8.03SC Physics III: Vibrations and Waves, Fall 2012
Transcript – Lecture 14: Generating EM Waves, Energy, Scattering

WALTER LEWIN: There is energy in an electric field, and there's energy in a magnetic field. You remember that from 8.02. And the energy density-- mind the units, in terms of joules per cubic meter-- for the electric field, we write for that a U equals 1/2 epsilon 0 E squared.

There's no such thing as a free lunch. You have to do work to create an electric field. You have to assemble charges and bring them together. That means work. That creates an electric field.

And the same is true for a magnetic field. And you have a solenoid, and you create the magnetic field inside the solenoid, that costs energy. And the energy density-- mind the word density, it is per cubic meter-- for a magnetic field is B squared divided by 2 mu 0.

Now traveling EM wave, traveling electromagnetic wave, so this is now a traveling wave. We know that the magnitude of B is the magnitude of E, at any moment in time, divided by c.

So I can write this as E squared divided by 2 mu 0 times c squared. But c squared is 1 over epsilon 0, mu 0. So this is also 1/2 epsilon 0 E squared.

And what you see now is so wonderful, so beautifully symmetric in electromagnetic waves. One is the same as the other. One cannot exist without the other. And look at the energy density in the electric field of a traveling wave is exactly the same as the energy density in the magnetic field.

And so the total energy density is the sum of the two, is epsilon 0 times E squared. But you can also write for that, epsilon 0 times E times Bc, if you prefer that.

This, of course, is only true in vacuum, when the speed of propagation is c. Now, there is a traveling wave, and this traveling wave moves. And so it carries energy with it. And now the question is, how much energy flows through an area, which is, say, one square meter area, perpendicular to the direction of propagation?

So suppose I have a box here. And this side is 1 square meter. And I want to know how much radiation comes out of there in one second. So the radiation is flowing in this direction with speed c.

So in one second-- this box is quite a large box, 3 times 10 to the 8 meters. And all that energy, which is in their, will flow through this one square meter, in the time of one second.

And so the dimensions that we're talking about now is joules per second per square meter, which is also watts per square meter. And that now is, of course, the total energy density times c, the speed of light.

For which you can write epsilon 0 times E times B times c squared, if you like that. But you also write for that EB divided by mu 0, because c squared is 1 over epsilon 0 mu.

And this should remind you of something that is in your far distant past, which is what we earlier have called, in 8.02, the Poynting vector.

And the Poynting vector S was E cross B divided by mu 0. And the units were watts per square meter, which is exactly what this is. And the reason why the cross disappears here, is that with electromagnetic traveling waves, E is always perpendicular to B. So that takes care of the cross.

Now both E and B are time variable. And so the Poynting vector will, obviously, also be time variable. E is going to be proportional-- you can write down E as some E0 times cosine omega t, and B is some B0 times cosine omega t.

So in the Poynting vector, you get the cosine square of omega t. But since we never interested in the Poynting vector on a time scale smaller than the period of oscillations-- we want to know the average over many oscillations-- what matters there is the average value of cosine squared omega t. One from the E and the other one from the B. And that is one half.

And so we can conclude then, that the average value of the Poynting vector, time averaged, is 1/2 that value, 1/2 EB divided by mu 0. For which you can also write, 1/2-- if you want to kill completely.

Oh, by the way, this is 1/2 E0 B0. It's important that you have to E0, because the cosine square average is 1/2. So here are the amplitudes. So you can also write for that 1/2 E0 squared. And so you write down for BE divided by c. And then you get, downstairs, mu 0 divided by c.

And the reason why I write it in this form is that it tells you that, if you know what the strength of the E field is, that alone tells you then what the Poynting vector is. Because B is coupled to E, through Maxwell's equations.

That is where B is E over C. And so all that matters then, if you want to calculate what the Poynting vector is, is E0. Of course B 0 alone would also be fine.

So let's take an example. Suppose we have an electromagnetic wave, whereby E0 were 100 volts per meter, then I can calculate, now-- and it isn't traveling electromagnetic wave-- what the average value of the Poynting vector is. So that would become 1/2 times 100 squared divided by mu 0 times c.

And if you do your homework on that, you will find that it is 13 watts per square meter. Now if you exposed yourself to 13 watts per square meter, visible light and infrared light, you take all your clothes off, and you expose yourself to that, your body will absorb that.

It will not go through your body. X-rays may go through your body, gamma rays, certainly. But infrared radiation and visible light get absorbed by your body. And so the question now is, will that harm you? And then the answer is, no, you would hardly notice 13 watts per square meter.

Your body, itself, radiates about 100 joules per second, because of the body heat that you have. And so the 13 watts per square meter that you absorb-- let's say your cross sectional area is about one square meter, just to round it off. So that means 13 joules per second would be absorbed by your body. And that will not affect you in any way.

But let's now take a situation that we have E0 equals 10 times higher, which is 1,000 volts per meter. Now the Poynting vector goes up by the square of E. So now you're going to get that the mean for Poynting vector is 1.3 kilowatts per square meter. And that will fry you.

If you walk naked in a field of infrared or optical, whereby the absorption, 1.3 kilowatts per square meter, that is very dangerous. You'll get skin cancer and worse.

Now, why do I mention that? And why do I focus on that 1.3 kilowatts per square meter? Because that is exactly what the sun does.

Here is the sun. And the sun is a rather powerful light bulb. About 3.9 times 10 to the 26 watts. And this is where you are on Earth. And the distance to Earth is 150 million kilometers.

