8.03SC Physics III: Vibrations and Waves, Fall 2012
Transcript – Lecture 18: Interactions of Light with Nonconductors

PROFESSOR: So we have discussed earlier the boundary conditions for ideal conductors. And today we're going to aim at dielectrics. In conductors we couldn't have electric fields, and we couldn't have a changing magnetic field. The situation is very different in dielectrics.

Dielectrics is an ideal insulator, It has zero conductivity, and there are electric fields and changing magnetic fields are certainly allowed inside the dielectrics. One adjustment that we have to make, which we discussed actually earlier, that wherever we have epsilon zero in Maxwell's equations, we got to replace it by kappa of e epsilon zero, the dielectric constant.

And wherever we have mu zero, we have to replace this by kappa of m times mu zero, that is the magnetic permeability. And that has certain consequences for the speed of propagation of electromagnetic radiation. I want to stress, though that kappa of m, except for ferromagnetic materials, which are rare-- but kappa of m is almost always very close to 1.00000, except for ferromagnetic materials.

So we know that the speed of light of electromagnetic radiation in vacuum was this, and so now that's going to change. We're going to have to take this into account. So now we get v, which is now the speed of light, speed of electromagnetic radiation, which is now c divided by the square root kappa e times kappa m. And that is also c divided by n, n as in Nancy.

And n is called the index of refraction of the dielectric. The dielectric constant is a strong functional frequency. I've mentioned that earlier, that is that whole basic idea behind dispersion. If I stick, for now to light, or 5 times 10 to the 14 hertz, and I'll give you an example, which you have seen before, for water, kappa of e for light. That means for that high frequency, it's approximately 1.77. And so the index of refraction is about 1.33, but it is different for different colors. A little bit different, but it is different.

And if you take glass, there's many different kinds of glass, there are many different indices of refraction, we will typically take today a number of about 1.5. So now I would like to take a look at the boundary condition of a dielectric from one medium to another medium.

And so here we have this boundary. Let this be the direction, which I call the z-axis. And let this be the direction, which I call the y-axis. I have here medium 1, which has index of refraction n1. And here I have medium 2, which has index of refraction n2. I have radiation coming in, which I call the incident wave. It's a plane wave. Just this is only the direction of the k vector, but it is a plane wave.

And I call that i, that stands for incidence. What is going to be reflected I'm going to call r, that stands for reflected. And what penetrates into this median 2 I will give a letter t, which stands for transmitted into. we also call it refracted. But now you get reflection and refracted, that's confusing. So I give it this letter t. And so there is here a k vector. And this then is k of the incident radiation.

I call this angle theta 1, and then there will be a reflection at this surface, and so I'm going to put in here k reflected. And this angle I will call theta 3. I will show you shortly that theta 1 is the same as theta 3. And then here some of that radiation goes into the second medium, and I have here k transmitted.

The magnitudes of these two k vectors is, of course, the same, because lambda is 2 pi divided by k, but the magnitude of the k vector here is different, because the two different indices of refraction. So I can write down now the general equation for a wave incident, reflected, and transmitted wave. I do it as generally as I possibly can.

So here is the E vector of the incident wave. This has an x, y, and z component, as general as I can do it. It is E0 of i, which also has three components, one in x, one in y, and y and one in z. And then if you prefer, I could write it down into complex notation. Omega t minus k of i dot r, and this is the vector. But if you'd like to write it down as the cosine, that's fine too, of course.

And so this is the incident wave. I can now change the i to an r here, and to an r here. And I can change this i to an r here. This r has nothing to do with that r. That is the position vector. This of means reflected, and I can put a t here, and a t here, and a t here, and I would have three equations then. For the incident one, for the reflected one, and for the transmitted one.

And now comes a key point. That at z equals 0-- that means really at the boundary-- the three waves have to be in phase with each other, except for possible 180 degree phase change. And this should remind you of something that we've dealt with many times before. You have a string which is connected to another string, and at the junction, they are in phase.

The incident one and the transmitted one are in phase with each other, and the incident one and the reflected one. Either in phase, mountain comes back as a mountain, or out of phase 180 degrees. And the same situation you have here, and that means that this dot product for these three cases k i dot r, k r dot r, and k t dot r, at z equals 0 must be the same, except for 180 degrees. But the 180 degrees we can always deal with in terms of a minus sign.

If you massage this a little bit further, which is really not much more than high school algebra, but it is worked out in Bekefi and Barrett. If you massage this a little further, you will be able to demonstrate the consequence of the fact that these three dot products are the same. At z equals 0, on page 472 this is it worked out in great detail.

You'll find then that they theta 1 is theta 3, and that n1 times the sine of theta 1 becomes n2 times the sine of theta 2. We call theta 1, the angle of incidence. We call theta 3 the angle of reflection, so the angle of incidence is always the angle of reflection.

This law you may remember from your high school days if you have any physics is Snell's law, and that is a law that does not follow from Maxwell's equations. Many of you think that Snell's law is the consequence of Maxwell's equations. That is not true. This law also holds for water, it holds for sound, it is simply the consequence of the fact that there's a different velocity in the medium here than there is in the medium there.

Maxwell's equations will come up later, it has nothing to do with Snell's law. Vilibor Snellius was as a mathematician, he was Dutch, and he discovered this relationship empirically, which is quite an amazing thing by the way. In 1621 he discovered it for light at the time that it was not known that the velocity of light in water was actually different from the velocity of the speed of light in air. Yep?

AUDIENCE: Is that one theta?

PROFESSOR: Thank you very much. Thank you, Amanda, this is theta 2. Thank you, much obliged. So now I'm going to apply Snell's law for two different situations. I will first go from air to water. So here we have air, and we'll just assume that n is very close to 1.00. And then here we have water, so this is n1. n1 is always where you are, and 2 is where you are going.

And so we have water, and so n2 is 1.33. And so here we have light coming in. And this angle is then theta 1. And here we have light being reflected. And we know that that angle is also theta 1. And then here we have light, so this is the incident one, reflected one, and the transmitted one, and this angle then is theta 2.

