divided by k is c. That is the speed of light. And so I can also write this now, let me write it down, B in the y direction, indicate that it is in the y direction, E0x divided by c because omega divided by k is c times the cosine omega t minus kz. And just to remind you here is your y again. But, of course, the By already indicates that.

And so now you have found the B field that is associated with this E field. As I used to say, and I said earlier, one is the other. One cannot exist without the other. So if we compare the two now you see several things which holds in general.

And that is, you see that the magnitude of the E field, this is the of the B field, the magnitude of the B field, you can call this B0y, is c times lower than the magnitude of E field. See this c here. Notice also that the E field and the B field are in phase with each other.

You have here cosine omega t minus kz and you have the same here. There is no phase difference between them. That means if one reaches a maximum, the other reaches a maximum. And if one is 0 the other is 0. Because they oscillate with that frequency omega.

Notice also that B is perpendicular to E because E is in the x direction and B is in the y direction. Notice also that each one of them is perpendicular to the direction of propagation. E is perpendicular to the z direction and B is perpendicular to the z direction. And all this follows from Maxwell's equations.

So now I would like to make a sketch of what this electromagnetic wave looks like and I will make an attempt to make you see it. it's not so easy, but I'll make an attempt.

So here is the x direction. And here is the y direction. And let this be the z direction, x, y. I pick the particular moment in time then I know that the E vector is a cosine function in z because I pick a particular moment in time. And so I'll try to put in here the cosine function. If you think it's a sine function then, of course, it's the same thing. And so, at this frozen moment in time, the E vector would be like this, like this, like this, like this, and like this. And this value here would then be E0x.

Associated with that E field is a B field that is in the y direction. So that's like this. It's in the yz plane. So the B vector is like so, like so, like so, like so, here, here, here, here. And this value then is B0y, that value.

And this whole pattern moves with velocity c in this direction, and the two are married together. They are stuck together. And if you take any plane perpendicular to the z-axis, a plane which is infinitely large, infinitely large in this direction, infinitely large in this direction, infinitely large in that direction, and infinitely large in that direction--

At that moment in time, the E vector is everywhere in that plane exactly the same value. Look at the equation. There is no dependence on y and x. That's why we call them plane wave solutions. How realistic they are is another matter, but they are consistent with Maxwell's equations.

And the same is true for the B field. So any plane that you take perpendicular to z, someone is sitting there, someone is sitting 25 miles away from you, but on that same plane at any moment in time you will see the same E vector, all of you, and the same B vector. And that whole pattern then moves with velocity c in space.

And so if I'm standing here, in one of the many planes perpendicular to the z-axis, and the electromagnetic wave comes to me, and there's another person standing there in the same plane, we will see the E vector go like this, and we see the B vector go like this, but they go in unison. When the E vector is maximum here the B vector is maximum there. And then they reach both zero at the same moment in time and then the E vector goes negative and then the B vector does that. So that's the way the oscillation works.

I would like to summarize for you what is actually the bottom line of all this. And when I have to solve B fields for given E fields or E fields for given B fields, that's really the only way that I used to think. So we have a traveling electromagnetic wave. So what follows only holds for traveling electromagnetic waves not for standing electromagnetic waves which comes in the future. Traveling electromagnetic waves.

The E vector is perpendicular to the direction of propagation. The B vector is perpendicular to the direction of propagation. E and B are in phase if one reaches 0 the other reaches 0. E is perpendicular to B. And as a result of the fact that they're both perpendicular to the direction of propagation, it follows immediately that E cross B is in the direction of propagation. And then last but not least, I have no room for it, I'll put it here, that the magnitude of B at any moment in time is the magnitude of E divided by c.

If the E vector is only in one direction as it is here in the x direction, we call that linearly polarized radiation. The word speaks for itself, linear. Now, of course, it is entirely possible which is also a perfect solution to Maxwell's equations and to a wave equation. You could easily have an E field which is in the y direction. At this moment in time it would be coupled to a B field which is in the minus x direction so that E cross B is still in the direction of propagation. And that would also be a perfect solution to the wave equation. And so the sum of the two or any linear combination of the two should also be a solution to our wave equation.

So, therefore, I can write now in somewhat more complicated form for traveling wave, in which I give it both the component in the x direction as well as a component in the y direction and I can even change the phase between the two. In other words, the following E vector is perfectly kosher. E0x times the cosine omega t minus kz. That would then be in the x direction. And then I would have another E0y times the cosine omega t minus kz.