So I can calculate now how many joules per second go through one square meter here. And this one square meter is held perpendicular to the direction to the sun.

So that value for S is of course the radiation that leaves here. That is my 3.9 times 10 to the 26. And now I have to divide it by the entire surface area of this sphere, which goes in all directions, that has this radius.

And that surface area is 4 pi times that radius square. So that is 150 times 10 to the 9th-- because I have to go to MKS units-- squared. And that is now the number of watts per square meter that we receive at Earth from the sun.

And that number is a very famous number. That is 1.4 kilowatts per square meter. And that's why I picked the 1.3 to show you that, if you walk around on the beach, and you don't take care of yourself, if you do that too long, that is very dangerous, because your body absorbs that.

This number is called the solar constant. It has major implications, of course, for people who want to harvest solar energy. The maximum that you can ever harvest is, for every square meter that you dedicate to solar energy, you only get 1,400 joules per second. You can never get more.

The electric power capacity of the United States is 700,000 megawatts, for which you need 700 power plants. A full size power plant is about 1,000 megawatt power plant. You need 700 of these power plants to have the capacity for the United States.

We are energy hungry. We consume more than 1/4 of all the energy in the world. So if you want to replace this by solar energy, that's a major problem. Because you can only get 1.4 kilojoules per second for every square meter.

You can calculate how many hundreds of square miles in the desert you would have to commit, with solar cells, which are extremely expensive, in order to get electricity. And the efficiency, of course, is never 100%.

And also, when the sun is low in the horizon, then you don't have this one square meter perpendicular to the direction of the sun. So all of that has to be taken into account. Solar energy is not very important in our lives, unfortunately.

And this is the number that is the ultimate limit. Now comes the question, is there such a thing as a electric field of 1,000 volts per meter in the solar radiation? Could we actually measure the electric field just from the radiation of the sun? And the answer is, no.

And the reason is that the radiation is not really in the form or our idealized plane waves. But more important than anything else is that there is no such thing as just one wave from the sun that has the amplitude of 1,000 volts per meter. In fact, the radiation reaches us in small packages, broken up in pieces, so to speak.

However, since the energy flow is 1.4 kilowatts per square meter, it is perfectly OK with me that you refer to this number as the Poynting vector. I have no problem with that.

But it is a little bit naive to associate with that an electric field that can be measured, that has a amplitude then of 1,000 volts per meter.

And so this now is the right time to take a close look at how electromagnetic waves are produced. In a nut shell, it comes down to this. You can create electromagnetic waves if you accelerate charges.

Charges that are stationary or moving at constant velocity, are surrounded by a radial field, pointing away, or pointing inwards, depending upon whether the charge is positive or negative.

And there's no kink anywhere in these fields. So whether it has a constant velocity, or whether it stands still, they are radially electric field lines.

The moment, however, that you accelerate it, as you will see today, you introduce a kink in those field lines. And that kink is responsible for that electromagnetic radiation. It manifests itself as electromagnetic radiation.

I will follow a classic derivation that is verbatim given that way in Bekefi and Barrett. I, therefore, advise you strongly, for the next 13 minutes, not to take any notes, but try to follow my arguments. That will help you way more than that you try to also take notes. Because it's really verbatim from Bekefi and Barrett.

It is a classical derivation. With many simplifying assumptions, but it gives a very nice result, which has great practical applications.

Suppose I have here a charge, q, which is located at position, a. So we have a charge, q. It is at a, at O, and it is at rest. And I'm going to accelerate that in this direction.

So I'm going to accelerate it with an acceleration, which is in this direction. I will not put the vector in there now. I will do that later. Otherwise the figure becomes too complicated. And I do that for delta t seconds. Only very brief.

And then it ends up at location O prime, which is here. So now it has a velocity, in this direction. And that velocity, u, in this direction, is now a times delta t, that's 8.01.

And so it's cruising now with constant velocity. And we just let it cruise all the way. It's now again a charge with constant velocity. And I look at it, where it is t seconds later. And so at time, t, we'll find it in O double prime.

And it's still cruising. And we just let it cruise. We're not going to interfere with it anymore. And here is then O double prime.

The whole exercise, from O to O double prime, took so many seconds. That means there is a sphere around point O. And that sphere has a radius, which is c times t plus delta t.

Outside that sphere, the world has no knowledge that this charge was accelerated. Because that message has to travel with the speed of light. So I'm going to draw a circle-- but in reality it is a sphere, in all directions-- which has a radius, c, t plus delta t. Delta t, by the way, is way much smaller than t.

And outside that sphere, there is no knowledge, the world doesn't have any clue about the fact that this object was being accelerated.

So I will mark again, to make sure that you can make a connection. So this one has its center at O and the radius is c times t plus delta t. And if you ask me, what is the electric field, right there, that is radially pointing outwards, if q is positive?

So it would be radially pointing outward. This world does not know yet that there was any acceleration. And if you ask me, what is the electric field here? The field line is like so.

And so the electric field, it is a positive charge, is pointing radially outwards and has no knowledge that there was any acceleration. And so I call this is my position vector, r. But you can take vector r in any direction that you want.

Now let's look at the world that does know that there was a change. When the object was at O prime, the acceleration stopped, and so it started cruising with a constant velocity. And so from that moment on, the field lines are, again, nicely, radially outwards.

