**8.03SC Physics III: Vibrations and Waves, Fall 2012**
**Transcript – Lecture 20: Interference**

PROFESSOR: Today we're going to talk about interference of electromagnetic radiation. I will start, as a warm up, with the famous historical experiment which was first done by Jung in 1801. By that time, the issue whether or not light was waves or whether it was particles were still unresolved. Newton always wanted light to be particles, but the Dutch physicist Huygens wanted them to be waves, and the issue is unresolved.

Let us agree that if light are particles and you have a screen that, say, has two openings and you throw particles through there like tomatoes, tomatoes are particles, and you collect them here, then those tomatoes that don't get stuck on the screen but that make it, would form a pile here of tomatoes. And they would form a pile there of tomatoes. That's very typical for particles.

But the situation is different when we deal with waves, because the moment that you have waves coming in-- say, for instance we have thin waves coming in like this-- then the wave can go through both openings simultaneously and that changes the picture quite dramatically.

It was known, of course, already in the 17th century that if you have water waves going through a small opening-- so here are water waves moving in like so, and here is also water-- that what you see coming out there are circular waves. The waves then look like this, and they're propagating out in a circular fashion. And if the velocity of the waves here is the same as the velocity there, then the wavelength here would be the same as the wavelengths there. But if the velocities are different, of course, you will see a difference in the wavelengths.

So Huygens, my countryman, suggested in the 17th century the idea which is now known as the Huygens Principle that we can think of this in a very different way. Huygens, by the way, is difficult to pronounce. Anyone of you who knows how to say Huygens correctly is also Dutch because you miss the R in your language and you missed the [HACKS] in your language. The combination of R and [HACKS] is a complete killer for you guys. You don't get extra 8.03 course credit if you can come to my office and say Huygens, but it will certainly put you in the Dutch category.

So in the 17th century then, Huygens came with an idea which was later amended by Fresnel in the 19th century, and is now known as the Huygens-- I will pronounce it your way-- the Huygens-Fresnel principle, which works as follows. If we have a plane, monochromatic wave and it's inserted on a screen with an aperture-- so this would be an aperture, an opening. Here you would have two apertures, two openings. Then the Huygens-Fresnel principle states the following: all points in the aperture plane may be thought of as secondary point sources of spherical waves, and the point sources replace the real source-- which is flooding the screen-- and the screen itself is a perfect absorber of the radiation falling upon it. And so I will repeat the Huygens-Fresnel principle, which I will need today. All points in the aperture plane may be thought of as secondary point sources of spherical waves. In the case that we have water, this is a two-dimensional surface they will be circles. But when you deal with light, you can think of them as three-dimensional-- that mean spherical-- waves.

Now, the Huygens-Fresnel principle is very powerful, though there is a wide range of opinion to its scientific merit. In a very famous book, The Principle of Electrodynamics by Melvin Schwartz, I read the following, and I quote verbatim from his book. He says, Huygens' principle tells us to consider each point on a wavefront as a new source of radiation and add the radiation from all the new sources together. Physically, this makes no sense at all. Light does not emit light. Only accelerating charges emit light. Thus, we will begin by throwing out Huygens' principle completely.

Later, we will see that it actually does give the right answer for the wrong reasons. I will proceed my lecture today to get the right answer but perhaps for the wrong reason. Suppose now we have an electromagnetic wave. I'm particularly thinking of light. And I have here an opening and I have here an opening, and plane waves are coming in from the left moving with the speed of light. And let this opening be A and let this opening be B. And here, the center of the two is O.

This could be a circular opening, it could also be a slit. Most of the experiments I will do today are with slits, which are perpendicular to the blackboard. So they are very narrow openings. Imagine now here a point, P. So the wave from point A will reach that point P, but the wave from source B will also reach that point P. So the electric vectors there are going to be added, of course, factorially.

Imagine now that BP minus AP, imagine that were 1/2 wavelengths. Then what you get is that the mountain of the E vector here will coincide with the valley of the E vector from the other one, and so you will get the situation that light plus light give darkness. We call that destructive interference. The wave from here and the wave from there would be 180 degrees out of phase.

