8.03SC Physics III: Vibrations and Waves, Fall 2012
Transcript – Lecture Lecture 17: Wave Guides & Resonant Cavities

WALTER LEWIN: Today we're going to talk about wave guides and about resonance cavities. Last Thursday, I discussed the boundary conditions of electromagnetic waves on the surface of an ideal conductor in a vacuum. And today, we will see some of the amazing consequences.

And I will return to something that I have discussed with you before but never fully explained. And that's the set-up, whereby, we have metal plate-- this is my coordinate system, x, y, z. This is the z direction.

And then I have here another plate, two parallel plates. And this will be the x direction. I call this x equals 0. And this x equal a. And this, then, is the y direction. So that's the set-up. You see it there.

I'm going to try to send electromagnetic radiation through this gap. What now comes holds always that k is always kx, x roof, plus ky, y roof, plus kz times z roof. k is the direction, then, of propagation. Also, lambda equals 2 pi divided by that k. And the magnitude of that k is the square root of kx squared plus ky squared plus kz squared. And omega equals kc, if this is in a vacuum.

I'm going to send, through this gap, linearly polarized radiation in the y direction. That's a choice that I make. I don't have to do that. But that's a choice that I make. And think about the y direction and the z direction, very, very large, infinitely large. Or, in practice, many, many, many times the wavelength of the radiation.

So to meet the boundary conditions, the electric field-- which is only in the y direction-- must become 0 here and must become 0 there, because the electric field cannot be in this plane. Remember, E of t, which was the tangential component, at the surface of a conductor must be 0. So E of y must vanish here. And it must vanish there. And so for this case, whereby we only have radiation linearly polarized in the y direction, E of y must become 0 for x equals a and for x equals 0.

Since there is no dependence of the E field in the y direction, k of y will be 0. You will see that come back many times in this problem. So now I would like to take a look at this geometry from above looking down on the xz plane.

Here is the z direction. And this is the x direction. And the y direction is coming straight out of the blackboard pointing in your direction. I'm looking down on that plane from above.

Let the k vector be this. This is a vector. This k vector has a component in the x direction, which is k of x. And it has a component in the z direction, which is k of z. But it does not have a component in the y direction, k of y is 0.

The waves-- and think of them as water waves, as far as I'm concerned. This is the direction of propagation in the gap. The water waves would be perpendicular to the direction of propagation, which are the planes with constant E fields. I'm going to put one in here. This is 90 degrees. And I'm going to put another one in here.

This is now propagating with c in this direction. And the separation between these two crests is lambda. That is my definition of lambda. Think of this surface, which is a surface perpendicular to the blackboard, as having a constant phase of the wave.

It means then, for instance, that the E vector is pointing in your direction and has reached a maximum. It would also have reached a maximum here, then. So, we go through one complete cycle, 2 pi radians, from here to here. That's the definition of wavelength.

Now, in one period that this wave moves from here to here, in that same amount of time, this wave here moves from here to there. And so I will give this here a name, as I have done before. And I call that L of z.

And so it is immediately obvious, by the definition of phase velocity, that the phase velocity in the z direction must be Lz divided by lambda times c, because it goes from here to here with the velocity z. But here, it goes much further in the same amount of time. So now that must be times c-- not equal c-- but times c.

So, it's larger than c. And this, by the way-- and that follows from the geometry-- is also k divided by k of z times c. And that is larger than c. The phase velocity is larger than c. And I have discussed that with you at length before.

Now look at what this wave is doing. The wave here-- these waves hit the wall. And they then reflect off the wall. I'll try to make you see that. They reflect off the wall. They come back this way. And then they reflect off the wall again.

And so these waves, which zig zag, slowly work their way through the gap in the z direction. And you can immediately see, therefore, that the speed with which they work their way through the gap in the z direction must be less than c. Because when they go from this point A to this point D, in that same amount of time, in the z direction, they only go from to B to D.

And so the group velocity in the z direction is then BD divided by AC times c. And that-- and that follows from this geometry-- equals k of z divided by k times c, and that is less than c. And it better be less than c, because you can just see how difficult it is for this wave that reflects to work its way through in the z direction.

We discussed earlier, and I explained that using the analogy with water waves that are hitting the shore, that a phase velocity larger than c has really no meaning. It's a geometrical effect of the consequences that the water slams on the wall everywhere at the same moment in time. However, the group velocity-- what is that, what did I do wrong?

AUDIENCE: Because the c--

WALTER LEWIN: Excuse me?

AUDIENCE: The c is less than 0.

WALTER LEWIN: Thank you, very much. This is larger than 0. Now, what do you-- ah. Boy, it's not my day. Is that what you want?

AUDIENCE: Yeah.

WALTER LEWIN: Thank you. All right. Notice that in this specific case, the product of group velocity and phase velocity happens to be c squared. It's a special case. It's not always the case. But what is important that you recognize-- and this picture is so nice. If you can really digest that picture and relate to it, it's so nice. You immediately recognize that the phase velocity, which is by way the speed of this distance is larger than c, and that the group velocity is smaller than c.