And so by the time that it reaches point O double prime, I can point, I can draw these field lines, radially outwards. And I think of this original field line as a stick that was connected to the charge in that direction. I could have picked another direction.

And so that field line is now here, outwards. And the world that knows about this is the world which has a sphere around O prime, with a radius c times t.

And so I'm going to draw another circle, which in reality is a sphere, about point O prime. And this sphere here has now a radius ct. And so this, to remind you what it is, it has the origin at O prime. And the radius is ct.

And so everything inside that sphere recognizes that the field is radially outward, from all point O double prime. And everything outside here, still thinks that the electric field is like this.

But this field line was this one, that was the stick that I attached to it. And so there must be a connection between here and there. And that connection is only in that very thin shell, which has a thickness c delta t.

Let me first make a drawing of a triangle. That's going to be important. Which is this triangle. This length here has a length, u perpendicular times t.

You can easily see why that is the case. Because this distance here is ut. Now, you may say, well, it wasn't going with velocity u, here. I grant you that.

But keep in mind, in whatever follows, that u is way, way smaller than c. Delta t is way, way smaller than the t. And so r, which is c times t, is way, way larger than ut. So when you see a little distortion in this picture, I cannot of course, make delta t way, way smaller than t. Then you wouldn't see any decent picture.

Let me first go to u perpendicular t. So there is here a velocity vector, u. And the component perpendicular to r is a component in this direction. It cruised for t seconds. So this the length is u perpendicular t, to a very good approximation. Forgetting about this little teeny weeny, little distance.

So we know what this is. So u perpendicular is the component of the vector, u, perpendicular to my position vector, r. And this little section here must, of course, have thickness c delta t. That is the difference, in this radius and this radius, c delta t. So look at this green triangle.

Now comes the key thing. This electric field line must connect with that electric field line. It was one and the same. It's been broken now. And the connection must go through this thin layer. I will draw here the connection.

And then here, you have to round it off a little. It cannot be sharp. And this runs off a little. So this point here represents the beginning of the acceleration. And this represents the end of the acceleration. And so then the field line goes [WOOSH] [WOOSH] [WOOSH].

And now comes my task. And that is to calculate the electric field inside that shell. Because that and only that is responsible for the electromagnetic radiation. And so the field line goes like this. So there must be a component of the electric field, which is in this direction. And there must be a component, which is in this direction.

I just decompose it. And I call this one E parallel. My notation, parallel, always means parallel to r. And my notation, perpendicular, always means perpendicular to r. And so this then would be called E perpendicular.

And this line here is a triangle. This triangle, that you see here, is congruent with this green one. That's why I gave you the dimension of the green one. And so now the question is, what is this component, E perpendicular?

You could just feel in your stomach that that is the one that is responsible, that is the electric field in the travelling wave that moves outwards. Because this whole shell moves outwards with a speed, c.

And we notice, this is perpendicular to the direction that I have chosen of propagation. You're watching here. And this shell comes over you. And it's always the E field, that is perpendicular to your line of sight, that is the E field in a traveling wave.

So our task is now to calculate the E vector perpendicular here. Well, if you look at the fact that this triangle is congruent with this one, then you see that E perpendicular divided by E parallel must be u perpendicular t divided by c delta t. But u perpendicular t-- we know that u is a t.

So u perpendicular is a perpendicular times t. a perpendicular is now the component of the acceleration perpendicular to r. I don't want to put it in here, because it becomes too cluttered. I will make a new drawing, shortly. I'll put the a in there.

Acceleration is in this direction. And so a perpendicular is like this. And so I can replace u perpendicular t, I can replace it by a perpendicular. Sorry, this is a perpendicular delta t.

You guys should have screamed, because the acceleration only lasts for delta t seconds. Yea? Only delta t seconds. So now I get a perpendicular times delta t times t divided by c delta t. And so that is a perpendicular times t divided by c.

If now I can calculate what E parallel is, I'm done. Because then I know what E perpendicular is. I put E parallel here, and I am in business. Before I do that, I want to eliminate t.

And I'm going to write for this t, r divided by c. So it is also a perpendicular times r divided by c squared. And so I can write down now that E perpendicular is this times E parallel.

How do we find E parallel? 8.02, Gauss's law. I make a pill box. And the pill box is going to be like this. I'll make a drawing of it. And this surface is here, in the world that doesn't know yet what happened.

And this surface here is in the world which is in turmoil, which is in the transition. So I'm going to make that pill box for you.

And so I draw here, again. This is a line that goes somewhere to this point here. It's not this one. It's somewhere there. And here is my pill box. That's my pill box.

And so in that pill box, I have in the outside world, the outside world here, I have that E field. So I will put that in as an E. It's in the world that doesn't know yet what happened. That's radially going through point O.

Inside the box, I have this E parallel, coming in like this. And then going straight through these sides of the box is this one, E perpendicular.

So this comes in, E perpendicular and that E perpendicular goes out. Now, since this is in vacuum, there is no charge density inside this box, so the divergence of E must be 0.

And that means that the E vector here must be exactly the same as the E vector here. Because the contribution to these two is 0, so this must also be 0. There's no charge inside the box.

But I do know what this E field is. That is 8.02. If someone tells you that I have a charge here, sitting there, what is the electric field at a distance, r? That is Coulomb's law. That's not Coulomb's law. It's a Gauss's law.

But in that case, you will find very easy that this E vector is q divided by 4 pi epsilon 0 r squared. It falls off as 1 over r squared. You people should remember that from 8.02.