If you take all the points for which the difference is 1/2 lambda, that is a hyperboloidal surface. It's like a bowl. Not only in the blackboard, it also comes out of the blackboard. It's like a bowl. Remember from your high school days that if the sum of the distance AP and BP is a constant, then you get an ellipse. But if the difference is a constant, you get a hyperbola. So you will get a hyperbolic surface, and everywhere on that surface BP minus AP would then be 1/2 lambda. And then you would get the amazing consequence that light plus light will give darkness. Destructive interference.

Tomatoes don't do that. One tomato on top of another tomato does not get no tomato. That is distinctly different for particles. Of course, there are also hyperbolic surfaces for which there is constructive interference. In other words, if I make this difference n times lambda, whereby n is either 0 or plus or minus 1 or plus or minus 2, then I would get surfaces for which we get constructive interference. Then the phase difference between the two ways is either 0 or 2 pi or 4 pi. So we have surfaces with constructive interference and then we have surfaces with destructive interference. And that's going to be at the heart of what I want to discuss with you.

So I will start a new drawing. I think we don't need this anymore. I will now have a screen with two narrow openings. They could be slits, as I said before. So here is one opening and here is the other opening, and that the separation between them be d. And the upper one we can call source number 1 and the other one source number 2.

Suppose now that we are looking at this at a distance which is very far away. Very, very far away. And where you are located in space, seen from this point, is an angle, theta. So the wave from this source will reach you. And since that point is very far away, these lines here are of course parallel to each other. Very close to parallel because it's very far away.

The wave that comes from this one has to travel over a larger distance than the wave that comes from this one. And that difference can easily be expressed algebraically, if this angle is 90 degrees. The difference is this much. That is how much further this wave has to travel than this one, and then they factorially add very far away at the location where you happen to be.

This difference here is d sine theta. That's immediately obvious. If this angle is theta, this angle is also theta. So the path difference is d sine theta. And that translates into a phase angle difference between the E vectors, theta. First of all, d sine theta, which is the path difference, this is how many wavelengths you can fit on that path difference. So I divide it by lambda. And for each wavelength that I can fit on there, the phase difference is going to be 2 pi. And so they phase angle, the difference between the wave from here and there, is therefore 2 pi times d divided by lambda times sine theta.

And so now, I can set the conditions for constructive interference. What I want is I want delta to be a multiple times 2 pi. I already wrote that down earlier, but I will do that again. n can be 0, it can be plus or minus 1, it can be plus or minus 2, et cetera. If n is 0, by the way, you are right at this plane, which comes perpendicular out of the blackboard. Clearly, when you're on this plane when n is 0, the distance to any point on the plane from source number 1 and source number 2 is obviously the same.

You can also write down that d sine theta-- and I'll give it the little n, which makes connection with this n. It has nothing to do with index of refraction. That d sine theta is now a multiple times one wavelength. That's saying the same thing. So this statement here is the same as this statement. There's no difference between them. And now we can write down the condition for destructive interference.

So now you want that delta is pi, 3 pi, 5 pi. So now we have delta equals 2n minus 1 times pi. And if you want to write it down in terms of the d sine theta, then you get that d sine theta of n, which is that n that you see there, is now 2n minus 1 divided by 2 times lambda. So this is at the heart of the idea of the interference, in this case, of two of these sources. Two slits or two circular openings. Let me check that we have that right. d sine theta, the n makes connection with the n, equals n lambda. Delta is a multiple of 2 pi. Destructive interference, we get 2n minus 1 times pi, yes, I can live with that.

Now I want to be quantitative because I'm going to do a demonstration very much with the numbers that you're going to see now on this blackboard. I'll give this back to, although you may not need this. We now have the two openings all the way on the left here. And they are so close together that you can't even separate them anymore. This is the point where the two openings are. We are going to look on the screen, which is far away. So here is a screen, which is going to be that screen. And we want to see how the constructive interference-- that means light shows up on the screen-- and how the destructive interference-- that means darkness shows up on that screen.