And I can give it any random phase angle delta. And that would then be in the y direction. And so clearly this satisfies the wave equation because each separately satisfies the wave equation.

If now we make delta equal 0, you still have linearly polarized radiation. Look at the xy plane. This is x, and this is y, and the radiation is coming straight to you. At a certain moment in time the E vector in this direction reaches a maximum, E0x. And that is if delta is 0 that is the moment in time that the E vector in the y direction also reaches a maximum because delta is 0. So this one E0y, occurs at the same moment as that one.

So what is the net E vector? That is the vectorial sum of the two. So E vector is this. So this is E total, which is the square root of E0x squared plus E0y squared. And if you see this coming to you, you will see the electric field going like this-- [CCHT CHHT CHHT CHHT]. Linearly polarized. No longer linearly polarized in the x direction, no longer in the y direction, but in this direction.

I can also make the phase angle between the two, 90 degrees. So I could make delta phi over 2. And now you get something very interesting. So this is now x and this is now y so that the radiation comes to you. x cross y is a new direction. And now I pick a moment in time that E0x is the maximum here.

So this is the vector, E0x. But now E0y, the E vector in the y direction is now 0 because they're 90 degrees out of phase and so, therefore, if this one reaches a maximum, this one is 0. A quarter period later this one becomes 0, but now you have here E0y. A quarter period later this one is back to 0 and now this one is here. And a quarter period later this one is again 0 and this one is here.

So now what you're going to see, there is never a moment that the E vector is 0, but the E vector rotates around in an ellipse. It goes like so, like so, like so, and so on. And so it rotates around like this. And we call that eliptically polarized radiation.

There's nothing very special about it. It's a perfect solution to Maxwell's equations. You have one component in the x direction, another in the y direction, and you offset them by 90 degrees. You can choose this angle any value you want to.

If E0x is the same as E0y, then it's a circle. And then we call it circularly polarized radiation. In this case it's going clockwise, but, of course, if you make delta minus pi over 2 it will go counterclockwise. Oh, I thought there was a question.

Now suppose you we're asked to calculate the associated B fields. That's a piece of cake. Because you just follow these simple rules. You take the component in the x direction and you calculate the associated traveling wave in B. And then you do for the y direction and you calculate the associated traveling B wave and you add them up. That gives you then the solution in B.

So this situation is nice and simple in 2D, but I think I owe you a more general description to widen your insight, and I want to go, at least in terms of the math, go in 3D. So we now have an xyz coordinate system. And now we want the option of having the E vector is not just in the xy plane or not in the xz plane, but in a random direction.

And so when we do that, we now get the E vector as a function of r and t. And r is a vector in space, which is x x roof plus y y roof plus z z roof. Any position vector in space that you prefer. And now I get this is E0 is now a vector, it's not in x or y or in z, it is in three dimensions, so this is 0. It has an x component, the y component, and the z component.

And now I can write down here the cosine of omega t minus k dot r which is now the most general way I can write this electromagnetic wave whereby k is kx in the x direction plus ky in the y direction plus kz in the z direction. And k, as you will see shortly, is the direction of propagation. And the magnitude of k is always 2pi divided by lambda. And that magnitude of k is the square root of kx squared plus ky squared plus kz squared.

And now I would like to give you some insight in the meaning of this k dot r. And for that I need a little bit more space. So now we have to make a tough decision what we're going to kill. I'm going to kill this.

To make you see what the geometric meaning is of k dot r, I will go first two dimensional and then you will immediately get the picture what it is in three dimensions. So I have here the x direction. And I have here the y direction. And here I have the k vector. And so this, kx and this is ky. It's a vector.

I'm going to draw a line perpendicular to the k vector. This line here is perpendicular to the k vector. This angle is 90 degrees. I will convince you that on that line k dot r is a constant. You can see that immediately if this is my vector r, which is the position vector in space given by your relation that you see there, then this angle here between k and r, I call that theta, then k dot r is a scalar is the magnitude of k times the magnitude of r times the cosine of theta.

And what is r cosine theta? That is this. This here is r cosine theta. And any vector r that ends on this red line will have the same value for r cosine theta. So, therefore, on the entire red line, which in this 2 dimensional plot is just a line, anywhere on this line the value for k dot r is a constant, is the same.