And so now we have accomplished-- the 13 minutes are almost up. E perpendicular now is therefore a perpendicular times r divided by c squared times q divided by 4 pi epsilon 0 times r squared. And you lose one r.

And so this is the classic derivation. Already were known in the late 19th century. Which is now the electric vector. The strength of the vector, which clearly is responsible for the electromagnetic traveling wave, because it's perpendicular to my direction, r, that I have chosen.

It is inversely proportional to r. I will come back to that. That is a natural consequence of the conservation of energy. And this field is very different from static electric fields, which fall off as one over r squared. This is a traveling wave, which has an E field that falls off as 1 over r, not as 1 over r squared.

Now, before I write down the final form, we also have to take into account the fact that there is a time delay. So I am an observer. At time t, the acceleration took place at an earlier time, because the radiation has to reach me. And so t prime is then t minus r divided by c. So there is always a time delay.

So that is a reason why we write down this equation differently. So I write it now then in all its glory, the way that you would find it in most books. So this is now the E vector.

Due to an accelerated charge, you may choose the direction, r. And it is a function of time. And that now becomes minus. I will get back to the minus sign. Then I get a perpendicular at time t prime, whereas this at time t. And that is the connection between the two.

Proportional with the charge, divided by c squared 4 pi epsilon 0 and just one r. You have the privilege to convert or not convert, to write down for me or for yourself, what the associated B field is. It's very easy of course. The strength of the B field is c times smaller. And you must make sure that E cross B is in the direction of propagation. So I leave you with that.

And then we get that the Poynting vector, as a function of rt, is then E cross B divided by mu 0. And there you have the entire set that tells me what the electric field is due to an acceleration. What the magnetic field is due to an acceleration. And now, what the Poynting vector is, how many joules per second per square meter, flow out.

a perpendicular is different for different directions. If I call this angle theta, then a perpendicular is 0, if I am looking down here. Because then a is towards me, there is no perpendicular component.

If I am in this direction, then a perpendicular is the same as a. So you see that the strength of the electric field is a strong function of that angle theta.

So what you're going to see is there are spherical waves going out from this accelerated charge. But the electric field strength, in that spherical wave going on in all directions, is the strongest in this plane here, perpendicular to a, and is 0 in that direction.

And so I would like to make a somewhat different, somewhat simplified picture for you, to stress the connection, and to also address the minus sign. So if this is the direction of the acceleration, a. So I put the a vector in here. Then this is a perpendicular. Let's first choose the direction of r. This is r.

You can choose it any way you want to. So then this would be a perpendicular. And if the charge is positive, which I have chosen here. Because look, the field lines go away. Then the E vector perpendicular is in this direction, is like so.

And that is the reason for that minus sign. Namely, if q is positive, then E is in the opposite direction of a perpendicular. If q is negative, then it is in the same direction.

And then you get that this angle theta is very important. The E vector is proportional to sine of theta, because a perpendicular is, of course, a times the sine of theta. And so the E vector and B vector have sines of theta in them.

And the Poynting vector have sine squared theta in them. Because this has a sine of theta, and this has a sine of theta. So the Poynting vector, S, is proportional to the sine squared of theta.

And so the spherical waves going in all directions, the Poynting vector is very different for the different directions. The amount of energy per second that flows out, per square meter, in the plane perpendicular to the acceleration is very high.

And it is 0 in this direction. If you looked here, you would see nothing. If you looked here, you would see a maximum. And anything in between, you would see less than the maximum, but more than 0.

So this is really nothing like a plane wave, infinitely at all. If anything, it's more spherical, then that it is a plane wave. However, if you're very far away from the origin, you can probably, locally, approximate it by a plane wave solution.

So now I want to summarize for you, basically, what I want you to remember, and which is all that I remember. And that is how do you find the E vector, if you know the acceleration, and you know where you are in space?

So we accelerate q. And we do that in the direction a. And you are at some position in space, r. Now you observe an electromagnetic wave, but you observe it with a time delay. Which I'll not further expand on.

But now comes the key thing. That the E field is in the plane of r and a. And you can confirm that with this picture there. So whatever r you choose, you may choose any r you want to. Also outside the blackboard of course, this is just for simplicity.

The E vector must be in that plane. But the E vector must be perpendicular to r. Maxwell's equations demand that E vector is perpendicular to the direction of propagation for a traveling wave. The magnitude of the E vector is proportional to the magnitude of the a perpendicular.

And the magnitude of E is proportional to 1 over r. And is q is positive, then E is in the direction minus a perpendicular. If q is negative, then it is in the direction of a perpendicular.

The 1 over r is an obvious thing, as a consequence of the conservation of energy, because the Poynting vector is proportional to E squared. Because it's proportional E cross B. So the Poynting vector is proportional E squared.

So imagine now that you have a sphere with radius, r. And you want to know all the energy that flows through that sphere. So you have to integrate over the entire sphere.

But if you double the radius, the surface area of that sphere goes up by factor of 4. So the only way that energy can be conserved, at the same energy flows through a small sphere then flows through a large sphere, is that the electric field goes down by factor of 2. So that the Poynting vector goes down by factor of 4.

So the surface area increases by factor of 4, but the Poynting vector goes down by a factor of 4, because E goes down by a factor of 2.

We have here an 80 megahertz transmitter. We've seen that before, wavelength 3.75 meters. And I demonstrated this before, to show you that the radiation is linearly polarized. And I've demonstrated it by having this receiving antenna and by rotating it like this, when the light went off, and rotating it like this, when the light goes on.