Now we can do the math. And I can write down the wave equation, del square E. There's only a component in the y direction, because that's the radiation I have chosen. So, I get d2Ey dx squared plus d2Ey dy squared plus d2Ey dz squared equals epsilon 0 mu 0 times d2Ey dt squared. And this term does not exist, because there is no component, no dependents of the E vector in the y direction.

The solution for the wave that goes from here to here immediately presents itself-- because we have a lot of experience now-- it's going to be a standing wave in the x direction and is going to be a traveling wave in the z direction. So E, as a function then, of x, z, and t-- because E itself is not a function of y-- is then some amplitude E0, to remind you that it's only in the y direction. I'll be nice to you and put a y there.

And then we have the traveling wave in the z direction. And then we have the standing wave in the x direction.

And so you see here-- it is staring you in the face, because I knew, of course, the answer-- that this is a traveling wave and that this is a standing wave. And if I use that solution, which has to be correct, and I put it in that equation-- which I can, it will meet the three-dimensional wave equation-- you substitute that in here, you will find something that, of course, you also know already, namely that omega squared, which is k squared times c squared.

But k now only has a kx and has a kz. And so you get this kx squared plus kz squared times c squared. That's what you will find if you substitute that in the wave equation. And you will find that c squared equals 1 over epsilon 0 mu 0.

Now the boundary conditions, which we haven't met yet, require that the vector, which is only in the y direction-- put this one here, and once more to remind you, it's linearly polarized in the y direction-- now must vanish when x equals 0 and when x equals a. That now means that kx must now be Nancy times pi divided by a.

And Nancy, then, can be 1 or 2 or 3 or 4. And if we look, we now look in a different direction. I make another cross-section. Now this is y. And this, now, is x. This is 0. And this a And the z direction is now coming straight out of the blackboard.

Then for Nancy equals 1, kx is going to be pi over a. And that is this wave. That's the sinusoid. And so at a particular moment in time, the E vector is would then be like this.

And you see they vanish here and they vanish here. There can be no tangential component in this plane here. And there can be no tangential component in that plane here.

This is, then, n equals 1. And, of course, it also waves like this. It's a standing wave. For n equals 2, that is another possibility, kx would be 2 pi divided by a. And I can put that in a different color. That would be like this.

And so now the E vector has, right here, a nodal plane. And now it goes like this. And so those are possibilities which are only restrictions in the x direction.

We can write down now the k vector. The k vector now is kx times x. But kx is n pi divided by a in the x direction. And then I have a kz in the z direction. And there is no ky. ky is 0.

And omega, which is kc, is therefore c times the square root of n pi over a squared plus kz squared. And this equation is one that I had on the blackboard earlier, before we understood why it was that way when I discussed with you dispersion. This is called the dispersion relation.

This, by the way, under the square root, is simply k. And so now you can ask yourself, what is now the phase velocity in the z direction? We already know what it is. We already know what is, it must be that. We already reasoned that purely from geometry. But it must come out now, too.

Well the phase velocity in the c direction is omega divided by kz. But omega, itself, is kc. So that becomes k divided by kz times c. And that is always larger than c. You see exactly the same stuff here that we had there.

When kz goes to 0-- and that is possible, I will show you that in a minute-- the phase velocity goes to infinity. So

Now, you can also calculate what the group velocity is. The group velocity in the z direction is d omega d kz. And since you have this term here, you should be able to do that. That's an 18.01 problem.

You take d omega d kz. And you will find now, that becomes-- and I will leave you with that-- kz divided by k times c. And that is always less than c. This is what we also knew already. I had already predicted that based on geometry arguments. That comes out now if you use this dispersion relation.

And this dispersion relation then has the absurdity that if kz goes to 0, that the phase velocity goes to infinity, but that the group velocity goes to 0. And so that now, I want to address further. And the best way that we do that is make what we call an omega kz diagram.

I'm going to plot for you this curve. It's extremely useful to always think of this as an omega kz diagram. Here is kz. Here is omega. And this line, this straight line, that would be a non-dispersive medium. That would be omega is kz times c. But that's not what we have.

What we have is that there is here for n equals 1, there is a particular frequency, omega c-- we call that the cut-off frequency-- below which no radiation can go through the gap. And that cut-off frequency is when n equals 1 and when kz become 0.

And so that is c times pi over a. c times pi over a. And this curve then for n equals 1 goes like this. This is n equals 1. If I radiate electromagnetic radiation linearly polarized in this direction, and I know the frequency-- let us say, this is the frequency that I radiate, that's a given-- then radiation will go through beautifully. And this will then be the value of kz.

The value of kx is non-negotiable. That value for kx must be the value that I had earlier-- where is it-- that must be pi divided by a. Let me write that down again. For n equals 1, k over x must be pi divided by a.

But it is kz that adjusts itself so that it meets the boundary conditions. kx does not adjust itself. kx has no choice. kx must meet the boundary conditions so that the E vector vanishes at x equals 0 and x equals a. And so any solution must lie on this curve. And so it is kz that pays the price and that adjusts itself.