Now at a given moment in time at t equals 0, we have an electromagnetic wave traveling in the direction of k and this line happens to be the line where the E field, at this moment in time is a maximum and pointing in your direction, say. So it is a crest in E. It's a mountain coming out of the board in maximum value.

And here I draw another line perpendicular to r. So here k dot r, is 0. The dot product is 0. R is perpendicular to k. To this point. I will also assume that the E vector here is a maximum pointing in your direction and then there is one here. So k dot r here is there for 2pi and k dot r here is 4pi. Because this now represents a wave. This is the full wavelength of the wave where E is a maximum in your direction. E is a maximum in your direction. E is a maximum in your direction. In other words, this here is by definition lambda. And this whole thing moves out in space with speed c.

And so you see, the lines k dot r, perpendicular to k represent the maxima of the E vector at this moment in time, and all that starts to move out. If now you go to the third dimension, then k dot r is a constant, are now no longer lines but they are planes perpendicular to the vector k. And this whole plane perpendicular to the vector k, in that whole plane at the moment in time, the E vector is everywhere the same whether it is linearly polarized, whether it is circularly polarized, or whether it's eliptically polarized, it's irrelevant. It's everywhere the same.

And that whole plane then moves out with the speed of light in the direction of k. And so k dot r in three dimensions are all planes perpendicular to the vector k. So if you stand anywhere in space and you look in the direction where the radiation is coming from, then in any plane that is perpendicular to k, go to infinity there to there, to there, at any moment in time the E vector is the same and the B vector is the same.

And I said, whether it's linearly polarized radiation or circular or elliptical that is a different matter. It could be either one of those. And so this is the best way that you can think of the general form of a E vector going in three dimensions. These are then planes perpendicular to the direction of propagation.

In our first case, which was so very simple, because I wanted to warm you up slowly, k dot r became simply kz. So kx was 0. ky was 0. Of course, it was only going in the z direction, kz was the only one which was not 0 which was k. And so my dot product, k dot r collapses into a kz. And so the wavelength lambda is 2pi divided by this k.

But if, of course, if you have a three dimensional case then the situation is a little bit more complicated because then k is the square root as you see here of x kx squared plus ky squared plus kz squared. So our first case was to make it simple for you.

This is a right moment to stop. After the break we can look at some demonstrations also so we have to relax a little to digest all this. This is not easy. And so let's start handing out this mini quiz to make you feel good about yourself. I made it easy this time. Don't start yet. Can you start handing it out?

I put here on the blackboard the xy plane. This is an electromagnetic wave going in this direction perpendicular to the direction of k. So this is by definition, a wavelength. This is where E is a maximum in your direction and this is where it's a maximum in your direction and so this is the wavelength.

Now look at the intersection of this wave in the y-axis. This wave interacts here, and it intersects there. And so the distance from here to here is Ly which is way larger than lambda. And the same is true for the distance in the x direction, it's also larger than lambda. And in the z direction in general it would also be larger than lambda.

Now this wave moves with the speed of c. And when it has moved a distance lambda in the k direction this crest is here. How far has it moved in the y direction then? All the way from here to there. So its speed in the y direction is a larger than c. And so that speed which we call the phase, the phase velocity in the y direction is very simple. It's Ly divided by lambda times c. And that is larger than c.

It follows immediately from the geometry because this angle here is the same as this angle there that this is also k divided by ky times c. And that is not only larger than c, but it can be way larger than c. ky is 2 pi divided by L of y.

I refuse to call L of y lambda of y. I don't want to have to think in terms of the wavelength in this direction, the wavelength in this direction, and another wavelength in that direction. For me there is only one wavelength and that wavelength is 2pi divided by k. So I refuse to call this lambda of y.

But ky is 2 pi divided by the L of y. And you can do the same, of course, in the x direction and in the z direction. And you will find then that the phase velocity in the z direction equals k divided by kx times c and the phase direction in the z direction is k divided by kz times c. And what is the phase direction, phase velocity in the direction k that this k divided by k times c that is c. Of course, in the direction of k it propagates with the speed of light. But this panel moves way faster than the speed of light and it can actually get completely out of hand. It can be very, very large.

Suppose these waves travel in the x direction. That means these red lines will be vertical. That means L of y will go to infinity. It means that k of y goes to 0. It means that the phase velocity goes to infinity. This pattern here, the intersection here, will then-- swoosh-- go at a speed which is infinitely high. And that is no violation of Einstein's theory of special relativity because no energy will flow with that speed. And I can best convince you of that by showing that something similar can happen with water.