So here are these two openings. So light comes in from the left. And let this distance be L. And so I want to know if I look on an angle theta, as seen from the slits, I want to know what kind of pattern I will see. Let's call this x equals 0, and we call this positive x and that's negative x.

You could also say that the sine of theta, if you want that, is approximately x divided by L. But that's only true for small angles of theta. And with this kind of interference, we sometimes have angles of theta which are not small at all. So I want to warn you that this is not something you can use universally, but today you will see that we can use it.

I start now with red light, lambda, is 600 nanometers. That's the light I will use. So I use monochromatic light. We will have a distance to the screen which is about five meters. The set up is here, this is about five meters, and we have red light from a red laser, which is a little larger than 600 nanometers, but I just want some round numbers.

And the separation here, d, between the two slits is 1/4 of a millimeter. You may think that is rather small. Well, you will see that that's what you need. You need very small values of d to even see this phenomenon. So now let's evaluate what n is, let's evaluate what delta is, let's evaluate what the sine is of theta of n, and then we will evaluate what x of n is. And we will do that for constructive interference. So the numbers that you're going to see here is for constructive. And of course, if you want to, you can do the same exercise for destructive interference.

I need constructive interference, so I have to turn to delta is that, and I make them n times 2 pi. And I start with n equals 0. That's easy. n is 0, delta is 0, the sine theta n is 0, and x of n is 0. That's where you will see maximum light, constructive interference. Obviously. That's the plane that goes right through the middle of those two openings. And the distance to each one of those sources everywhere on that plane is, of course, the same. And so you expect here to see light maximum.

Now I go to plus or minus 1 and I calculate now what delta is. Then I get plus or minus 2 pi. And so the sine is now 2.4 times 10 to the minus 3. You can check that because you know that the d sine theta is n times lambda, and I told you what lambda is. d sine theta is now n times lambda. And we take n equals 1, and you know what lambda is, you know what d is, so we can calculate the angle sine theta. And that translates, then, into an x value of about plus or minus 1.2 centimeters.

So now I know what the sine of theta is and I use the small angle approximation, which is more than adequate. And I know now where on the screen I get again maxima. And that is then here, 1.2 centimeters on this side, and 1.2 centimeters on the other side. Because you get, of course, plus as well as minus. And then I can continue for n equals 1, n equals 2. I will just do plus or minus 10. So then you here plus or minus 20 pi, so then you get here 24 times 10 to the minus 3, and so you get plus or minus 12 centimeters. It's almost linear because the sine of theta for these small angles is still the same as theta in radians.

Before I show the demonstration, I want a little bit more information on the shape of the dark and light areas that I'm going to see. I'll lower this and I will raise it later because I want to work above my head so you can see what I'm doing. So I'm plotting here the sine of theta. And keep in mind that is linearly proportional with x, but I always prefer to plot the sine of theta. And so here is 0. Let here be lambda divided by d, that is where my first maximum will come, you can see that. And then here, the same distance, I have 2 lambda divided by d and on the other side, I have minus lambda divided by d. And all of these are maxima constructive interference, and the destructive interference clearly falls smack in the middle. I did not calculate them.

And so the destructive interferences are here. And so you get light curves, light intensity just like this. Light intensity, remember, is always the Poynting vector. So this is light intensity. And that is in watts per square meter. The first thing I want you to notice, and you will see that today in various demonstrations, is that the location of the maxima depends on the wavelength.

So that means if red light, 650 nanometers, if that falls here as a maximum and it falls here with a maximum and it has here a maximum and it has there a maximum, that blue light-- which would have a wavelength, say, I pick 400 nanometers-- would not have its maximum at the same location, therefore not the minima either. Because look at the relationship. d sine theta is n times lambda. And so the angles of theta are different. And so you would get here, roughly-- it's about 2/3 down the way-- you will get here roughly the maximum for blue. You would get here the maximum for blue. Notice that for n equals 0, the red and the blue have the maximum at the same location. As you would get the first the blue maximum here, you will get the second maximum here, and you will get the third maximum almost exactly at the location where you get the second maximum for red.