Now, keep in mind that all radiation, with frequency above this value here, can go through this gap. There is often a misunderstanding among students. They think that there are resonance frequencies. There are no resonance frequencies. Any frequency above that value can go through the gap.

You can immediately see here that the phase velocity is larger than c, because the phase velocity is omega divided by kz. And so when I draw this line, then this slope here, omega divided by kz, is larger than this slope. And that corresponded to a speed c. And so you see, since this slope is steeper, the phase velocity is larger than c.

And you see that when you reach the situation that kz becomes 0, that then you have this is omega and kz is 0. When you reach this crazy cut-off frequency, the phase velocity goes to infinity. Also, notice that the tangent here is the group velocity-- that is the omega dk-- and notice that this slope is smaller than this one. That is why the group velocity is lower than c.

And also notice that when you reach this cut-off frequency that the tangent is like this. That means the group velocity is 0. Nothing can go through the gap anymore.

I now want to write down for you something that may help you later, if you want to understand your notes. I will put this back up again. I'm going to do the following experiment now with you, in my head.

I start with a certain frequency, omega. And I'm going to lower that frequency. I'm doing it in my head. It's a Duncan experiment. I'm going to write down step by step what's going to happen.

This point will slowly go down. And so we will come down this line here. And finally, we reach here. I probably can write it here. It is nicer for you, because then you have it all together.

I started with omega, which is larger than omega c. That's how I start. It's a given. And now I'm going to lower omega for the next thing that I'm going to do. But when you lower omega, kx can not change. It must remain pi over a, because it must meet the boundary conditions. Therefore, kz gets smaller.

When you're going down this line, and the kz gets smaller. Omega is kz. Omega is always k times c. So, clearly, if omega comes down, k must come down. And that means kz comes down, in this case, because kx is not going to give up.

And so k gets smaller. And if k gets smaller, then, of course, lambda increases. Because lambda is 2 pi over k. Until disaster strikes, omega becomes omega c. And when you reach that point, kz has become 0, and kx is still pi over a.

So kz is now 0. And kx is still pi over a. So now k, itself, is kx. And say for lambda, now, is 2 pi divided by that value kx. 2 pi divided by kx means that lambda now equals 2 a.

It means if I have a given frequency, which we do have here. Which I cannot change. I have 10 gigahertz. And that 10 gigahertz has a wavelength of 3 centimeters in vacuum.

It means the moment that I make a any smaller than 1 and 1/2 centimeters, there is no propagation anymore in that direction. Because now I can no longer meet the boundary conditions for kx. And I will demonstrate that as I did before. This is not the first time that you will see that.

Before I do that, I want to discuss with you the meaning now of n, Nancy equals 2. What is does that mean now? In other words, we had here the n equals 1. And we also have Nancy equals 2. And so I'm going to make a new plot, because it becomes otherwise too cluttered.

I will leave this on. Make sure you get that sequence. That's very important. And I will put it up later again.

I'm going to make a new plot, otherwise it becomes too confusing, too many lines. kz omega, and here is your line non-dispersion. Omega is c times k of z. And here now is your n equals 1, omega c equals pi divided by a times c. And here, exactly twice as high, is n equals 2, whereby allotted omega c is 2 pi times c divided by a.

And so I have one line here. And I have another line here. So this is now the plot of this equation for n equals 1 and, independently, for n equals 2. This is n equals 1. And this is n equals 2.

Imagine now that this is our omega. That is a given. That's the one that we happen to have. This frequency can obviously go through the gap. But now it has a choice out of two possibilities.

It can either go through this way, with this value for kz, which corresponds with n equals 2. That's the mode in green there. Or it can go through in this mode, with this value for kz, for n equals 1. Or it can do both.

It can do any linear combination between these two, because each one is a solution to Maxwell's equations. Each one obeys the boundary condition. And each one meets the wave equation.

And so there are two ways that it can go through. If it decides to choose this route, then kx is a given, is pi over a. If it decides to go this way, then kx is 2 pi over a. The only difference between these two is that kx is different.

k is not different, because they have the same omega. If they have the same omega, they have the same k, the same wavelength. But this one has a different k of x, therefore, a different k of z. And you see that. And this one has a different k of x therefore a different value for k of z. I cannot be more specific.

Before, I want to demonstrate to you-- which is a repeat-- that if I have here electromagnetic radiation, 3 centimeter waves polarized in this direction, and I'm going to send it to this receiver which can receive it. Before I'm going to demonstrate that when I make the gap smaller than 1 and 1/2 centimeters, that it will all of a sudden disappear, the radiation. Before I do that, I want to ask you a question, which is important. You can test whether you understand this concept.

My radiation is linearly polarized in this direction. Out of that came all this misery. Is there a way that I can send electromagnetic radiation through that gap when the gap spacing, for my transmitter, is smaller than 1 and 1/2 centimeters? I know I cannot do it when it is polarized in this direction. It will immediately disappear. You will hear it. Is there a way that I can still get the radiation through when the opening of the gap is smaller than 1 and 1/2 centimeters?

I realize it may take you a little bit of time to think of it. But I help you by saying, all this misery is the consequence of the fact that I chose the direction of linear polarization in the y direction.