And what you see here and what I will demonstrate is not the most exciting thing, but if you increase theta 1, then you will see that theta 2 also increases. That follows immediately from Snell's law. So you make this angle larger, and so this opens up, and this angle also opens up. But what is interesting is that you cannot make theta 1 any larger than 90 degrees. And when you do make say theta 1 90 degrees, theta 2 has a maximum. And that maximum follows immediately, of course, from Snell's law.

And in this case the angle theta 2, which is the maximum value that you can have, is then 48.7 degrees. So that's the consequence of the indices of refraction the way I have chosen them. And I will show you that. Now, what you could also do, which makes it way more interesting, you could also send light from water to air. So now you must be careful. If you are in water, that is now your n1. If you go to air, that is now your n2.

So I will be repetitive, but I do that for good reason. Now n1 is 1.33, and now this is air. And so now n2 is 1.00. So here is now light coming in. This now, by my definition, is theta 1. That's the angle of incidence. And so this now by my definition is the angle of reflection, which is also theta 1.

And then here, in this medium here, in air you will have light coming out at an angle which we now call theta 2. You can apply Snell's law to find these angles. And it is now immediately obvious that if you make theta 1 larger than 48.7 degrees, then there is no longer any solution for Snell's law because the sine of theta 2 cannot be larger than 1. And so now there's a crisis for nature, and what nature now does, nature reflects all the light off this surface. Nothing comes out any more in the air, and we call that total internal reflection.

And that happens at a critical angle, theta critical, which in this case is, of course, the 48.7 degrees. And so that's 48.7 degrees for this transition, and you'll find that value, the sine of the critical angle is n2 over n1. And this is only meaningful if n1 is larger than n2. There is no critical angle in this situation. There's only a critical angle that means total internal reflection if n1 is larger than n2.

We say that as physicists you have to go from an optical dense medium to an optically less dense medium. That is when you can have total reflection here at the surface. And so I would like to demonstrate that. We're going to do that here, going to make it a little dark for you so you see it better.

I have here a laser light coming out. Shall we turn that off for-- yeah I think you-- can you turn that off, it will be nice. Thank you very much. So you have here light coming from above. And it strikes the water, and you see that some of that light is reflected. And one of my major goals today, which we haven't reached yet, is to actually calculate the light intensity of that reflected light, and see how little is reflected.

But for now it is enough that you notice that the angle of incidence is the angle of reflection. And notice how much goes into that second medium of water. So that is that angle theta 2, which is clearly smaller than the angle theta 1.

And what I can do now is, I can increase the angle. I can increase the angle theta 1, provided that I get my hands out of the way. There we go. And this is not so exciting, you still see reflected light coming here, and you see that the angle theta 2 increases. Is this exciting? No, but this is a consequence of Snell's law, the breaking of light in water.

Now I do something that is way more interesting. I now have light coming from below. So we manipulated the light using a prism, but it comes in now from below. So it doesn't come in from above, but it comes in from the water here. And that is the situation I had on the blackboard there on the right side.

So now you see a reflection in the water, and you see some of that radiation coming out into the air. The intensity of the light in the air is actually quite strong. Notice that. But if now I make the angle of incidence larger than 49 degrees, then this, of course, will approach 90 degrees, and then will suddenly disappear, and 100% will be reflected. And that is what's coming up now.

I'm going to make the angle larger, larger. Notice that the light intensity of the beam in air is already decreasing. And now I reach that critical angle of 49 degrees. And now we have 100 percent internal reflection. All the light that came out from here, 100 percent is now reflected at that surface, because we have exceeded the critical angle.

And this, as you can imagine, plays a major and an interesting role in fiber optics. This allows you to transport images over thousands of miles without any loss of light intensity. What in a nutshell, it comes down to this. If I had here some kind of a glass fiber or a plastic fiber, index of refraction of say 1.5, which would have a critical angle of about 42 degrees. And if we send here light in, then it's very easy to arrange it that the angle there is larger than the critical angle.

And if that's the case, then it is for 100 percent reflected. But if it hits here, it's again larger than the critical angle, so it's again for 100 percent reflected. And you can send it around five times around the earth, and it will come out unaffected. So this is a very powerful way, and a lot of companies who try to make money out of this do use this for image transfer.

I have here an example of the 5,000 of these fibers. In this little metal hose are 5,000 small fibers, each of those fibers I believe is a 20th of a millimeter diameter. And we have a laser beam which is shining into it. There's a laser beam that should be shining into it now. And the light that comes out here-- you see it on the wall there. And I can put a knot in here. I can do anything with it I want, and it will not change the intensity. I can bend it over in this direction, there it is.

I could do anything with this I want to, and there is no-- here, look. Look how tightly this is wound. I will see it here at the ceiling. So that is the idea of fiber optics. And to show you that you can actually also send a message, that means an image, that's of course where the big companies are beginning to be interested in. That is, I want to demonstrate that too. I have a special message for you.

We have here this special message which I wrote this morning. And then we have here the fiber optics, a few thousand of these fibers, and you can put a knot in it. The message will go through here. The fibers are arranged in the following way, this is about two centimeters by six millimeters, and here are all these fibers. And they come out at the other end, of course, in the same sequence.

You don't scramble them. If you scramble them then you can't read it, which the military people use, by the way. In the old days they used it to transmit messages which would be decoded. But in any case, we will not decode it, so we have the same image that comes in here. The fibers come out in the same way there. And here we have a television camera. And then you can see very shortly there the message that I wrote for you.

Let me see whether we can get that there. There, you see the individual fibers. So each one of those little honey combs is a fiber. And here comes the message. I cannot read it because I have to concentrate, but maybe you can read it and let me know what it reads. Can I hear you? Oh boy, you can read it, eh?

AUDIENCE: Coming up shortly.