Today, that's not my objective. Today, I'm interested in the sine squared theta term, for which I need a little bit of assistance from someone.

So I want someone to hold this here. And you will see that the light will be on. And then I will rotate that, so that the angle of theta changes. If I have it here, I have maximum. Because, you see, the charge is accelerated in this direction.

So this angle is theta 90 degrees. That's optimum. That is theta 0. It's 90 degrees. That's optimum. But the moment that you rotate that 90 degrees, or that you rotate this 90 degrees, then it's 0. And that's exactly what I want to demonstrate.

So I need assistance. Can you help me? Make sure you hold it here. If you don't, you get electrocuted. So you have a choice. Hold it above your head.

Now you see the light is on. So now, this is the direction of acceleration. And I'm going to point the direction of acceleration towards him. If the acceleration is like this, there's no radiation going in his direction, and that light will go out.

Move very slowly. Move carefully. It's already dimming. It's already dimming. And now it's [INAUDIBLE]. And now, I'll get it back. There it goes on. And there it goes off.

If you come a little closer, then the light is a little brighter. Hold it a little further down, a little down. That's fine. I'll do it once more. So I change the angle theta, and now it goes out. And now it's in.

Great. You're lucky you survived this one. Some students don't. So this was very qualitatively, to show you the effect of the sine squared theta relationship.

Now, in order to calculate the total power that is produced by-- the total power in the electromagnetic wave that is produced by the oscillating charge, you will have to integrate this Poynting vector over one complete sphere.

Which is not a total trivial integral, because the Poynting vector changes with sine square theta. So I will leave you with that exercise. But I will give you the result, which is not so surprising. At least, it is not surprising that the quantities that you will see.

Can most of you see this part of the blackboard? Still that I put it better there. Oh, I can erase some here. Let me erase some here.

So this is now the integration of the Poynting vector over a complete sphere that I put around the charge. And the result then is that the power, which is no longer watts per square meter per second, which is simply now in watts. How many energy flows per second through a sphere.

And that now becomes q squared. That's no surprise, because q is in a, in the E vector. And q is in the B vector. so you get a q squared. You got an a squared here. No surprise, because the a is in the B. And the a is in the E.

And then you get this divided by 6 pi epsilon 0. And I think it is c to the power 2.

Did you see my hesitation? I finally arrived at c squared. But it really is c to the power 3. So the downstairs is not c squared, but it is c to the power 3. Sorry for that.

And so this is the famous Larmor result. It takes energy to accelerate a single charge in vacuum. And I'm not talking here about kinetic energy, in terms of 1/2 mv squared. But I'm talking here about the creation of electromagnetic fields due to the motion of the charge.

Now comes the question, how do we accelerate charges? And I thought about that for a few days, how I was going to tell you that. And the answer that I have to give you is a little bit embarrassing.

And that is, we accelerate charges by exposing them to electromagnetic radiation. And then you say, well, isn't this Catch-22. Because you have to accelerate them to create the electromagnetic radiation.

How do you accelerate them? Well, you have to expose them to electromagnetic radiation. For whatever that's worth, of course, there is electromagnetic radiation that comes out of all kinds of sources.

And so I will show you now that, if we take an electromagnetic wave, we'll take a linearly polarized traveling wave, and we will expose an electron to that radiation. And then we will see what the electron will do with that incoming, traveling wave, a plane wave.

So here is an electron. It has charge, q, which is now negative. It has mass, m. And from the left, comes linearly polarized radiation. That's what I have chosen. E0 cosine omega t. So it just comes in. It's a plane wave. Infinite in size, it comes in like that. And this electron begins to shake.

And the force that the electron will experience, F is q times E0 cosine omega t. That is the definition of electric field, as it is connected to force. I do this for bound electrons. So these electrons are bound in an atom or in a molecule. So they're bound electrons.

So when you try to move them away from equilibrium, they don't like that. They experience a restoring force, because of the Coulomb interaction in the atom. And they want to go back. So there is a restoring force that acts, to some degree, like a spring.

And so there is also then a resonance frequency associated with that restoring force. And so that resonance frequency, I will simply call, omega 0 squared, which is then K over m. And K is then, the equivalent of the spring constant, which now acts in the atom.

I will ignore, purposely, for the reason of simplicity, any form of damping. And so I'm going to write down now Newton's Second Law. And Newton's Second Law becomes now x double dot plus omega 0 squared times x equals this driving force. Which is q times E 0 divided by m times the cosine omega t.

And the reason why there is an m here, because I have already removed the m here. And I have removed the m there. And so this is an equation that we have seen before, when we were dealing with force oscillations.

And it has a very simple solution that x is A times the cosine of omega t. This is omega of the driver. And so the object will follow, in steady state solution. And this A is the amplitude.

And the amplitude I even remember, if you ask me three months from now, I will not remember anymore. This is always upstairs, remember. And then downstairs, you get all that stuff, omega 0 squared minus omega squared plus omega squared gamma squared, but gamma is 0.

So you have no damping. And so you get downstairs here, omega 0 squared minus omega squared. So that is the amplitude of the oscillating electron.

But now we want to know how much radiation it's going to produce. So now we have to calculate what x double dot is, because x double dot is the acceleration of the electron. And then we want to see how much radiation comes out.