AUDIENCE: Rotate it 90 degrees.

WALTER LEWIN: Excuse me?

AUDIENCE: Can you rotate it back 90 degrees?

WALTER LEWIN: Exactly. If I put in-- and I will do that, believe me. If I rotate my transmitter by 90 degrees, and I sent in radiation whereby the E vector is linearly polarized in the x direction, there is no misery at all. Because all boundary conditions will always be met by mother nature.

Because all that mother nature has to do is to make sure that the normal component at the conductor is sigma s divided by epsilon zero. All it has to do is rearrange the charges on the surface. But nothing has effort to go to 0.

The moment that I make the radiation polarized in this direction, there is no dispersion, the phrase velocity c, the group velocity is c, and this is the line that holds now. It's only the component in the y direction that gives you this misery. That is very interesting, isn't it?

Because what that means then is that if the radiation that you send in had a component both in the y in the x direction-- I can do that, right? I can send radiation through that is literally polarized like this-- then something unbelievable is going to happen. The component in the x direction will go straight through at the speed of light.

And the component in the y direction has to suffer like this. And so they completely decouple. If x direction is non-dispersive, group velocity is c, phase velocity is c. And it is the component in the y direction that decouples, and that can go so slowly that it can even have a group velocity of 0.

When I show you that I can change the size of that gap, what that means in terms of this line is the following. I cannot lower my frequency. My frequency will be somewhere here. It is 10 to the 10 hertz, 10 Gigahertz. Lambda is 3 centimeters. The moment that a becomes less than 1.5 centimeters, there is no longer a way for the y component to go through.

What I'm doing is to keep this point constant, but this line I am moving slowly up. Because when I make a smaller, it is this point that omega c goes up. And so I make omega c go up until it hits my 10 to the 10 hertz.

In our thinking here, we lowered omega. And I showed you what happens. Now, to do the experiment, I lower a. And I move this line up until I hit that point.

I think we ready for that now. You will be able to hear it. I would like someone to witness it. Could you come and witness it? Because I want you to see that the separation between the gap is now 2 centimeters.

You stand a little bit on the side here and not in the beam, yeah? Look here, come here. What is your name?

CRYSTAL LEE: Crystal Lee.

WALTER LEWIN: Crystal Lee, wonderful name. My name is Walter, by the way. This is at 2 centimeters, you see that? This is a transmitter. It's polarized like this. Here is the receiver, which receives the polarized radiation like this.

We modulate it with 550 hertz so that you can hear it. You cannot hear 10 to the 10 hertz. That's not the way that nature designed you, nor can I hear it. This is 550 hertz. The gap is 2 centimeters.

I'm going to make the gap smaller with my hands. You watch it when it gets to 1 and 1/2 centimeters. It's gone. All of a sudden, it disappears. That is the consequence of the fact that this point hits here now.

But now it this is 1 and 1/2 centimeters apart, I can now rotate this 90 degrees. And rotate this 90 degree, so that now we have the E vector in this direction. There are no longer any restrictions and the radiation go through. Because now I have this red stuff.

And now I can make the gap smaller. And what do you think when I make the gap smaller, then smaller, then smaller? Well, very slowly will you hear nothing. Because if the gap has no cross-sectional opening anymore, no radiation can go through.

Don't expect miracles. If I now make to gap smaller, and smaller, and smaller, nature has no problem sending radiation through. But nature is stuck to the cross-sectional opening. And as, of course, you make this opening narrower, and narrower, and narrower, then less, and less, and less, radiation will go through.

And so I will now make the gap smaller. She is my witness. I will put this on top. It's now about 1 and 1/2 centimeters. I'll make it smaller. It's now about 1 centimeter. Still going well, about 0.7 centimeters, about 1/2 a centimeter, it's about three millimeters. And, obviously, we slowly lose density.

So you see-- thank you very much for assisting me. My pleasure, actually. And so you see how bizarre it is that the behavior is so fundamentally different for these two different modes. And then the ideal of the decoupling, that if you oscillate in a direction that has an x and a y component that the radiation decouples.

I think this is a nice, natural moment to have our break. And I was just wondering why the attendance was so high. But then I was reminded of the fact that it's Tuesday. So I'm very flattered that you all came.

And we will hand them out. And then we will not start yet until the whistle blows. I will raise all the blackboards so that you can see everything. [? Kristina, ?] can you help handing these out? Can you also help? Could you also hand? Make sure that everyone has one.

OK, now I'm going to change the situation very dramatically. And now I'm going to give you a closed box conductor. There's a conductor. Our coordinate system is the same, x, y, z. And I simply call this a. I called this b. And I call this c. That is the length. And that's the coordinate system.

And now comes the question. If I now have electromagnetic radiation in this box, what is going to happen? Now there is no way that it can get in or out, if it's in there. Clearly, you now get a situation that you have a cavity with very discrete resonances. It's totally different from what we did before. You only get standing waves with very discreet resonance frequencies.

