Massachusetts Institute of Technology OpenCourseWare

8.03SC **Fall 2012**

Notes for Lecture #15: Doppler Effect, Sound, EM Radiation

The initial discussion about whistles should be familiar, for example with train whistles. The perceived change due to motion is called a Doppler shift. In acoustics, the speed of the medium is important, and it does matter who is doing the motion. The situation is asymmetric. Recall that the speed of sound is about 340 m/s. The frequency f' observed by a receiver moving with speed v_r, for sound emitted by a transmitter of frequency f moving at speed v_t is:

$$f' = f\left(\frac{v_s - v_r}{v_s - v_t}\right)$$

where v_s is the speed of sound (**3:15**). If v_r is the sound speed v_s, then the receiver is moving at the speed of sound away from the direction of the transmitter, and the received frequency goes to zero: no sound is received since it cannot catch up with the receiver. If, instead, the sound source is going away at this speed, then $v_t = -v_s$ and the formula gives $f' = \frac{1}{2}f$. This is a huge difference. A demo with a 4000 Hz tuning fork moved with about 1 m/s shows a variation from about 4012 to 3988 Hz, which is readily enough heard (**6:00**).

Rather than straight line motion, sources often travel on a circle with tangential speed $v_0 = \omega R$. There is no frequency change when the source is moving perpendicular to the line of sight, increased frequency when it is moving toward the observer, and decreased frequency when it is moving away from the observer. The component in the observer's direction is called radial velocity and is $v_{rad} = v_0 \sin \omega t$ if the time zero is chosen when the source is furthest away and moving transverse. A lot can be deduced about the motion by studying this frequency shift. One can get the period, T, and from the extrema f'_{max} and f'_{min} one can get v_0. From T and v_0, one can even find the radius of motion, R. This is demonstrated with a whirling whistle (**9:40**).

Electromagnetic radiation also shows a Doppler effect, with the same basic phenomenology. However the result requires special relativity, and the velocities are no longer absolute, so only the *relative* velocity matters. (Note, in acoustics, the medium can be used as a reference). The Doppler formula for EM radiation is expressed in terms of wavelength as

$$\lambda' = \lambda\left(\frac{1 - \beta\cos\theta}{\sqrt{1-\beta^2}}\right)$$

where $\beta = v/c$. For $0° < \theta < 90°$, the object is approaching the observer, $\lambda' < \lambda$, and the light is bluer (higher frequency) so it is called a blue shift. For $90° < \theta < 180°$, the object is receding from the observer, $\lambda' > \lambda$, and the light is redder (lower frequency) so it is called a red shift (**14:15**).

These terms are commonly used in astronomy. If β^2 is a lot less than 1, the denominator is about 1, and the simpler form $\lambda' = \lambda(1 - \beta\cos\theta)$ (or $f' = f(1 + \beta\cos\theta)$) can be used. Radar works by emitting a known wavelength and checking the reflected wavelength received. Stars show absorption lines at known wavelengths, so measuring these allows easy determination of radial velocity of stars. For example, Delta Leporis has a Ca absorption line with known wavelength when at rest of $\lambda = 3933.664$ Å(one Å, or Ångstrom, is 0.1 nm, or 10^{-10} m), but in this star this line is observed at $\lambda' = \lambda + 1.298$Å. From this $v\cos\theta$ is 99 km/s (relative) (**19:15**). In astronomy, the ability to measure Doppler shifts has been important and two examples are now examined.

First, black hole binaries. Many stars are binary, and sometimes one of the two cannot be seen. Nevertheless, the absorption lines in the visible star will show Doppler shifts as the stars orbit each other, appearing to move around their common centre of mass. The period can be derived, and if two sets of lines, the radius is easy to get. In a black hole system, assumed to have circular orbits, the two objects must be on opposite sides of the centre of mass, so that $m_1 r_1 = m_2 r_2$. It is easy to get T, v_1, r_1 and T, v_2, r_2. From Kepler's Third Law these quantities are related by (**24:50**) $T^2 = \dfrac{4\pi^2(r_1 + r_2)^3}{(m_1 + m_2)G}$ where G is the universal gravitational constant (value $\sim 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2}). This allows to get the sum of the masses of the stars, and with the centre of mass formula, $m_1 r_1 = m_2 r_2$, one can solve for the masses. All this is assuming one is in the plane of the orbit. The radial velocity will be lower if there is a tilt but in the case of eclipsing binary systems this is not a problem.