And so the x double dot becomes minus omega squared times x. Just taking the second derivative. You've seen that before. And so that becomes then, minus q E0 divided by m (omega 0 squared minus omega squared). And then I got upstairs that omega squared.

And that is the acceleration of this electron. But now, I know how much power goes into that field, because the power that goes into that field-- where do I have? Did I erase my-- the Larmor? I wanted it for that reason.

So the total power that is radiated is proportional to A squared. I hope you remember that. I erased the wrong part of the blackboard. And so the A squared is what I really want to know.

And so the power that is radiated by that shaking electron is proportional-- forget all the constants-- omega to the 4th divided by omega 0 squared minus omega squared, squared.

Forget all the rest. All the rest is constant. And now, if omega is way below resonance, this is proportional to omega to the 4th. And that's a very famous result. Which is known in the literature as Rayleigh scattering.

And the reason why we call it scattering, Rayleigh scattering, that you must understand that the radiation that comes in has this frequency. But the radiation that comes out has exactly the same frequency. Look. That is the oscillation of the electron.

So there's no change in the frequency. Radiation comes in a certain color, red light. And then there is an electron which starts to shake, gets very nervous, because of the E field. And it radiates, again, red light. The color does not change. But it scatters it. It goes off in different directions. And that's why we call this Rayleigh scattering. And if you take oxygen and nitrogen, in our atmosphere, then the resonance frequency is way above the frequency of visible light.

The resonant frequency for oxygen and nitrogen is in the far UV. And so we meet that condition that a visible light strikes oxygen and nitrogen molecules, and for that matter, many other molecules, this relationship holds.

And what that tells you then is that blue light has a way higher probability to be scattered by moving electrons then red light, because the wavelength or the frequency, I should say, of blue light is substantially larger than the frequency of red light.

And so let me try to stay on the center board then and show you what the difference is in frequency. So omega blue divided by omega red is approximately 1.5. Taking either side of the spectrum, about 6,500 angstroms is red and 4,500 angstroms is blue.

And that means that 1.5 to the power of 4 is about 5. And so the blue light has a 5 times higher probability to be scattered then the red light. And this holds, as long as the particles, off which the radiation scatters, are smaller than the wavelength of light, typically a few tenths of a micron.

So then we have Rayleigh scattering, then we have the dependence on wavelengths to the power of 4. If the particles grow in size, this dependence weakens. And then the difference between the red and the blue becomes less.

And by the time you reach 5 microns, particles that have a size of 5 microns and 10 microns, then the probability is the same for all colors. And that is what I would like to demonstrate to you, very shortly.

There is an extra bonus. And the extra bonus is-- this is not so intuitive, but I will convince you of that-- that radiation that was scattered over 90 degrees becomes linearly polarized, 100 percent. Unpolarized radiation comes in, here are scatterers.

Light goes off in all directions. But what goes off at 90 degrees is 100% linearly polarized. I will first convince you of that, and then I will do a demonstration to-- that shows that, indeed, it is linearly polarized. That's why I wanted you to bring your polarizers.

You see radiation coming from this side. And here is this electron that starts to shake. And if you have very small particles, 1/10 and 2/10 of a micron, there are electrons in those particles which begin to shake.

And so this electron starts to shake up and down. And the electric field must be in the plane of r and a. Look very carefully. That this electric field is indeed in the plane or r and a. And this electric field is in the plane of r and a. And this electric field is in the plane or r and a.

And so this is the direction of oscillation of the E field. In the direction of theta 0, there is no radiation going, because the acceleration is like this. And if you go off at a different angle, then the electric vector oscillates perpendicular to its direction of propagation.

Why now is radiation 100 percent polarized if it scatters over 90 degrees? Well, let's have a beam of unpolarized radiation that comes to you. Linearly polarized light comes to you, [WOOSH] [WOOSH] [WOOSH]. There it is.

There's the next wave, and the next wave, and the next wave, and the next wave, and the next wave. We call that unpolarized light. So one comes in like this. The next one comes in like this. One comes in like this. We call that unpolarized light.

You are looking here, in the blackboard, at this radiation. And there are scatterers here. There's dust here, very fine dust. And so some light comes to you. The angle is 90 degrees. Because it came like this, and now it does this.

The whole board is 90 degrees. Someone who is here, also 90 degrees. What is the direction of the E vector when this light reaches my eye? It's in the plane of a and r, that means it must be in the blackboard.

Remember, the E vector is in the plane of acceleration. This acceleration is like this. That's the plane of the backboard. This acceleration is like this. And r is also in the blackboard.

So the E factor must be in the plane of the blackboard. But it must also be perpendicular to r. So it must also light like this. And here it must also light like this. And if you look here, it must light like this.

So whenever you look at scattered light under 90 degrees, if it is Rayleigh scattering, you will see that the radiation is 100% linearly polarized. An amazing thing. And that is my goal, to demonstrate this to you now.

It was a very dramatic experiment in which I put my health on the line. I'm going to smoke a cigarette. And I advise you not too. And in cigarette smoke are extremely fine dust particles, a few tenths of a micron in size, ideal for Rayleigh scattering.

And then we have light coming from below. And that light is unpolarized light, light bulbs there. Here we will have dust. And this very fine dust scatters the light in your direction.

What will be the color of that light? Blue. Because the white light contains red, green, yellow, blue, everything. But the blue light has a higher probability to be scattered. So my smoke, if the particles are small enough, will look blue.