Suppose we have here a shoreline. And we have some water waves coming in like this. And maybe I should put them in red so that you begin to make the connection between the electromagnetic waves and water waves. So here is a wave rolling in nicely. This is, per definition, the wavelength lambda and all of that moves with velocity v. But the intersection of these two waves here at point A and here at point B, I call this distance L of x. I could have called it L of y. Just as I did here. It's the intersection with this axis that I gave the symbol L of y. I called it there L of x.

Now the difference in arrival time between wave 1 and wave 2, the difference in arrival time between wave 1 and wave 2 at point B is the period of the wave which is simply lambda divided by v. That is trivial. But that's not only the case for point B. That is also the case point A. This wave 1 reaches A before coarse wave 2 reaches point A.

And this is the time in between the arrival time of wave 1 and wave 2 which is a completely different question from what is the difference in arrival time of wave 1 alone at A and B. That difference in arrival time between one wave, between point A and B, depends on this angle theta. And when theta goes to 0, that difference in arrival time goes to 0. Both A and B will at the same moment in time see that wave.

So that means, that in that case, when theta is 0, this L of x becomes infinitely large, and so if you express that in terms of a phase velocity in this direction, the phase velocity then becomes infinitely high. So it is this pattern that moves with the velocity that can be way larger than c, but no one water will move with that velocity. It's very clear. There's no water going from here to there. And so this is not a violation of Einstein's theory of special relativity.

And so several very dedicated students wrote me email that after last lecture they could not sleep. And I didn't even feel guilty. And the reason why they couldn't sleep is that we had this wonderful demonstration whereby I have here an aluminum plate and had another aluminum plate and this was the z direction and the separation between the plates was a. And we had electromagnetic radiation going in that direction. And we concluded, I will not go over the reasoning again, that the phase velocity in the z direction was omega divided by k of z and that was a larger than c. That's why you guys couldn't sleep. And, in fact, I even demonstrated that if you make the radiation, the frequency, close to the cutoff that this phase velocity even goes to infinity.

Now you know that there is no problem. There is no energy flowing with that speed. It's no different from the water. In fact, the energy flow is with the group velocity which is the group velocity is d omega d kz and that was always less than c, as you perhaps remember. Not 0, my enthusiasm is getting carried away. Less than c.

And remember, at the cutoff frequency where no longer radiation would go through, this phase velocity went to infinity and the group velocity went to 0. So I have fulfilled a promise to those of you couldn't sleep to tell you that the meaning of phase velocity is larger than c, are natural, there's nothing wrong with it. You have it with water. You have it with electromagnetic radiation. You cannot transport any mass with that speed. You cannot transport any energy with that speed. So there's nothing obscene about it. Very straight

I now want to do a demonstration to show you that electromagnetic waves can be linearly polarized. Next lecture we will discuss how we generate electromagnetic waves. We do that by accelerating charges, in this case electrons, and I have here a transmitter.

This is an antenna, through which we are going to oscillate a current at a frequency of 80 megahertz. So F is 80 megahertz. So the wavelength lambda is about 3.75 meters. The wavelength is the speed of light divided by the frequency comes out to be about 3.75 meters.

And as we oscillate 80 million times per second back and forth electrons in this antenna, that means a current is going back and forth. Electromagnetic radiation is produced. Next lecture you will exactly see how much and why. And that electromagnetic radiation is polarized in this direction, linearly. That should not surprise you. If the antenna is like this and the electrons move like this, it should not surprise you.

But you will see next Tuesday why it is linearly polarized in this direction. And here I have a receiving antenna which is a copper wire which is cut in half and where the two copper rods connect there is a light bulb. And so any current that flows in here must go through the light bulb.

When I turn on this transmitter I want to show you that as long as I hold this receiving antenna like this that I will see these lights go on, but when I do this I won't. Because the electromagnetic radiation, the E field goes like this, it's going to slosh a current in here with a frequency 80 megahertz and the light will go.

But when I do it like this, the electric field is like this. So there is no current flow in this direction. And so this is a dramatic way of demonstrating that there is such a thing as linearly polarized radiation.

Now what we decided on the lights? We were going to TV, right? We made it as dark as we possibly could.