Well, I will write down for you-- because I want to ram it down your throat, but I could have left it here-- k equals kx times x plus ky times y, plus kz times z. k equals the square root of kx squared plus ky squared plus kz squared. Omega equals k times c. Lambda is 2 pi divided by k. None of this is negotiable.

Now in this box I can have an E vector, which has an x, y, and a z component. I have an E vector which is something in the x direction plus something in the y direction plus something in the z direction. And our task is, now, to find the resonance frequency for this cavity.

Let us only look for now at the component of the e vector in the x direction. So everything that follows now is only true for the x direction. E of x, which is the component of the E field in the x direction, must be 0. E of x is this component, in the x direction, must be 0 at y equals 0 and y equals b. If it were not 0, there would be a tangential electric field in that top plate.

It must be 0 when y equals 0, and also when y equals b. But it must also be 0 when z is 0 and when z is c. Because if the e vector is like this, it must be 0 not only on the bottom plate and on the top plate, but it must also be 0 in the front plate, which is this plate, and in the back plate. Because it cannot have any E vector in that surface either.

So the Ex alone must meet now four special boundary conditions. And we are so experienced now that we can write down, just like that, the solution for that component E of x. And that solution must have this form. You can test yourself whether you would have come up with something similar. This is only the component now in the x direction.

This must have a sine of ky times y times the sine off kz times z times cosine omega t. And to remind you that it is in this direction, I put to the x there. So what you have now are two standing waves, one standing wave in the y direction, and one standing wave in the z direction.

And to meet the boundary conditions, which I wrote down here, it is immediately obvious that k of y must now be m pi divided by b. And k of z must be Nancy pi divided by c. And then m equals, in this case, sine. So it is 1, 2, 3, et cetera. And Nancy equals 1, 2, 3, et cetera.

And as long as I meet this condition, the E of x component is quite happy. It means that k now only has a component in the z and in the y direction. kx is zero for this component. And so we get k of y times y plus k of z times z.

The wave equation-- del squared E. If I apply the wave equation now only to the x component-- of course, we should also apply it to the y and the z component-- then we get d2Ex dx squared plus d2Ex dy squared plus d2Ex dz squared equals epsilon zero mu zero times d2Ex dt squared. That's now the wave equation.

And this term is not there for that x component. But the other terms are there. And if I substitute my solution, which I know deep in my belly has to be correct, in this wave equation, then what you find, of course, is no surprise. That omega squared equals k squared times c squared. And so that is going to be ky squared plus kz squared times c squared.

But my ky and my kz are quantized, because they are now only allowed for certain values. And so I can write then now that omega squared is then c squared times-- and I get m pi over b squared plus Nancy pi over c squared. Excuse me?

AUDIENCE: You mean d. You've been writing c for d.

WALTER LEWIN: b is for the y direction. And c is for the z direction.

AUDIENCE: [INAUDIBLE].

WALTER LEWIN: What is the problem?

AUDIENCE: On the box--

WALTER LEWIN: Only one person, please.

AUDIENCE: On the box, your c looked like a d, so everyone was getting d, when it is actually c.

WALTER LEWIN: The z direction is c. The x direction is a. And the y direction is b.

AUDIENCE: Walter, the speed of light, e is the dimension of your box.

WALTER LEWIN: b is in the y direction. Oh, yeah. We have a boundary conviction in the y direction and a boundary conviction in the z direction.

AUDIENCE: [INAUDIBLE].

WALTER LEWIN: Why won't you come here, [INAUDIBLE] and just make the change. Did I miss something?

AUDIENCE: [INAUDIBLE].

WALTER LEWIN: Oh, I leave it c. I cannot change that. Because if I change that, then it becomes a terrible chain reaction. That is c. Oh, I'm very sorry that I called that c. Yeah, thank you very much. Can I get my chalk back, by the way?

So, we agree that there was no mistake on the blackboard, but there is confusion about c. My apologies. Yes, if I had called it d, there is another problem, dx, dy. And then you have another d. Indeed, I have followed this convention. This was a c. This is a b.

You see the same c here? That is that same c. z equals c has nothing to do with the speed of light. It's one of those things. Yeah, that happens. m stands for Mary. And n stands for Nancy. It doesn't stand for normal this time. n, it stands for nancy. My apologies.

All right, can we go on? Yeah, [INAUDIBLE] I have your permission? OK, thank you for trying. Because, indeed, it could have been a mistake. But it wasn't meant that way.

What you see there now is that you have a whole family of frequencies which meet all the conditions for the x direction. And that whole family, then, you can give Mary and Nancy here as a subscript. There's an omega 1,1. There's an omega 1,2, an omega 2,1, an omega 2,2, and so on.

I try to see what, actually, this system is doing. It's awfully difficult. But I think I can make an attempt.

Suppose we look at the system in this direction. We're looking at the yz plane. Here is the z direction, which has this length c. Sorry, I will repeat it once more to avoid confusion this.

This is b. This is the y direction. And this is the z direction. x is coming straight out of the blackboard.

We're looking now, from the side, like this. Yeah? That means that the component of the e vector in the x direction must be is 0 everywhere. It must be 0 here, because this is a plane and there cannot be any tangential component in that plane.