If the particles are not small enough, then it won't be blue. It will be blue, believe me. But not only that. The radiation comes up like this, and every thing, every person in the audience is in ideal position, you're all at 90 degrees relative to the scattering, because that's the way I'm made the arrangements. Light comes up like this. And then it goes like this.

So all of you are very close to 90 degrees. So get your polarizers out. And you should see that the E vector is in this direction. For you there, like this. For you there, like this. And for you, like this. So we're going to kill two birds with one stone. It will be blue. And it will be linearly polarized.

So you ready for this? Now, look at the smoke. Do you agree that it's bluish? Who agrees, just say, yeah.

AUDIENCE: Yeah.

WALTER LEWIN: Who doesn't agree, just say no. Thank you. Now, use your polarizers. Can you see that when you rotate you polarizers that it's polarized. It's polarized in this direction.

Who can see that, say yes.

AUDIENCE: Yes.

WALTER LEWIN: Who cannot see it, say no. Thank you. Now comes the double header. If I keep that rotten smoke in my lungs for a while, the vapor in my lungs will precipitate onto these small particles, which initially are smaller than a few tenths of a micron, so you get Rayleigh scattering.

But now, when I puff out the smoke, you will have teeny-weeny, little water drops, because of the water vapor in my lungs. And now the particles are too large for Rayleigh scattering, and so the scattered light will now be white.

In other words, there's no preference anymore for blue over red. And so, what I will do is, I will hold the smoke in my lungs for while. Just before I puff it out, I will show you, again, this smoke, so that you can have a reference of the color, so that you really believe this is blue.

And then when I puff it out, you will see there is a distinct difference in color. It becomes white. And so you've seen three things then.

Number one, that light that scatters on very small particles, a few tenths of a micron, prefers the blue. Blue has a higher probability. 90 degree scattering, 100% linearly polarized. But if the particles grow beyond a certain size, there is no longer any preference for the blue.

Big difference. Who saw the difference? Say yeah, otherwise I'll do it again.

AUDIENCE: Yeah.

WALTER LEWIN: Who didn't see the difference in color? Ah, thank goodness.

OK, we'll have a break, so I can recover, and you can recover. Five minutes. Four minutes.

So the sky is blue, because light scatters on very fine dust and even on density fluctuations in the atmosphere, due to the thermal motion of the molecules.

And the clouds are not blue, because the clouds have very small water drops, like I have in my lungs. And so that's why the clouds are white.

Let's take a look at you're standing on Earth, and let the sun be in this direction, midday. And here is the atmosphere, thickness of about-- depends on how you measure it, how you define it-- 60, 70, 80 kilometers.

And you look in this direction, and there's white light coming in. But the probability that blue is scattered in your direction is larger than red, and so the sky looks blue.

If you look in this direction, wow, this angle is 90 degrees. Not only is the sky blue there, but the light, from the sky at a 90 degree angle away from the sun, is 100% linearly polarized.

And it is linearly polarized in this direction, perpendicular to the blackboard. And you should be able to figure that out for yourself now. And so you have your linear polarizers. And you can go out and look at the sky.

There is always a great circle in the sky, which is 90 degrees away from the sun. And anywhere on that great circle, the light from the blue sky is 100% linearly polarized.

It's very easy to show with your linear polarizers. Now, when the sun is high in the sky, the amount of light that is scattered in the atmosphere is only 1%.

Thank goodness, otherwise there would be rather dark on Earth, right? But if the angle of elevation above the horizon is 5 degrees, then already the amount of scattered light goes up to about 10%.

And the lower the sun is on the horizon, the longer it has to travel, the more dust it has to see in the Earth's atmosphere. And so the more the blue will be scattered out. The green will be scattered out. The yellow will be scattered out. And the only color that hangs in there, that survives is the red. That's the reason why sunsets and sunrises are red.

So here now is the sun near the horizon. A huge amount of dust on the way to you. And there is a nice cloud here in the sky. And the light that finally gets filtered through the atmosphere is red.

And so this side of the cloud is red. And you look at the cloud, and you say there's a red cloud. Because all the light is red. And so the sun is red when it sets, when it rises.

The moon is red. The planets are red. The stars are red. But if a planet rises, and it is red, it doesn't make your clouds red. So you don't notice that. But it's the same effect.

And you can see that. I often watch planets just coming above the horizon, and they are just as red as the sun is, when the sun sets.

So needless to say, that the dirtier the atmosphere, the more pollution we have, the more beautiful sunsets are. Because you get more crud into the atmosphere. After volcanic eruptions, it's well known that people see spectacular sunsets, spectacular sunrises.

I now want to show you three slides, which in a rather dramatic way, make the point that I was trying to make. And we're going to make it very dark for that. So Marco will come with the first slide, which is a picture of the Pleiades.

The Pleiades is a small group of stars in our galaxy. And these are these bright stars, very hot stars. They produce white light. But there's dust around them. And the dust manifests itself by this blue light. Very fine dust, you can even conclude that it must be very fine, otherwise it wouldn't be blue, the light.

And it scatters in news directions, so it takes off in a different direction. It comes to you, and it is blue. That's Rayleigh scattering that tells you that there is very fine dust there.

The next picture is a man walking on the moon. And this man is walking like this. And as he does that, there is some dust from the surface of the moon, which he throws up, comes up.

In addition, as I learned from my friend Jeffrey Hoffman, who was an astronaut, Jeffrey told me that there is also water vapor that is released from this package. His body is being cooled, because it is very hot in the sunlight on the moon.