So here is the transmitter. And here is the receiver. And I'm holding it like this and you see that I'm receiving a signal. And so there's a current sloshing back and forth and the light is on. And now it's off. So the E field comes in like this, the light says, tough luck, I don't see a current going through it. But now it does. So this is sensitive to the polarization of the incoming signal.

And when I go a little bit to this side then as we just saw, electromagnetic radiation, the E field must be always perpendicular to the direction of propagation so the direction of propagation now is in this direction. So that's why I hold it like this for maximum effect because now the E field is like this whereas here the E field is like this. Now the E field is like this. And the light is not as bright. And you will see Tuesday why. But clearly when I do this, swoosh, nothing. Because of the linear polarization of the electromagnetic radiation.

If I come too close to this you may love that, then the light will go, [PPPPT]. That will burn the light. You want to see that? Who wants to see that? Oh God, you children. You children.

[APPLAUSE]

There is another way that I can show you this and that we do with radar with the same set up that we used last time except we did a different kind of experiment last time. This is 10 gigahertz. So it produces a 3 centimeter radar.

And this is the transmitter where the antenna is in this direction. It's not only very small antenna but it's only 3 centimeters wavelength. So the E field goes like this. And here is the receiver. And if we put the antenna of the receiver like this it will receive the signal. If we put the antenna like this, it says sorry. I can't receive it.

And this 10 gigahertz signal we modulate with 550 Hertz, with a triangular modulation so that you can hear it. And triangular audio signals are always not very pleasant because if you do a Fourier analysis of this you have many, many high harmonics. It's not just a beautiful 550 hertz sinusoidal. So you hear very sharp tone. But in any case, the period is such that the frequency is 550 hertz.

I will make you hear it and I will make you see it. You can see it there. That is the signal as it is sent by the transmitter. So that is the triangular shape. And now I will turn on the receiver and you will hear the sound. This is the loudspeaker. And you will see the signal of the receiver there.

[RECEIVER SOUND]

So the radiation comes in like this. And now I'm going to rotate this one 90 degrees, the receiver. And it's gone. Same phenomenon that I just showed you is the 80 megahertz. How can I get it back? One way I can get back is rotating this back, but I can do something better, something more convincing. Yeah? I can rotate this by 90 degrees. So now the transmitter--

[HIGH PITCHED NOISE]

--and now we have it back. So you see here two dramatic cases of linear polarization. Ah, my hand absorbs it. Nice feeling. Let's make sure we have some light back.

We will discuss extensively in 8.03, actually one of my favorite parts of 8.03, how you can turn unpolarized light into linearly polarized light. There are various ways that we can do that.

The cheapest, which is really a cop out, is to buy a linear polarizer. You have three of them in your little envelope. Don't take them out yet. This is one of them. They were invented by Edwin Land. And what they do is they change unpolarized light into 100% linearly polarized light at the expense of the light intensity. An ideal linear polarizer would reduce the light intensity by a factor of 2.

Let us first discuss what is unpolarized light. The light from the desk lamps and the light from the sun is unpolarized. Imagine that there is a desk lamp here and light comes straight to you. And here is a plane wave solution by the E vector oscillate. Linearly polarized, it's coming straight at you in this direction. And it's linearly polarized.

But a little later there is one that comes in like this, also linearly polarized. And then one like this, and then one like this, and then one like that. Chaos. All possible angles of linearly polarized light. And we call that unpolarized radiation.

Now what is this magic piece of plastic doing that Edwin Land invented? It's doing the following. There is one direction, not always indicated on the plastic what that is, but there is one direction. I'll just put it arbitrarily vertical, but I can put it in any direction. And that is the direction that the E vector will have when it emerges from this linear polarized.

So now let us suppose that linearly polarized light comes in, which is one of the many unpolarized radiation. And this has an amplitude E0 and it's oscillating like this, cosine omega t. And let this angle be theta.

And one way to explain the reduction in light is to project this E on to this preferred direction. Is the only direction in which the E vector will emerge. And so you see there is a reduction of the E vector. And the reduction is the cosine of theta.

Now comes the question, if there is a reduction of the cosine of theta in the E field, what is the reduction in light intensity? Well light intensity means energy. And energy is always proportional with the square of the amplitude. If that doesn't convince you, next Tuesday we will talk about the Poynting vector you remember from 8.02.