PROFESSOR: Well we knew that anyhow. So you see, it's not so bad the quality, and this is only a few thousand fibers. So this is actually a very nice way of showing you that you can transport that image. And as I do this, I hope you realize that nothing changes. You see no change there and here. I make a crazy knot in here. It all goes through, and there's no loss of the light intensity.

Now comes the key question that's really at the heart of this lecture, and that is light intensities. I now am interested in light intensities. We have an incident beam, we have a reflected beam, and we have transmitted beam. And now in order to get the light intensities, I now must use Maxwell's equations.

And the first thing we have to do is we have to derive the boundary conditions at the surface of the dielectric. We did that before for conductors, and now we have to do that for our dielectrics. So here we have the boundary. This is medium 1, index of refraction, n1, kappa of e1, kappa of n1. And here we have medium number 2, index of refraction n2. And here we have kappa e2, and we have kappa m2.

Needless to say, I want to mention it once more, that for all dielectrics kappa of m is 1. And so here is the boundary, and there is an electromagnetic wave. And when it reaches that boundary, the E vector happens to be in this direction. So this is E1. That means in that medium, 1. That's the total E vector of the electromagnetic wave coming in, highly time- variable frequency omega, but that speaks for itself.

And I'm going to decompose this now as I did before into two components, one that is parallel to the surface, we called that the tangential component. So I'm going to call this E1 tangential. Earlier I wrote down a t, but we already have a t today, so that's why I change it into tangential, not to get confused with the t that we had for transmitted. That's why I also changed the ideal conductor boundary conditions. I changed the t there into t-a-n, to avoid any possible misunderstanding.

And then here you would have the normal component. So this would be E1 normal. N is not so confusing. This n has nothing to do with that n, that means normal. I mean I still feel bad about my two c's last time, but there's not always a way that you can bypass using the same symbols. I could have done that last time, though.

And now in medium 2 when the wave gets through, there is an electric vector which is now E2. and I'm going to decompose that now also into one component, tangential. So this is now E2 tangential, and this is now E2 normal.

Now we have four Maxwell's equations, two divs and two curls. The divs means pill boxes, the curls mean closed loops. Last time I did two out of four. Today I'm a little more greedy, I will do one out of four, and you'll do the other three. None of them are difficult. It's just a straightforward 8.02 application of the divs and the curls. So I will do only one.

So I will do the curl of E is minus dB dt. It's my favorite one, it's Faraday's Law. It runs our economy. Whenever I see a chance, I use Faraday's law. And that means that the closed loop integral of E dot dl, dl is a little vector along the path that I take in the direction of my path. It is a dot product, it has to be a closed loop that now equals minus d phi. Let me put a B there. It's a magnetic flux change.

This is the magnetic flux change. Through a surface that is open and that is attached to that loop, you can choose any loop you want to. You could even choose any surface, provided it is everywhere attached to that loop. And then Faraday's law holds. It's an amazing law, truly incredible, always holds.

So here is going to be my path, no different from what I did earlier when we derived the boundary conditions for conductors. And I'm going to march around like this. And let this be here dz. I will avoid the letter L, because we already have a dl here, so I call it just z. And let this from here to here be just capital L.

And so now we're going to do the closed loop interval. So now follow me closely. I start here, and I go here. So little dl is in this direction. So this one here in medium 1 has no effect, because it's a dot product. But this one has an effect, dl and this component are in the same direction. But when I go down here, it's exactly the same value as a minus sign, because dl is in the down direction. So this contribution here

PROFESSOR: And here will exactly kill each other. So there's no need writing them down. And the same is true for medium 2. When I go from here to here, I get one component of the normal E vector with the dl, and here I get another one, and the two will cancel out each other, because it's a dot product.

So all I have to worry about is the path from here to here, and from here to here. And the path from here to here-- this component-- has no effect because it's at 90 degrees. So it's only the horizontal component that I deal with. And so I get that E1 tangential times L-- that is in this direction.

This is my positive direction of E. This is my positive direction of E. But now I go in this direction. Now my little dl is like so, but my E is now in the positive direction. So I get the product of L times this, but now I get a minus sign, because the L and E are in opposite directions.

So now I get minus E2 tangential times L equals 0. And so-- oh, it's not 0. Not yet. It is 0, but not yet. So now this is minus d phi dt. But now I'm going to make dz 0. That is always the trick we do. When we go to the boundary, we must make this 0, because we want to know exactly what changes at the boundary.

So we make dz 0, but once you make dz 0, the surface here-- that is, the surface through which I want to measure my changing magnetic field-- goes to 0. So this part goes to 0. And so now you see that this equals 0. And now we have derived our first boundary condition.

And the first boundary condition then for a dielectric is the following, that E1 tangential-- E1 tangential-- equals E2 tangential. It's not too different from a conductor. The only difference is that E tangential was 0 for a conductor, and so it was 0 on both sides.

So this one and this one look alike, provided you keep in mind that, for a conductor, it had to be 0. So now there are three Maxwell's equations left. They are in your court, and I will give you the results. But you can derive them, and it's really not difficult. No more difficult than what I just did.

So we now get that kappa e 1 times E1 normal minus kappa e 2 times E2 normal is rho s divided by epsilon 0. Now I want to warn you for something. You will say, ah, that is the same rho s that we have here, this is this surface charge density. Remember, coulombs per square meter.

The answer is yes. However, there's a huge difference. This rho s was time variable as to your electromagnetic wave came in. It changed like crazy with the frequency of the incoming wave. This one can not change, because it is an ideal insulator. There's no conductivity. So no charges can move on the surface.

In other words, this rho s is the surface charge density that somehow ended up on that surface that you may have put there yourself, and it will never change. It can also be 0. There's a big difference, but it is of course, physically, the same rho s, namely coulombs per square meter. But the function is very different.

It's very easy to do a transition whereby this is 0, but if you put a lot of charge at that surface, then this is, of course, a large number. And so that is the number 1, and then we have B1n equals B2n . And then we have the last one, and that is that B1 tangential divided by kappa m 1 is the same as B2 tangential divided by kappa m 2.