And his body is being cooled with water, with an ice pack. And that water is released into the nothing. Because I wouldn't even say atmosphere, because it's vacuum on the moon. And there must also be water vapor here. And then there is the dust.

But the net result is that this person creates, around him, his own blue sky. You are looking at the light that's from the sun that comes in this direction.

And it is scattered to you. And if the particles are fine enough, then you see them blue. So this is dramatic example of Rayleigh scattering.

The next one is a picture of a piece of aerogel. I don't know whether any of you have ever had aerogel in your hands. But aerogel has a density which is only four times the density of air. And it is 99.8% porous.

And there are extremely small silica particles, which typically have only a size of 1 to 2 nanometers. It's way smaller than the wavelengths of light. And now look at this. This tells you why the sky is blue.

But it also tells you why sunsets are red. Because light comes from above. So the light that is scattered to your direction looks distinctly bluish.

But the light that makes it through there, the blue has been removed, and maybe the green has been removed. So whatever comes through is red.

And so you can even see this in the laboratory. With just a piece of this very amazing material, aerogel, you see the effect of Rayleigh scattering.

Who has seen the total lunar eclipse last night? Did you notice that when the moon was completely in the shadow of the Earth, that the moon was red? Who noticed that? Who knows the explanation for that?

AUDIENCE: [INAUDIBLE]

WALTER LEWIN: You know. And I believe you, that you know. Tell me.

AUDIENCE: The light is going through the Earth's atmosphere, and because it has to go through so much atmosphere, it turns red.

WALTER LEWIN: A plus. What you see is light from the Earth. The Earth is four times larger in the sky then the sun when you are on the moon. Because the moon is four times smaller than the Earth.

The moon can just cover the sun, seen from the Earth. But seen from the moon, the Earth is four times bigger than the sun. So here is the sun during a total eclipse. So you are now on the moon. You are a moon walker.

You have this beautiful halo around you, blue. And you're looking at the Earth. And here is the Earth, dark. And the sun is behind it.

And here you have this extremely thin layer, only 1% of the diameter of the Earth. It's a very thin layer of atmosphere. But the sunlight that hits that atmosphere, that gets scattered.

And so it can make it to you. It can change direction. But it has to go through an extremely thick layer. It's like having the sun at the horizon.

And so the light that makes it through it is red. So imagine, if you stand on the moon, and you see that the Earth, four times larger than the size of the sun, there's this gorgeous red ring around it.

And it is that light, from the earth, that goes to the moon, makes the moon red. And then reflects to you. And that's why the moon is red brown. Remarkable phenomenon. Rayleigh scattering in the Earth's atmosphere illuminates the moon.

Now comes the demonstration that I think is the best of all demonstrations, the mother of all demonstrations. I'm going to show you-- it is one demonstration, kill three birds with one stone.

I'm going to create the blue sky. I'm going to create a red sunset. And I'm going to show you that the sky at 90 degree angles to the sun is linearly polarized. All of that in one demonstration.

I have here a bucket of sodium thiosulfate. It there are chemists in my audience, they can tell you and me what that is, sodium thiosulfate.

I will put in the sodium thiosulfate some sulfuric acid. Then a chemical reaction will precipitate, small particles of sulfur, in the beginning, very small, smaller than a micron. The light that comes from this direction will be scattered off these very small dust particles, which are sulfur. And they come in your direction.

Now very little is coming in your direction, because this is very clear. But now when the sulfur precipitate, it comes in your direction. What color will it have? Blue. The blue is preferred.

For those of you who are sitting here, the scattering angle is 90 degrees. Get your polarizers out. The chance of a lifetime. It will be polarized like this.

Those of you there, who didn't pay as much as they do, partially polarized, not 0, but not 100%. But there is ideal, 100% polarized in this direction.

This is the plane perpendicular to the direction of the radiation, and so if I look here it will be polarized light here. If I look here, it will be polarized light there. If you look there, it will be polarized light there.

So you can actually turn the light on and off. It's blue. You can turn it on, and you could turn it off. You, turn it on a little bit and turn it off a little bit.

But now, as time goes on, the sulfur will become more and more and more. And so more and more of the blue light will scatter. What do you think the sun will do? The sun will turn red.

And as we approach 7 o'clock in the evening, it will be really red. Because that means the atmosphere is thicker and thicker and thicker. Let me first put this stuff in, in daylight. And then I will make it dark. I will get also my polarizer out.

And then we will observe this extremely romantic way of ending this lecture. The sun is just normal. It is probably, maybe, 3 o'clock in the afternoon. And the sulfur, ah, the sulfur is already beginning.

Look, I already see blue light. And boy is it polarized, look. For those of you, I use my polarizer for you. See the difference? See the difference?

Ah, look at the sun. Oh, I think we are approaching 5:00 PM already. The sun is already turning a little bit, in the direction of the red. Oh man, I always feel butterflies in my stomach and ants in my pants. So romantic. Walking on the beach, pinky-pinky, it's wonderful.

You want to see the polarization again? And you can use your own linear polarizer. Let's now enjoy this wonderful sunset. The sun gets redder. More and more light, sky is nice and blue. Sun gets redder.

Ah, it's a cloud coming in front of the sun. Do you see that? That happens sometimes. Or was that just my imagination? I would guess it's now 6:30, roughly. What time does the sun set? Ah, you must be kidding. Maybe in Australia, but not here. I think we are very close to sunset now. Yeah, yeah I knew it. I knew it.

See you Tuesday.

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