The reason being that 2 times 650 is very close to 3 times 400. That's the reason. So each lives a life of its own. The light intensity that you see is a cosine square function of delta divided by 2. And this is so fundamental that first I thought I will ask you to derive it, but then I decided to spend one minute on it and derive it for you. Because it's extremely important that you-- we use it later in our predictions.

So let us assume that source number 1, and the other one I call source number 2, that source number 0-- E1 has an amplitude, E01 when it reaches the screen. And then, of course, you have cosine omega t, and this is the frequency of the light, that's the electromagnetic radiation. E2, that is the other source, has E02 times cosine omega t. And here is that crucial delta, which is the difference between the two in phase. That's why you get maxima and that's why you get minima. That's the whole idea of the interference pattern.

Now if you are very far away from the two openings, then these E vectors are, of course, very close to the same. Remember, E is proportional to 1 over r, but if the distance, r, is very closely the same, then we can assume to good approximation that the E vector and the amplitudes are the same.

So that means the total E vector that you get, the factorial sum of the two is then going to be 2E0. I simply call it E0 now. And then I get the cosine of 1/2 the sum. So I get the cosine of omega t minus delta divided by 2 times the cosine of half the difference. So that becomes the cosine of delta divided by 2. And now you see why the light intensity is proportional to cosine square delta over 2. Because when you calculate the mean value of the Poynting vector, you have to take E cross B.

So B also has this term and also has this term, and so you get the square of this term and the square of this term. But the average of the square of this term is 1/2, so forget that for now. And so you can see it is going to be proportional to the square of delta over 2. So this function that you see, delta is literally proportional to sin theta, this curve that you see is a cosine square curve.

The experiment that I'm going to show you has a separation of the slits, which is 1/4 millimeter. My wavelength is 633 nanometers, which is very much the same. And the distance to the screen is about five meters. And so the way that we have this arranged is that the two slits, they are vertical, they are like this. So this separation is 1.4 millimeter, and then we put over there a laser beam. And then the result is what you're going to see there. So I now have to think about it. This is where it is. You're going to see it there. And in order for you to see it, I have to make it completely dark.

When you look here on the screen, you will see areas which are distinctly dark, and you see areas whereby you see the light. In other words, you see the pattern that we just calculated. You see constructive interference, and you see destructive interference. You're looking here at an experiment that is historically of great importance. It is one of the most mysterious and puzzling issues in all of physics.

This interference pattern, which was first shown by Jung in 1801, is clearly convincing evidence that light is a wave phenomenon. However, 20th-century physics has shown that light comes in the form of individual photons with well-defined energies, and photons can behave like bullets. They have momentum, they produce radiation pressure, and they can be localized in space. We can detect individual photons in a way that we detect individual tomatoes. Photons behave like particles.

However, the interference pattern that you are staring at on that screen can only be explained if we assume that each photon went through both slits. But how on earth can one particle go through both slits? It will have to be one slit or the other. And here lies the great mystery. Light is both. It is a wave if you want it to be a wave, and it is a particle if you want it to be a particle. If you manage to determine for every photon through which of the two slits it went-- and that is possible if you do it at very low light intensity, you can really determine through which of the two slits each photon went-- then you will not see any interference anymore. The pattern on the screen will be like that of the tomatoes. You will get two piles. So the moment you establish the particle character of the light by determining through which slit it went, you have destroyed its wave character and it will not behave like a wave, it will behave like a particle.

However, as long as you do not determine through which slit the photon went, just as long as it remains your guess through which one it went, light will reveal to you its wave character and you will see interference. The choice is yours, but you cannot have it both ways. So you're looking now at something that had a huge impact on physics. Clearly, at the time that this experiment was done, it was accepted that light are waves. But we now look at it in a very different way.

I want to show you a slide of what we call double-slit interference in white light. We did it in red light. That's the easiest, and you get very sharply defined maxima or minima. And the reason why you get them so sharply defined is because you don't have the interference of the other color. But of course, if you do it with white light, then you will see the blue light has its own spacing and the red light has its own spacing, and the net result, of course, is that you see a somewhat different pattern. And that I will show you here. I will make it a little darker for that.