And you-- kappa 2 m? Yeah, OK, well, maybe I should-- let me use a more consistent notation, that I will first call the 2. This is the 2 and this is the m, and this is the 1 and this is the m. That's the notation I have there, too. So now you have the complete set of boundary conditions, and I have only done one out of three.

And now comes really the hardest part of this lecture. The goal is intensities of light. We want to know the intensity of the reflected light compared to the incident light, the transmitted light compared to the incident light, and that it leads to four equations, which are known as Fresnel's equations.

You have them on your handout. They're also on the web. And I will derive two of the four, and I will leave you with the other two. For that, I have to make some room. And I think this is really what we don't need anymore. So let's do it on the center board.

This is not done as thoroughly and as well as I would have liked to see in Bekefi and Barrett. They do not derive, in all its glory, all four Fresnel equations. Yet they are absolutely fundamental to an understanding of what happens with the light when it strikes from one dielectric to another.

So try to follow me closely. I'll try to go slowly, but if it confuses you, I can understand that. So here we have, again, the transition from one medium to another. So we have here medium 1 with index of refraction n1 and here we have medium number 2 with index of refraction n2.

We have incident radiation coming in here, and this angle is theta 1. We have reflected radiation, and this angle is also theta 1. And then we have radiation that goes into this medium and this angle is theta 1. Incident, reflected, transmitted. The radiation that comes in, the E vector, must be perpendicular to the direction of propagation, which is this.

It must be perpendicular. So it must be somewhere in this plane, perpendicular to the blackboard and perpendicular to this line. That's non-negotiable. Therefore, I can always decompose that if I want to-- and you will see shortly why we do that-- I can always decompose any electromagnetic wave that comes into this direction in two components.

One component, which is perpendicular to the blackboard, which we will give the symbol perpendicular, Ei perpendicular, and one component in the blackboard which we will call E of i parallel. We run out of symbols, so we try to do it all kinds of different ways to give names.

So this is a component of an incoming electromagnetic wave. It has an electric vector in some direction, it could even be circularly polarized light. I can always decompose that in two components, and I can always decompose it in one that oscillates like this, and one that oscillates like this.

I will now march through this problem, and I will now show you that I can predict what this component will come out here, and how this component will come out here. And then I will let you do the job on this component, how that comes out here, and how this component comes out here. And then the job is done.

Because you can always then here combine this component with this component, and that will tell you, then, what the transmitted wave looks like. And you could do the same here. And so that is my next task. I'm going to do it for the perpendicular component.

So once we have done it for these two components independently, we are safe for all cases, whether elliptically polarized light comes in, circularly polarized light, or linearly polarized light, we can then tackle every, any possibility. OK. Follow me closely. Here is an enlargement of the incident electromagnetic wave.

This is the direction that it comes in. I will only do the component perpendicular to the blackboard. So I'm only looking at this component, Ei perpendicular. That component must have an associated B vector. It is married to a B vector. The two go hand in hand. One does not exist without the other.

And the E cross B of that component must go in this direction. And the B vector there must lie like so. There's no other possibility. This must be the B vector of the incident beam. But it just so happens that lies in the plane of the blackboard.

Therefore, we give it the symbol parallel, because if the E vector is oscillating like this, the only way that the wave can propagate in this direction if the B vector oscillates like this. So it is the B parallel component that is married to the E perpendicular component. And I'm now going to-- this angle is theta 1-- I'm now going to decompose this one.

I will do it in green. Bi parallel times the cosine theta 1 and the vertical component is Bi parallel times the sine of theta 1. This is the one, when it reaches z equals 0-- that is this one-- that's why I'm doing it that way. I want to get a component that hits the surface that I can handle with Maxwell's equations.

And this component, when this component hits the surface here, that is also the tangential component that is this one. So you see now why I'm doing it this way. I need components that I can handle with the boundary conditions. So that is the incoming stuff. So now we'll do the reflective stuff.

So again, this angle is theta 1, and I'm only dealing with the perpendicular component, so here is that perpendicular component. So that is E reflected perpendicular. Now I want to know the B vector with which it's married. This is the B vector perpendicular in the blackboard that is associated with that E vector.

So it is B reflected, but it is parallel. It's in the plane of incidence. The plane of incidence is always defined as the incident light and the normal to the surface. So the blackboard is what we call the plane of incidence. And so I can now decompose that into a horizontal component, and I can decompose that into a vertical component.

I'm not interested in the vertical component, because I'm only going to use this equation. So I will only do the component in this direction. And so this one here is then B reflected perpendicular times the cosine of theta 1. Now, we have to go into medium 2. So there's one more to come. There it is.

And this angle now is theta 2, and the light goes like this. Electromagnetic radiation. I'm only dealing with E perpendicular, because you are going to do-- so this is the E perpendicular component of the transmitted wave. So this is E transmitted perpendicular. It is married to a B vector.

And that B vector must be in this direction to make sure that E cross B is in the direction of propagation. So this is B transmitted parallel, and the horizontal component is then B transmitted parallel times the cosine of theta 2. All of this we need. But we are close.

I'll give you a minute to digest it, and I will erase this so that we stay on the center board. Now I go to the boundary conditions. Before I go to the boundary conditions, there's one thing that I want you to remember, and that is that there is a relationship between B and E. They're married to each other.

And the ratio of their amplitudes is c. Well, not quite. It used to be c, but now it's v, because we're now working in dielectrics. So, since B is married to E, we can always write down-- and we're going to need that-- that the magnitude of B parallel must always be the same as the magnitude of E perpendicular divided by v, which is also the magnitude of E perpendicular divided by c times n.

That is the index of refraction. And we're going to need this. This is completely kosher. This is what we knew from Maxwell's equations way before we dealt with this. We're close. At the boundary condition, I now must make the E vector or the tangential component on the left side the same as the tangential component-- not on the left side, on the top side-- the same as on the medium 2.