The Poynting vector which is responsible for energy transport which is proportional to E cross B. But B is proportional to E. So E cross B is always proportional to E squared. And so therefore, the light intensity that emerges from this plate is proportional to cosine squared theta. And that is the reduction of the light intensity and that is referred to as Malus's Law.

Now, if all angles are present, randomly angles of theta, then the average value of cosine squared theta becomes 1/2, So for an ideal polarizer, now they don't exist, but an ideal polarizer would turn unpolarized light into 100% polarized light all in this direction, if that's the direction, and then the intensity of the light would be twice as low as the incoming one.

In practice, however, you may not get 50% but you may get 40%. So there's always some additional absorption. And in problem set number seven, you will be able to answer some interesting questions and use the polarizers that you have in your envelope, in your optics kit.

Clearly if I have one linear polarizer like this. This is the preferred direction. And the other one is like this, for which we have a name in physics. We call them cross polarizers. Then, of course, independent of this absorption phenomena that I mentioned, no light can emerge because now the cosine of the angle is 0. So cross polarizers will turn unpolarized light into darkness. And that's something that I would like you to see now.

Well let's make it a little darker. I have here several sheets of that linear polarizer. And the first thing I want you to see is this absorption phenomenon that I just mentioned. Here is a linear polarizer. So the light that comes through here now is linearly polarized. The light that reflects off the screen is not linearly polarized. That's a different thing, because you go through a reflection process. But this light that comes up here is linearly polarized.

You and I have no way of telling. There are animals who can see polarized light, who can distinguish it from unpolarized light, and that even can see the direction of polarization. Bees can see the direction of polarization. And I have learned, also to see it under ideal conditions. If any one of you is interested, I can teach you, but it is not easy. You have to come to my office. There is a way that humans can actually recognize linearly polarized light and see the direction of polarization.

But apart from that, we have no way of knowing that this is linearly polarized light. And if I rotate this the direction of linear polarization will change but we have no way of knowing. In fact, the preferred direction is indicated on the sheet is like this.

Now I take a second one which is identical and I'm going to put it on top of it so that the two directions are aligned. If they were ideal then there would be no light reduction anymore. But there is a light reduction. Look at this middle portion where they overlap. There is still a further reduction in light intensity even though the direction of polarization has not changed. It is not the results of Malus's Law that there is a reduction, but there is an absorption phenomenon that I mentioned. So only here are two plates and here is one plate.

Now I have here a black stripe. That is on top of the overhead. Here now is one linear polarizer. And here comes the other. They are now aligned. And I'm going to rotate them. The angle of theta is increasing and increasing and increasing. And I'm approaching slowly 90 degrees. And there you are. And it is as dark as you can have it. So the two cross polarizers now, turn unpolarized light, first into linearly polarized light, that is sheet number one, and then sheet number two kills it altogether. Nothing can get through. Cosine of the angle is 0.

Now comes the great miracle. Which is something you can also do at home because you have three linear polarizes in you optics kit. Suppose I stick a third linear polarizer in between the two at a random angle. Will I see light coming through or will I not see light coming through?

AUDIENCE: [INAUDIBLE]

Very good. If I do it in front of number one darkness will remain darkness. If I do it above number two, darkness will remain darkness. But if I do it in between one and two, there's my line again, light comes through. And there is an immediate consequence, of course, of the fact that you have to apply now the cosine square theta twice, but you never will see it go down to 0.

Now I want you to get open your envelope. Don't put your fingers on all the beautiful pieces that are in there which we will have to use in the future. But open it and get out of it one linear polarizer. And you will recognize them. They are green plates and they have this shape. And you just get one. Don't drop them on the floor and don't make them too dirty.

Can you see me? Who cannot see me? Good. Now you can rotate your polarizes until you are purple in your face and you won't see much difference. You will see Walter Lewin no matter what. Use your polarizers and rotate them around. And look at my face. Not going to change very much.

Now I'm going to hold in front of my face this linear polarizer. So now you're looking at Walter Lewin in polarized light. And that now gives you the option to make me come and go.

You can rotate in such a direction that you say, there he is. But if you rotate it at 90 degrees you say, thank goodness he's gone. Now, keep in mind that if you rotate your linear polarizers such that you can see me, that the reverse is also true. I can then look through your linear polarizer and I can see your eye. And if somehow, in evil way you prefer 90 degrees so that you don't see me, then you have a black eye. And who wants a black eye? See you next Tuesday.