So the upper panel is then the double-slit interference in blue light. And you see the clearly dark lines, which are the destructive interference locations. And you see, as I already indicated on the blackboard, that the separation of the red is larger because it's wavelength dependent. And if you then do with white light, then you see something like this. it's more difficult to actually see the maxima and the minima.

Now what you can do with light, if you can turn light plus light into darkness, then obviously you should also be able to turn sound plus sound into silence. It's just a matter of scaling up the whole experiment. And that is what we have set up for you here. We have two loudspeakers. And so all the equations that you have here apply verbatim, except that we scale it a little bit. So our d is no longer 1/4 millimeter, but our d is now 1 and 1/2 meters.

So here is a source of sound, and here is a source of sound, and they are fed by the same electronics. And so the d, the separation, is 1.5 meters. And the frequency is 3000 hertz. So we do this experiment now for sound. And so lambda is about 11.3 centimeters if you take 340 meters per second for the speed of sound. So we have, in a way, monochromatic light, monochromatic sound. More or less one wavelength. And so you can calculate what lambda divided by d is. It's about 0.075.

And so you can calculate where now those surfaces are. They are really hyperbolic surfaces. They're coming out from the center here, and they bend in your direction, where all those hyperbolic surfaces are where there is no sound, and then you can calculate where all those hyperbolic surfaces are where there is sound.

Let's first calculate how many hyperbolic services there are. So in order to do that, we simply have to stick in our equation, d sine theta equals n lambda. We just put in theta is the maximum value possible and that is 90 degrees. So you put in theta max, and that is 90 degrees. Pi over 2 radians. And so when you do that, you'll find that n max, that is the highest number of n in that series 0, 1, 2, 3, 4, 5, that n max is then d divided by lambda, which in our case is about 13. It is 1 over this number.

So that means there are 13 of those surfaces going in this direction, but also 13 going in this direction. Remember, we have plus and minus. So you really have to multiply this by 2. And so there are 26 surfaces, no more and probably no less, in this audience of constructive interference. And then there are probably also around 26 destructive interference.

So now I want to be as qualitative as I can be. I would like to know the students who are sitting about five meters away from me, what the distance is between one maximum, the 0 or the maximum at the center, and the next maximum. We get an idea where to search for those maxima and where for those minima. So I will go now for the maxima. So I know that at theta 0, right here, all of you sitting here must hear a maximum in this plane. Now, the question is where is the next maxima?

The next maxima depends, of course, on the distance, how far you are sitting away from me. So I take the students which are about five meters away from me. The sine of theta 1-- this 1 makes reference to n equals 1, but is the first surface away from the center. So the sine of theta 1 is then 0.075. And that translates into a value of theta 1, I think it's about 4.3 degrees. That's right. So now you can calculate what the separation is between one maximum and the next maximum that separation x1-- I call it x1-- is roughly L times that 0.075. And that is about 38 centimeters.

So right there, those guys that I'm looking at now, you will have a maximum sound here. And when you move 38 centimeters, you will have another maximum. And there is a minimum in between. So the minima and the maxima right there, where Elizio is sitting, is about 19 centimeters. For you there in the back, it is larger. And for you here in the front it is smaller. Remember, those surfaces go like this, and so here the separation is small.

I'll just give you this as an example, that you know roughly an order of magnitude of what you are looking for. I'm going to turn on the sound now. And then I want you to wiggle back and forth as you did when you were in nursery school, and I want you to find the locations of maxima and minima. And I can assure you that the ones was minima, the ones with sound silence, are extremely well-defined because, of course, we have done this several times ourselves. Don't move too fast. Move very slowly. And there will be points where you hear no sound, and there will be points where you hear a lot of sound. So you're ready for this?

[HIGH-PITCHED SOUND PLAYS]

Keep moving. Most of you move too fast. Who was able to clearly distinguish the maxima from the minima? Not too many, eh? The reason is that all of you are lousy scientists. If the difference in space between maxima and minima is 19 centimeters-- that is this much-- what is the separation between your two ears? It's about 20 centimeters. It's not an accident that I chose that. So if you have a maximum here, you would have a minimum there. So what do you have to do? You have to close one ear, and then you will be real scientists. So I'm going to turn it back on again. I want you to close one ear and then you will be able to find those minima and those maxima.