Medium 1 and medium 2 must have the same. But medium 1 has two. Medium 1 has this component when it reaches here, and it has this component when it comes-- sorry-- this component when it reaches here, and it has this component when it comes back. So I always must add the incident and the reflected E fields, of course.

So we get E incidence perpendicular plus E reflected perpendicular must now be E transmitted perpendicular. Of course, transmitted perpendicular component means in this plane. That is that tangential thing. All these components are tangential. You see now my problem with T-A-Ns and Ts. It's awkward.

But this is the perpendicular component. So that's a given. That's the boundary condition. So now I go to this boundary condition. And just for simplicity, I will leave the kappas out, because the kappas are 1 as far as I'm concerned. If you want to put them in, be my guest. What are those components?

Well, I already alluded you to that. This is one of them in medium 1, this is another one in medium 1, and this is one in medium 2. And the two in medium 1 together must be the same as the one in medium 2 to meet the boundary condition. So I'm going to write this one down.

So I'm going to get that B of i parallel times the cosine of theta 1-- that's in this direction. Then I have to subtract this one, because it's in the opposite direction-- minus B of r parallel times the cosine of theta 1. That is on one side. Plus and minus must now be the same as B transmitted parallel times the cosine of theta 2.

And now, I'm going to use this. I want to convert all my B's into E perpendicular. And once I have done that, I'm spitting distance away from knowing what the ratios of those E vectors are, and that's my goal. If you know the ratio of the E vectors over the reflective one and the incident one, you know the ratio of intensities, because that's the Poynting vector where you get squares and things like that.

So I'm going to replace now the parallel components by the E perpendicular divided by v, or I can do it E perpendicular times n divided by c. So each one of these terms now is going to be replaced by this. They all have the c downstairs, so the hell with the c's, I will not write the c's. So there we go.

So now we get the E incident perpendicular times n1, because this is medium 1, times the cosine of theta 1 minus E reflected perpendicular times n1 times the cosine of theta 1 equals E transmitted perpendicular times n2-- because I'm now medium 2-- times the cosine of theta 2.

And I have killed all c's. And now we're practically done. Look at this equation. And look at this equation. If you know n1, n2, and theta 1, Snell's law gives you theta 2. So now you have two equations with three unknowns. This intensity, this electric field, this electric field, and that electric field.

So what you can do now, you can find the ratios of these electric fields, and that's the only thing that is meaningful, because clearly, you can make the intensity of the incident one as strong as you want to. So by combining this equation with this equation, it is clearly a high school exercise to now find what is E transmitted divided by E incident, of course, for the perpendicular component.

I admit, I've only done the perpendicular component. You're going to do the parallel component. And I can also find now what the E vector of the reflected one is divided by the E vector of the incident one, but only for the perpendicular component. And that is my goal.

Once I have ratios of E vectors, I can calculate light intensity ratios, and that's why when we were looking at the demonstration there, when I wanted to show you Snell's law, there's no way that we had to calculate why there was so little light being reflected in that first demonstration. And now you take your handout.

Take your handout and look at page 1. It's also on the web. And there you see, in all glorious detail, the four Fresnel equations, two of which, I am within spitting distance to convert this to these two ratios. It's no more than another three minute job. But it is so trivial that I don't want to do that.

At the top, you see Snell's law. So that means, you tell me what n1 and what n2 is, and you give me theta 1, I will immediately tell you what theta 2 is. But that's non-negotiable. That's easy. Once you know n1, theta 1, n2, theta 2, these four equations will tell you what the ratios are of the incident electric vector over the reflected one.

But there's a parallel component and there is a perpendicular component. And so the only two that I derived, or I came very close to deriving, that was this one, and that was this one. I was within spitting distance of these two. Had I made it one step further, you would have seen this equation coming out.

The fact that I put a 0 here means that this is-- I make it the magnitude of the E vector. And I give it a shorthand notation, a little parallel, a little r is the ratio of r over i, and the t is the ratio of t over i. Just a shorthand notation. I give you two columns. They are the same, except that one is sometimes easier to use than the other.

This is extremely consumer friendly, though you may not think that. It is extremely consumer friendly. You can not go wrong if you simply apply these equations. No one in the world could just, in five minutes, derive these equations. It's a lot of work. I spent 20 minutes on one, and I only came within spitting distance.

I didn't even finish it. So it's a lot of time. Once you know what they are, you just use them. You use them. I give you theta 1, I give you n1, I give you n2, you calculate theta 2, and out come these ratios. There are things in here which are amazing, which are very opaque. They're not very transparent. And I'm going to work with you on that after the break.

I just want to know you have them. They are worth gold. They are not in Bekefi and Barrett. Bekefi and Barrett only mentioned 1 and 1/2 or so for a very special case. This is the complete set. And so I would've thought-- but let me check my notes-- I would've thought that this is an ideal moment for a break.

And then after the break we will try to digest some remarkable consequences of these equations. And so, if you can help me handing out the mini-quiz number nine, then-- the idea was, but I could be wrong, that you can answer this in 30 seconds. That was my goal. And if it takes you any more than 30 seconds, you might as well not do it. Believe me. You will see why.

So if you can help me handing it out, wait until the whistle blows. Can you help me dealing it out? And you? I hope you all have a handout, right? There are handouts at all three entrances.

So, as I promised, I now want to give you a little bit of insight, but I will only scratch the surface of the consequences of Fresnel's equations. And let's first do a case whereby theta 1 is theta 2, and is therefore 0. Snell's law tells you if theta 1 is 0, that theta 2 is also 0. We give that a name. We call that normal incidence.

It then so happens-- which is by no means obvious when you look at these equations-- that the first two equations get you exactly the same result. And so you'll find then-- and I'll just write that down in terms of Er over Ei. So Er over Ei, regardless of whether it is the parallel or the perpendicular component, is the same. It's n1 minus n2 divided by n1 plus n2.