In some systems, one star is an ordinary star, and the other is a neutron star or black hole quite close to it. There is a point, the Inner Lagrange point, where the attraction of each star is equal. If this is inside the bigger star, matter will stream from it onto the companion, spiraling due to orbital motion. The star providing mass is called the *donor*, and the star getting the mass is called the *accretor*, with the spiraling material forming an *accretion* disk. It is a mechanics question to calculate the energy with which the material will arrive at the donor. Potential energy going to kinetic energy, we get $mMG/R = (1/2)mv^2$ or $v = \sqrt{2MG/R}$ which coincidentally is also the *escape velocity* from that same radius (**30:00**). The mass of a neutron star is about 1.5 that of the Sun, but the radius is tiny, only 10 km. The energy falling into such a potential well is enormous, so infalling material gets very hot and radiates mainly X-rays. Even a marshmallow would give the energy of an atomic bomb. Often the mass of the accretor turns out to be 1.4 times that of the Sun (**35:00**). Chandrasekhar used quantum mechanics to show that this was the limit on stability of white dwarf stars (planet-sized stars), past that they collapse, so it is not surprising that this is a typical mass for a neutron star. Even neutron stars have a limit beyond which they collapse into black holes. General relativity predicts that this upper limit is about 3 solar masses.

The gravity of a black hole is so strong that the escape velocity is greater than c and therefore light cannot escape. The *event horizon* is the point where the escape velocity reaches c: $c = \sqrt{2MG/R}$. If the Earth was compressed to be a black hole, this radius would be 1 cm (rather than 6400 km). For the Sun, it would be 3 km, and for 5 solar masses, it would be 15 km (**40:00**). In 1971, Bolton and others concluded that the double star X-ray source Cygnus X-1 must contain a black hole.

In the 1920's, astronomers were measuring the radial velocities of many stars, typically up to a few hundred km/s. Some nebulae in the skies had large red shifts (over 1000 km/s). Eventually it was realized that these were galaxies, having about 10 billion stars each and showing an average spectrum from them (**45:00**). Hubble found a correlation of the distance of galaxies and their redshift. The distances were found in a complex way (from variable stars) and were initially underestimated. The linear proportionality of distance and redshift is now known as Hubble's Law: $v = Hd$, with H being about 70 km/s/Mpc, where an Mpc is 3.26×10^6 light years, or $3.1{\times}10^{19}$ km. The dimension of H is 1/s or 1/time. Thus, distances, and therfore how long ago the light left the galaxy, can be calculated from redshifts, . Several examples are given (**50:00**).

As the distance gets bigger (note the images of the galaxies appearing smaller), the redshift of absorption lines shifts toward the red compared to those in laboratory spectra (on the edges of the observed spectra). More recently Hubble Space Telescope data led to the most accurate value for Hubble's constant. Wendy Freedman concluded it is about 72 km/s/Mpc. The range of Hubble's original data was tiny compared to that of the most recent results (**55:00**). Despite all of the galaxies appearing to recede from us, and the inference that there was an initial giant explosion called the *Big Bang*, we cannot conclude that we are at the centre, but we can estimate the age of the universe using $d = vt_u$. With $v = Hd$, $d = Hdt_u$, i.e. $Ht_u = 1$, or $t_u = 1/H$. With the value of H and doing unit conversion, the age of the universe is $t_u = 4.3 \times 10^{17} s$ or ≈14 billion years.

The oldest stars are about 10 billion years old, and the subject of the evolution of the universe is actively studied. It is now believed that the universe had a very early expansion due to so-called *inflation* (first proposed by Alan Guth of MIT in 1979), decelerated for some time, but then began *accelerating* due an unknown mechanism dubbed *Dark Energy*. (**1:00:00**) Redshift can significantly change the range in which a line is seen. For example, UV light can end up in the visible if the redshift is high enough. To understand why the Earth is not the centre of the universe, imagine expanding raisin bread. Seen from *any* raisin, all the others are moving away with a speed which depends on distance. Thus, there is a Hubble law for raisin cakes (**1:05:00**). A similar things happens on the surface of an expanding balloon if distances are measured only *along* the surface. The conclusion of the lecture discusses changes in our understanding of the time dependence of cosmological aspects of the universe, including inflation and Dark Energy.

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These viewing notes were written by Prof. Martin Connors in collaboration with Dr. George S.F. Stephans.