[HIGH-PITCHED SOUND PLAYS]

Move again, back and forth. There may be some reflections from the wall which may interfere with his experiment, no pun implied. Go very slowly. Remarkable. Unbelievable. I hear nothing here. But here I hear a lot of noise. So who heard it now? There you go. This is the great moment for the mini-break, the mini-quiz. So if someone is going to help me.

I owe you something regarding exam two. Here you see a histogram of exam two. And what is interesting, and in a way helpful to me, it's almost bimodal. We have 47% to go in the course, of which 40% is for the final alone. So clearly I can't say much about dividing lines between A,B, C, D, and Fs.

I want to make only a global statement, and only warn those of you who I put in the danger zone. The danger zone, for me, are students whose cumulative-- assuming that they've taken all the exams then-- whose cumulative is less than 30 or near 30. And those are really the students almost exclusively who have less than 45 for exam 2, or approximately 45. So this is the danger zone.

That doesn't mean that you will fail the course. It's just a matter of probability. I would say it's high. And then those of you who are dying to know whether they are going to get an A, all I can say is that if your cumulative now is more than 45-- assuming that you've taken all exams-- then you will probably end up with A. Will you end up with an A? I do not know. It's all a matter of probability, but the chances are you may. But that's as far as I can go regarding the dividing lines of course grades.

So now I will continue the idea of interference, which has very far-reaching consequences, and I will turn to what we call thin film interference. Normal incidence really doesn't change very much if you change to a different angle. The concept is the same. Thin film. I have here a thin film of oil. And you will shortly see what I mean by thin. So this is air. So we have plane waves of light coming in this way. So it's normal incidence. This is n1, which is air, so that's about 1. And this is n2, which for oil is about 1.45. Let's make it 1.5 just so we round it off. And this is n3, which is again air in this case, so let's make it 1. It doesn't have to be air, but I will make it air. And let the separation, or the thickness I should say, of the oil be d. And I call this surface surface A, and this surface between oil and air, I'll call that B.

So that light come in, normal incidence, comes in from above over a large surface. I'll just put in one arrow here. And let that intensity be I0. And now I'm going to calculate how much is reflected back into the air. And I put the arrow here. So I offset it, just for the purpose of clarity. Some of it is reflected, and some of it goes straight through. And all of you are more than capable of calculating how much is reflected. At that point, A, the r, the reflectivity of the E vector, is n1 minus n2 divided by n1 plus n2, and that is minus 0.2. The minus sign tells you that there is a flip here in the E vector by 180 degrees. The intensity of the light that is reflected is then 4%, the square of this number. And if 4% is reflected, then and clearly 96% goes through. So if this is 100%, then this is 4.0 and then this is 96.

This light here hits the surface at B, the boundary between oil and air. And some of that light goes through, which I'm not interested in, and some of it comes back. And so I can calculate the r value for that surface near B, which is now n2 minus and n3 divided by n2 plus n3. And so that is plus 0.2. So there is no flip in the E vector as it returns.

The fraction of the light that is reflected is again 4%. However, it is 4% of the 96%, because only 96% goes through. And 4% of 96% is 3.84. When it reaches this surface here, again, 4% will be reflected and 96% will go through. So what comes out here, ultimately, is 96% of this number, which is 3.7.

I now want to ask the question, what happens when you are looking here from above and you see this light coming back straight from A, and this light coming back at you, which made this journey through the oil, whether or not the combination of those two can constructively interfere and whether they can destructively interfere. Now as far as destructive interference is concerned, let me make a qualitative statement. The intensity of this is 4, and the intensity of this is 3.7. So it can never be completely darkness because you can never kill 3.7 intensity with 4. But if they're 180 degrees out of phase, which we will be able to do, there's very little light left. There will only be the difference, which is 0.3, left. So it will be quite dark. But I will still call that constructive and destructive interference.