And you'll find that Et divided by Ei-- notice we always compare with the incident beam-- that E t divided by E i is then 2n1 divided by n1 plus n2. And now I take a special case, which is on your page number two, that is to transition from air to glass. That is the upper panel in going from air to glass.

So air to glass. So n1-- where you are is always n1. Never forget that. So n1 is then very close to 1.00, and n2 is very close to 1.5, if we take 1.5 for glass. We'll find, then, that for this case this is minus 0.2. What is the meaning of the minus? Come on. We've seen that with strings.

AUDIENCE: [INAUDIBLE]?

PROFESSOR: What?

AUDIENCE: It's the opposite way.

PROFESSOR: It means that there is 180 degrees phase flip. That's all it means. So it means that the E vector, if it is in a positive direction, as it arrives, it reflects, it flips over. So that minus sign is what we see very often when we hit boundaries where there is a change. So the minus sign is 180 degree phase flip.

And this is plus 0.8. So there's no phase flip in going from medium 1 to medium 2. Of course not. That was the same with the string. If you have a string which is connected to another string, if the junction goes up, then the wave that goes into the second medium also goes up. There is no way that it could go down, remember? It is the reflected one that can change phase, but not the transmitted one. So that's not too surprising, by the way.

All right. If now you look at that panel, that upper panel, which is a plot of Fresnel's equations, four Fresnel equations have been plotted here for the transition from air to glass, horizontally theta 1, and vertically is the ratio of the E vector in those four cases: reflected parallel, reflected perpendicular, transmitted parallel, and transmitted perpendicular. And look at theta 1 is 0. What do you see there? You see minus 0.2, which I just calculated for you, and plus 0.8 at an angle of 0 degrees.

Now clearly, we can also calculate the light intensity now. That was my goal. I don't give a damn whether that light changes phase by 180 degrees or not. That's a later issue. I want to know light intensities, and light intensities means I deal with Poynting vectors, E cross B's. Well B in magnitude is always E divided by v. So whenever you compare light intensities, all you have to do is take E squared and then you divide it by v.

But since the reflected one and the incident one are both in the same medium, I don't even have to worry about v, because v is the same in both cases, which is not true when I go from medium 1 to medium 2. So therefore, I can now calculate for you the intensity, I stands for intensity, of the reflected wave divided by the intensity of the incident wave. And I claim that that is the E vector reflected, divided by the E vector incident squared. That is all it is, because the velocity in that medium is the same for reflected and incident waves. This takes everything into account. Mu zero is taken into account, B is taken into account, the fact that we have a square, that's the result of the B, remember it's E cross B.

And so this is 0.04. That's a famous number. Every astronomer knows that number. That means when light strikes glass surface at normal incidence, 4% comes back. There's nothing you can do about that. 4% of that radiation is lost. If you want to get it through a lens, there's 4% that comes back.

So therefore, in order to conserve energy, I of t divided by I of i must be 0.96, right? Well, you do that, the answer is yes, it is 0.96. But I want you to calculate it, because now you must take into account that the velocity in medium 2 is different from the velocity in medium 1. If you overlook that, you will never find 0.96. If you take this number and you square it, you don't get 0.96. But you cannot do that, because in medium 2, the B vector is E divided by its own v, and that own velocity is different from the velocity in medium 1. So I've set you off on the right track. This is still correct, but you cannot just take this ratio.

Very well. There is something so incredibly bizarre about Fresnel's equations. Look at equation number one, and look at the right column. Suppose you make theta 1 plus theta 2, suppose you make that 90 degrees. I will show you, it's very easy. Then that thing goes to 0, because the tangent of 90 degrees is infinitely high, so r parallel, the way I have called that there, r parallel goes to 0. That is amazing. So that means there must be one angle theta one if I hit it just right that none of the parallel component is reflected. So the only thing that is reflected is then the perpendicular component. Now it may not be reflected for 100%, but some of it is reflected. Therefore, the reflected light is now 100% polarized.

There's only one and one angle for which that happens, and that angle has a name. We call that the Brewster angle. And the tangent of theta Brewster-- so that is that theta 1-- the tangent of theta Brewster is n2 divided by n1. I want to remind you, 1 is always where you are and 2 is where you're going, and I can easily show you that this is an immediate consequence of that. I will prove it to you.

For one thing, if theta 1 plus theta 2 is 90 degrees, then the cosine of theta 1 is the sine of theta 2. That's high school. But n1 times sine theta 1 is n2 times the sine of theta 2. That's my countryman Snell. And if you combine these two, in 30 seconds you can prove this. And so this is the angle that then will automatically give you that theta 1 plus theta 2 is 90 degrees. And that, of course, is so wonderful to demonstrate, and I can demonstrate that in various ways.

And look at that panel, that upper panel that you have on your handout on page two, and look at that Brewster angle. If you go from air to glass, so now we go again from air to glass-- and we know what n2 is, n2 is 1.5 and n1 is 1-- then you'll find that theta Brewster, which is this equation, is 56.3 degrees. Of course, that's only correct if n2 is exactly 1.5, which it almost never is. And if you use Snell's law, then you'll find that theta 2 is 33.7 degrees. Of course, the sum must be 90, and it is 90 if I did not make a mistake.

And then you get the crazy situation that r parallel is 0, and so now you can calculate what is now r perpendicular, for which you have to go to your Fresnel equations. There's no way that anyone can immediately see what that is. So now you have to go to this equation and substitute in here the values for theta 1 and theta 2 and for n1 and n2, and you'll find then that r perpendicular-- and I'll just make sure that they have it right-- is minus 0.385. That's what it is.