But this is also a plane and there cannot be any tangential component in that plane. And this is also a plane and there cannot be any tangential component here. So everywhere here must be the E vector, and the x direction must be 0.

And so what does it mean the 1,1 mode. It means that the whole thing-- think of it as a membrane. The whole membrane comes to you, the E vector is like this, goes away from you, E vector is like this, goes towards you and goes away from you. That's what the 1,1 mode is. Think of it as a membrane.

But keep in mind that, of course, it extends in the x direction. And it's everywhere the same in the x direction over the whole length a. I cannot change that. And I cannot make you see that.

And so, for instance, the 2,1 mode would then be if I make a Mary 2 and I make Nancy one. Then in the y direction, I would have here a complete nodal surface which runs all the way through the box-- entirely through the box, no x component of the E field-- and this one would then be an E field in your direction. And this would be a E field in this direction. And it would go like this.

And then if you have the 2,2 mode. This is the 2,2 mode-- x is in this direction-- then you would get this. You would get one whole nodal surface here-- no E component, no x component of the E vector-- throughout the entire box, throughout complete x. And then this would come to you. This would come to you. This would go away from you. And it will be this standing, oscillating wave.

Now independently, I can now look at the y component of the E field, because I only looked at the x component. Let's now look in the y direction. And let's now ask what is the problem with Ey? Well, Ey also has to become 0 when it, in a plane, becomes the tangential component.

And therefore, Ey must be 0 when x equals 0, and when x equals a, and when z equals 0, and when z equals c. If Ey, in this direction, must vanish for x equals 0, it must vanish for x equals a. And it must also vanish in this front plate, z equals 0, and in the back plate, z equals c.

Now you can come up, immediately by parallel, with a relationship. I will call it now l as in lion for x. And then for z, I will keep the n as in Nancy so that we don't get confused. So that is now c-- I will do it in terms of squares-- and then I will get lion pi over a squared plus Nancy pi over c squared, and that's it. Now I have another infinite family of values for l as in lion, n as in nancy, which gives me the resonance condition in the y direction.

And so if I had linearly polarized radiation, I could have, in the x direction, linearly polarized radiation would then have to obey this. I could have linearly polarized radiation in the y direction, which would have to obey this sequence of frequencies. And I could do one in the z direction, which, of course, you can make up for yourself.

However, it is also possible-- and any linear combination of those would be fine. But, of course, it is also possible that I have an E vector which has a component x, y, and z and that, simultaneously, all boundary conditions are met. So I don't just radiate linearly polarized radiation only in the x direction and only in the z direction.

But now I say, aha. If now I make omega squared l, m, n, if that now is c squared times l pi over a squared plus Mary-- l m-- Mary pi over b squared plus Nancy pi over c squared then I can have an E vector, which is in some random direction. Well, maybe not too random, because I have to meet this condition.

And now I have a whole family of infinite number of resonance frequencies, which are not only E vectors in x, y, or in z direction. Did I make a slip? Thank you, very much. I have a c squared downstairs, right? Isn't that one over epsilon 0 mu 0?

AUDIENCE: [INAUDIBLE].

WALTER LEWIN: Thank you. I appreciate it, really, I do. Because it's so nasty, these slips. We are all happy now, right? Yes, I am.

All right, now I'm going to change gears, because I wanted to find a way to demonstrate this to you. I'm going to change gears. And I'm going to make this a box. And I'm going to ask myself what are the resonance frequencies for sound. Sound is a longitudinal wave, so sound has no linear polarization.

Now, I can immediately write down the function of the overpressure p-- over and above one atmosphere or below one atmosphere-- because now I know that at all surfaces I must have pressure anti-nodes. The particles cannot move. They're stuck against the wall. The pressure can build up. The pressure can get low. The pressure can build up. But the particles, at the walls themselves, cannot move through the wall.

And so now we have pressure anti-nodes at the walls. It is easy now for me, without any further thinking, to write down the general solution for the pressure in the box as a function of the dimensions of that box. And so this p-- which is either a little bit above one atmosphere or a little bit below one atmosphere-- is some amplitude p0. And then now we get the cosine kx times x times the cosine of ky times y times the cosine of kz times z. And then the whole thing times cosine omega t, which is now the frequency of my sound source.

And so now in complete analogy with what I wrote on the blackboard there, I must have pressure anti-nodes at x equals 0 and at x equals a, at y equals 0 and y equals b, at k equals 0 and k equals c. Now I have that my boundary conditions require that kx is l pi over a, that ky equals Mary pi over b, and that kz is Nancy pi over c. Then, I'm happy that I have all surfaces. I have met the boundary conditions that the pressures are at maximum.

And so what always holds is that omega squared is always k squared times v squared. But this v is now the speed of sound. And the wavelength lambda is 2 pi divided by k.

I'm going to rewrite that equation on the blackboard in terms of frequency in hertz, because it's going to play a key role in the demonstration that is coming. It is exactly the same result that you see here, but I'm going to write it now as f as a function of l, m, n, which is omega l, m, n divided by 2 pi, which is therefore v, which is the speed of sound, divided by 2 pi.