So if we want to know intensity, all we have to do is square it because we are still in the same medium, the reflected medium and the incident medium is one and the same medium. So 0.385 squared is about 0.15. What this means is that if you have an E component perpendicular that comes in, then 15% of that will be reflected, and 0% will be reflected of the parallel component. That means if unpolarized light comes in, unpolarized light means that half of it is perpendicular and half of it is parallel, that 7 and 1/2% of the incoming unpolarized light will come out 100% polarized in this direction. Imagine, you now have a wave. You start with unpolarized light, you shine it onto a glass panel, and if the angle of incidence is close to 56 degrees, 7 and 1/2% reflects of the total intensity, and it is 100% polarized, and it is 100% polarized in the direction perpendicular to the plane of incidence.

And that is what you see there on that panel on this plot. You see at this angle, the Brewster angle of 56.3 degrees, you notice that r parallel goes to 0. And therefore, everything that comes back-- now of course, some of it penetrates into medium 2, I didn't calculate how much that is. But here, it's the minus 0.385 that I just calculated, that is this component. That is reflected. And you can imagine that this is, of course, great stuff to demonstrate.

Before I demonstrate that, I want you to appreciate that if you go from glass to air, that there is also a Brewster angle. There's a different one, of course, because then n1 and n2 flip roles. And so if you look at the bottom panel of your page two, then you will see that there is a Brewster angle at 33.7 degrees. So that means if you are now inside glass and you bounce the light off the medium with air, that then, at an incident angle of 33.7 degrees, that then, you can create also 100% polarized light.

Notice also that in this case, but only in this case, there is such a thing as a critical angle. Remember earlier I mentioned you only have a critical angle of total internal reflection if you go from an optical dense medium to a less dense medium. That is the case where you go from glass to air. There was no such thing when you go from air to glass, and that's what you see in this panel. You see that the moment that you hit this critical angle, that 100% of the radiation is totally reflected. You see this is 1.0 and the r parallel is 1.0. Nothing penetrates into the second medium.

All right. So the Fresnel equations, I want you to appreciate, are unbelievably powerful, they are extremely consumer-friendly, because you can now calculate light intensities of reflected and transmitted light for any angle of incidence theta 1. All you have to know is what n1 and what n2 is. Whatever the incoming light is, if it is elliptically polarized, you decompose it into an E perpendicular and an E parallel component, and then you turn the crank, four equations. And then you get the reflected component, and you get the transmitted component. You can do it for any incoming light, whether it's circularly polarized, or elliptically polarized, or linearly polarized.

And you can now also evaluate to what degree the reflected light is polarized. So far we always said, 100% polarized or it's unpolarized. Life is not that way. There is also something, of course, of being partially polarized, and I'm going to give you now one possible way of defining the degree of polarization. The degree of linear, I stress that, of linear polarization. And I will define that for you. I give it a capital letter V. I don't think that's universal, I just gave it a letter V, the degree of polarization. And I define it as the intensity of the parallel component minus the intensity of the perpendicular component divided by the intensity of the parallel component plus the intensity of the perpendicular component, and I take the absolute value. This number can be anywhere from 0 to 1. If it is 0, it's totally unpolarized, and if it is 1, then it is 100% linearly polarized.

So you can have a situation that the answer is 0.5, then it is partially polarized. It's not completely unpolarized. That means one of these components, then, dominates, and then you have partially polarized light. And when you have your linear depolarizer, you can actually tell that as you rotate it around, there is a change in light intensity. Only if this is 1 is it 100% linearly polarized.

OK, a small test, but this test may come too early for you, but just see how you react to this test. You know that there are some glasses which are made of polarized polarizers. And so here is one of those sunglasses, and here is another one, and here is another one. And you are the manufacturer, and you have to decide on the direction of polarization of the linear depolarizers. Would you do it this way, or that way, or this way, or any other way? And if so, why? And now think about it. Think about it. What would be the goal of having sunglasses that are polarized? What would be the goal? Why would you want that? Just because it looks sexy and it is in? No. There is really a reason for that. What is the reason? Excuse me?

AUDIENCE: So I'm a fisherman. If you look into the water, it's all glare. But if you wear polarized sunglasses, you can see the fish underneath.

PROFESSOR: The goal of polarizers is that light that has been reflected, for instance, off water, when you drive your car and there's water, or ice, that the reflected light is linearly polarized to some degree. Now if the light comes in at the Brewster angle, it would be 100% polarized. That would be very special. But suppose the sun is there, and I'm driving my car here, and the sunlight hits there water, and it comes up in this direction. Then it is highly polarized at the very minimum, partially polarized in the perpendicular direction, perpendicular to the plane of incidence, this is the plane of incidence. So it's polarized like this to a large degree. So how do I want my sunglasses? Like this, because now I killed this component, and that's exactly the way you make them. So this is the correct answer.

And that, of course, calls for a demonstration. I have here a light beam, and here, I have a pane of glass, and the light reflects off that glass, and we have aimed it roughly at the Brewster angle. You can't do that, of course, exactly, because the beam is not so well-defined. And so when I look here at the light that reflects off that plane, it's painful. I'm not even joking. It is enormously annoying. There's a tremendous amount of light that comes as it reflects from it. Who has seen The Matrix? You've seen The Matrix? Remember me? Now I look, and I have no problems. I can just see, there's an astronaut walking on the moon.

Now you may say sure, you can say that, but what do we know? OK, I take that. I accept that. The light that comes back is polarized in this direction. This is the plane of incidence, and it's almost 100% polarized in this direction. OK I'm holding this in front of my face. This is the direction of polarization of this sheet. Look at me. Now look at me. Big difference, right?

What I'm doing now, this is a super pair of sunglasses. It's very expensive, specially made for me. So this one is super. It cuts out this component. And when I wear them like this, the sunglass, it isn't doing anything. I mean, you could look at the wall there, you see the same phenomenon. So that is what sunglasses are doing for you. And this is an amazing thing that you cannot even read what is under that glass plate. You cannot even see what it is. But the moment you do this, you have no problem. So that is the power of the Brewster angle, and then the linear polarizer that takes care of it.

I would like you to take out of your envelope, if you have it with you, one linear polarizer, and all I want you to do is relax for one minute and look around in this room. And we have objects that are made of metal, and we have objects that are made of glass. If unpolarized light strikes metal, it will not become polarized. If unpolarized light strikes a dielectric, it can become polarized to some degree, in an extreme case, for 100%. So if you take your linear polarizer and you rotate it in front of your eyes, I want you to see that when you look at dielectrics, like the glass, or varnished surfaces, or leather, that you may actually see a change in light intensity, which you will not see when you look at metal. If you do you see it at metal, it means that the light that struck the metal was already partially polarized. And then, of course, it remains polarized. So metal, an ideal conductor, doesn't change the degree of polarization. It keeps it as it was, but it is the dielectric that can change it.

AUDIENCE: [INAUDIBLE].

PROFESSOR: Excuse me?

AUDIENCE: [INAUDIBLE]

PROFESSOR: Oh, you have almost the Brewster angle? But it's only for you, not for other people, depends on where you're located. So you're saying if you look into the direction of the projector there, that you-- it doesn't do much for me, but because the angle of yours is different from me. Yeah, you can see the effect. Yeah, yeah, yeah. I can see it now very well. Yeah, absolutely. One of those lamps goes out. Boy, getting dark, yeah. That's what you had in mind? Thank you very much. But look at some objects here. Look at this, for instance. I bet you, if you rotate this, that certain lights change their intensity. And it also looks at varnished surfaces. Anything that is dielectric has potentially this possibility.

And now comes the ultimate demonstration of a Brewster angle. What I'm going to do now is create out of 100% unpolarized light, I'm going to create 7 and 1/2% linearly polarized light, the famous 7 and 1/2% that I mentioned to you earlier. And the way I'm going to do is as follows. We have here a light beam. Actually, it goes in this direction, but big deal. And this is unpolarized light, and I'm going to put in here one, and only one, pane of glass. Here it is, one pane of glass. And I bounce that light unpolarized off this surface, and I'm going to project it here. And when I do that, then the light will go like this. If this angle is near 56 degrees, then the light is polarized for 100% in the direction perpendicular to the plane of incidence. That means in this case, since this is the plane of incidence, that is the incident light and the normal, it will be polarized in this direction. And as I rotate that pane of glass, the reflected beam will move on the board here, and when I approach the Brewster angle, I will show you that I have it 100% polarized in this direction. When I'm over the Brewster angle, it's partially polarized, when I'm under the Brewster angle, it's partially polarized, but I will come very close to that.

Once I have done that, you're going to get even more for your money. I'll show you something even more remarkable. Most of the light will go through. For one thing, the parallel component doesn't come out at all, so that means it goes through. And of course, most of the perpendicular component will still go through, because only 7 and 1/2% of the total radiation comes out here, so most of it goes through. Well, I put another plane here, and then I put another one, and another one, and another one, and another one, and Markos and I don't even know how many, but I think there are 200 in here. And every time that the light strikes that surface at the Brewster angle, whatever comes out, if anything comes out, it's polarized in that direction. Every time it's polarized in this direction.

What do you think then happens with the light that ultimately makes it through? 100% polarized. I have sucked out every perpendicular component, and so what remains then is the parallel component. So imagine what I have done now. I start with 100% unpolarized light. With one reflection, I can get 7 and 1/2% linearly polarized, but with hundreds of reflections, without being too specific, I can get 100% polarized light here, which you're going to see there, and I get 100% polarized light here. The light that comes out here is polarized in this direction, that's the perpendicular component. The other component is in the plane of incidence. Remember one is perpendicular, the other is incident. In the plane of incident means this is the normal to the glass, this is the incident beam. So this is the plane of incidence. It's in the plane of incidence, so it's polarized like this. So when I use those 300 or 400 glass plates in a row, the light that you see here will be polarized like this. And I can show you that because I have a big polarizer, remember?

So if we can lower the screen. Markos, would you do that? Then I will turn on this. There it is. We'll make it very dark, or not completely dark. We're going to set it at TV level. All right, so first of all, this only goes through one plane, one pane of glass. You can't use your polarizers, because whenever the light reflects off a screen, it loses its degree of polarization, at least to a high degree. So this is the light that goes through the plane, and 7 and 1/2%, if I'm near the Brewster angle, comes out here. So 93% percent goes through, about 7 and 1/2% comes out here if I'm close to the Brewster angle. And I may be actually close here. Actually, let me be purposely not be close. So now I'm not very close.

So here, you see it is already polarized to some degree, but you see clearly light behind it. Now I'm going to go to the Brewster angle, and now I kill it. I'm very close to the Brewster angle now. This light is now almost 100% polarized. This is the direction. The direction of polarization is like this. That is the direction of my polarizer, and here I kill it. And that, of course, is hardly polarized at all. I can show it by rotating it, it's hardly polarized at all. It has lost some of its perpendicular component, but not so much that you can really see the difference when I rotate it.

But now, I'm going to make the light go through my hundreds of panes, and that is going to be even more interesting. I think that is this one. Yes. OK, so now I do the same thing, except I don't have one pane of glass, but I have many. A few hundred, Markos? We don't want to open it, you see, when you open it, you break it, so we have no idea. But the idea is, we agreed we are close to the Brewster angle here, right? So this now is polarized in this direction. So that means if it is polarized in this direction, I should be able to kill it like this, and I can. This is when I let it through, and this is when I kill it, and I claim that this is polarized now in this direction, 100% polarized. So that means I can let it through like this, but I can kill it like this. There you go.

So now, I have made 50% of my incident light goes out there, 50% comes out here roughly, 100% polarized, 100% polarized. This is an absolutely remarkable thing. So we now have a method to convert unpolarized light into 100% polarized radiation, component like this, and a component like this. And one of the amazing things of physics is that it makes the impossible possible. And that is the power and the beauty of physics, and that's why I love physics. To make the impossible possible. It's like the impossible dream. It's also to make difficult things easy. Physics makes difficult things easy. See you Thursday.