And then I get here the square root of l pi over a squared plus m pi over b squared plus n pi over c squared. And notice that you lose your pi. This pi here eats up this pi eats up this pi, eats up that pi. And this is now in hertz.

Now you have an equation that if you make a box, and you know the dimensions of that box-- then you know that a is a dimension in the x direction, b in the y direction, and c in the z direction-- then you can now predict at what sound frequencies this box will resonate. Marcos Hankin, sitting there, and I have spent the last two weeks of our lives-- he more time than I did-- to build such a box. And the box is here.

It is a marvel. He even gave it colors. He loved it so much he could hardly sleep at night, he told me. And there is the box. And here is a loudspeaker on this side. He has a hole in there of 1 inch. And so the sound comes in there.

And inside the box is a microphone. And this microphone will tell us whether we hit resonance. Because then we will see a very high signal from this microphone. You have no idea how much fun we have had with this.

And I'll show you here. You have plenty of time to copy it. Because this is going to take at least 15 minutes this demonstration.

You see here the dimensions of the box. They were meant to be 30 by 40 by 50 centimeters. But the accuracy of our measurements became so stunningly high that Markos said, well, maybe it's not 40, maybe it is 39.9. Because there was one particular frequency which was a little bit off. And he was right. We measured it and it was a little bit less than 40 centimeters.

You see the sizes there. And you see the uncertainties. The accuracy of our measurement was so enormous that we began to worry about what the speed of sound was. That means we have to know the room temperature. Markos and I agreed that we will just accept for now that it is probably very close to 344 meters per second, which is the speed of sound at room temperature of 20 degrees centigrade.

But keep in mind, it is linearly proportional with the square root of the temperature in degrees Kelvin. That means that 1 degree centigrade difference will make a difference. 2 degrees centigrade will be a noticeable difference. We would be surprised, though, if it is any more different than 1%. But it could be different by 1%.

Here you see the modes that Markos computerized and predicted with increasing frequency. Notice that when n equals 1, that is when c-- which is the largest dimension in the z direction-- is the largest value, that gives you the lowest frequency. That should be immediately obvious. Because if you make lion 0, and Mary is 0, and Nancy is 1, if this one is 50 centimeters-- which is half a meter-- you will get the largest value possible. Oh, we are here now.

If c is 0.5 meters, and you multiply that by 2, then you get 1 meter. This is 0. And this is 0. And so 344 divided by 1 is 344. And so that is why this one is 344, because one dimension happens to be 1/2 a meter.

And then from there, from that moment on, it becomes way more difficult to see what's happening in the box. The next the frequency that we predict is 0,1,0. And we went all the way up to 812. In fact, Markos went even past 4,000. But they become so enormously close together that it's no long fun. Here, they are quite well separated.

We're going to make you see the results. And we're going to make you hear the results. Hearing is easy, because we turn on the sound as we receive it from the microphone inside the box. I'm going to turn that on now.

AUDIENCE: [INAUDIBLE]

WALTER LEWIN: Yeah, I will do that. So now you, in principle, should be able to hear the frequency of the microphone inside the box. We're also going to show you the results on our-- did we do TV or 5?

Where is it, Markos?

MARKOS: [INAUDIBLE]

WALTER LEWIN: Oh, sorry. The one at the bottom is the speaker. That's the driving frequency of the speaker. The frequency as I've said it now, you can also see here-- it's very important that you see that-- is now, you'll see it come up, 325 hertz. Way off resonance.

The one on top is the one that the microphone records. And if I speak, you see nonsense at the top. Because the microphone inside can hear me. It's important that we are relatively quiet.

And I'm going to increase the frequency and search for the first resonance. And you can see how accurate it comes, because I will go over and under the resonance. And you can see there the response of the microphone.

That's it. Did you notice? I went over it. And then I went under it. Look, what an incredible signal. I shouldn't be talking. 344. I'm going to find the next one.

434, less than 1/2 a percent off. Next one, 552. 548, less than 1% off. The next one, 575.

Amazing what mother nature is doing. All the time making sure that there are pressure anti-nodes on all the walls. And only then, do you see the resonance.

The next time, 670. Yeah The next one. You may get bored, but Markos and I don't. We love this.

691, unbelievable. If you say physics works, that's the understatement of the day. 721. Next one, 792. Only about 1/2 a percent off. And now, I'll go only for the next one, 812.

What Markos also did, he scanned very slowly from 300 hertz to 850, extremely slowly, so you don't see the wave structure anymore. And he recorded only the amplitude recorded by the microphone. That means, the largest value then of the sinusoid there, the peak value.

And he made a plot of that. And you see that plot here which will blow your mind. Horizontally, frequency. This is the experiment you have just seen. It is presented in a different way. Here you see, as we scan with the frequency very slowly over it-- which we did, but now you see it all in one picture-- that when you hit the 344 hertz, that is the amplitude that is recorded inside by the microphone due to the resonance.

And you see the 434. These are the values that we measured this morning. 547, OK, it was 1 hertz off. 574, one hertz difference. 696, you see them all there. And you also get a feeling for the height.

We tried to understand the difference in height, but we are not acoustic engineers. And we contacted some people at MIT who are acoustic engineers. They didn't have a clue either. So we're not alone. It's very, very difficult, because now you have to understand where your microphone is located in the box. This, already, is so amazing, absolutely so amazing.

Then one afternoon, when we knew this was going to work, and both of those were very proud, Markos called me. He said, Walter, you have to come down. I'm going to show you something that you won't believe. I said, well, you've got tell me first, because--

He says, we can show the students transient behavior in a way that no one has ever demonstrated in class. What did Markos do? Markos said, when we turn on, all of a sudden, the sound, then it is like starting to drive all of a sudden a coupled oscillator, which has a resonant frequencies.

What happens when, all of a sudden, you start driving a coupled oscillator? Well, then you get steady state solution plus a transient solution. The system is going to oscillate in it's normal modes. And then, ultimately, the transient will die out, because the driver, of course, is the only one that survives. Because that's the frequency that you impose on the system.

And so what Markos then did, as a function of time, he chose a frequency-- we will show, shortly, what he has chosen-- but is somewhere around 580 hertz. That is the driving frequency. And here, he shows the amplitude of the microphone during a 1.5 second pulse. And he shows only the amplitude when it is at its highest and when it is at its lowest value of the sinusoid.

Imagine that at this moment in time, this is the zero value. This is the highest value. And this is the lowest value. This is a huge sinusoid, highest and lowest value.

But a little later in time, because of the transient phenomenon which is going to interfere with the driver, often you get even beat frequencies. A little later, it may be that the amplitude of the microphone is lower here and, therefore, lower here, too. And then ultimately, given enough time, this one and this one will no longer change, because this is now the steady state solution.

Now we have the amplitude of the steady state solution. But in the beginning, you will see the interference of the transient with the steady state solution. And so we were wondering what the transient frequencies would be that would start up all, of a sudden, when we start with this pulse.

And we had no clue. We had no prediction. We asked acoustics experts, they say, well, just try it and see what happens. And we're going to show you what happens.

And we have now all reasons to believe, that the moment that you turn on the 580 hertz, that this one-- which is only 7 hertz away-- apparently becomes very potent, not so clear why. And that one is going to set up a transient oscillation. But that transient oscillation, which ultimately has to die out, is going to mix now with our 580.

And it will cause a beat phenomenon. The two beat against each other. And so you will see this do this. Of course, it goes in time. You will see it do this, this, and then it will die out.

You will be able to see, for the first time, the transient phenomenon. And you even get an idea of how long it takes for the transient to die out. So you can even get an idea of what gamma is. And you read 1 over gamma is the 1 over e dk time of the transient.

And Markos is going to demonstrate it to you in person. He is unbelievable. He worked on this day and night. And he has the touch to do this. Markos, I think we don't need that anymore. Shall I turn that off?

MARKOS: Uh, we do need that.

WALTER LEWIN: Yeah, yeah.

MARKOS: But I need to--

WALTER LEWIN: You do your thing. Take your time. Ooh, you make it darker than Jeffrey would have liked. Here is the frequency that he is going to use as the driver.

You're now at 812. Oh, yeah, you're going lower now. Oh, yeah, you're going to roughly near 580, right? Or 581, or whatever. It's very important, again, that none of you talk, and that I don't talk. Because talking-- all the nonsense you see now is due to my talking.

MARKOS: There it is.

WALTER LEWIN: Can you just freeze it?

MARKOS: OK.

WALTER LEWIN: Now, from here to here is 1 and 1/2 seconds. He drives in 581 as a driver. And here you see the transient phenomenon. From here to there, we measured it, is exactly what you expect from the beats. In other words, if he has 581, and we measure here 575-- which is this resonance-- then the difference is 6 hertz.

And we measured that from here to here is, indeed, 1/6 of a second. 1/6 of a second, that means 6 hertz. The oscillation that you see right at the beginning, when we start the driver, that oscillation is 6 hertz. That means it must be the beat frequency between the 581 driver and a normal mode frequency, at which this system is going to fight back, so to speak. That must be the 575 hertz.

And so the 575 hertz and the 581 give you a beat frequency of 6 hertz. But the 575, of course, cannot survive. It dies out. And the 581 is the one that survives.

Now there's something else, which is an extra bonus. And that is something you haven't missed here. At the end, we turn, all of a sudden, the driver off. The moment that the driver goes off, the system can only oscillate in a superposition of its normal modes.

Now which normal modes, I do not know. And the experts may not even though. But we can look, at the moment that we turn it off, and what do we see there? We see a signal that is about 27 hertz. We've measured it. It's very close to 27 hertz.

I therefore conclude that that must be the beat frequency between the 575 normal mode and the 548 hertz normal mode. That difference is 27 hertz. And you see that beat phenomenon obviously dies out. It is a transient.

Tell your parents that you have seen this. Tell your friends that you have seen this. Have a good vacation. I'm leaving this afternoon for Amsterdam. I will be back Sunday morning.