So the question now is, what is the phase difference between this light here and this light there? Phase difference, that's what matters. Let's first look at the difference in path. The difference in the path is clearly 2d. Because this light has to go in this direction, that is d, and it has to come back. So that's 2d. So now there's the question, how many wavelengths fit on that separation, on that distance 2d? Wavelengths in oil, of course. That's what matters. So that is this many.

So what now is the phase difference? Now we have to multiply this by 2 pi. Because if this were 3, then the phase difference is 3 times 2 pi. For each wavelength you get a phase difference 2 pi. So now you have to multiply this by 2 pi. So now you may think, and that would not be unreasonable, that this is the phase difference between this radiation and this radiation. But what have you overlooked when you do that?

AUDIENCE: [INAUDIBLE].

PROFESSOR: That's right. You have overlooked the fact that there is here a flip of 180 degrees, which does not occur here. So there's an additional pi in the phase difference. So I'm going to write down here now the total delta. So delta is now 4 pi times d. I can write this as lambda oil. That's perfectly fine, I can do that. I prefer to write it as lambda air, and then all I have to do, of course, is divide the lambda air by the index of refraction of oil.

And so, yes, I could have written lambda oil here, but I prefer to write here lambda air, and then all I have to do is multiply this by n2. That is that 1.5. It's the same thing. So this is then the delta that we calculated from here. And then we have to add pi. And the pi, then, is the consequence of this minus sign which you don't see here.

Keep in mind, if someone wants to be nasty to you during an exam-- of course, not I-- if someone wanted to be nasty to you on an exam, they could make n3 larger than n2. What would change in this equation? The pi would disappear, that's all. The rest would remain the same. So don't think of this as a universal equation, there's no such thing. So this pi is the result of the fact that here, n2 is larger than n1, but that here n3 is smaller than n2. That's why you get only pi.

So now we're in business. We now have here the criterion for constructive interference.

[SIDE CONVERSATION]

PROFESSOR: It is d, right? Uh oh. My Greek heritage somehow. So we agree, right, that in this case, this is the phase angle. So now we're going to ask ourselves the question, what thickness are required to get to minima and to get maxima? The first very interesting case is that if you make d equals 0 that you kill this whole term and you only have pi. So that's destructive interference. Destructive.

Now you may say, yeah, zero film. What kind of nonsense is that? Is that meaningful? Yes, it is very meaningful. I will demonstrate it to you. I cannot make an oil film zero thickness. But that's not necessary. All that is necessary, that this d overlapped our oil becomes exceedingly small. For instance, 1/100. So if I make d 1/100 of the wavelength of light in oil, then I'm close enough. And you will indeed see then-- I will do this with soap-- that in reflection, you will see no light. You get completely destructive interference. Apart from the fact, which I mentioned, that there is an imbalance between the 4.0 and 3.7. But that's a different issue.

So we will even be able to enjoy the d equals 0. But now I want to be more general. I want to give you a feeling, qualitatively, for what you're going to look at and also a feeling for what thicknesses are really necessary to see these colors. And so I am going to address now only the case of destructive interference.

I'm going to choose a lambda in air of 400 nanometers. That's my choice. That means that my lambda in oil is 400 divided by 1.5, which is about 267 nanometers. Now I want to calculate for what thicknesses, d, I'm going to get destructive interference. So I'm going to set this equal to 2n minus 1 times pi. That's all I do. I choose n equals 1.

And what do I find? I find d equals 0, of course. We knew that already. We just discussed that. Because if n equals 1, then that requires that delta is pi. And if delta is pi, then this term is 0. So you see that d is 0, you can immediately see that. Now we take n equals 2. If you substitute for n equals 2 in this equation, we get 3 pi. This is already pi, so this has to be 2 pi. And so now if you calculate what d is, you'll find that d is 133 nanometers. That's obvious.

It obviously must be 1/2 of the wavelength of the light in oil. Because if the wavelength of light in oil is 267, you have to travel this distance twice. And if you travel this distance twice, that is exactly one wavelength. And one wavelength corresponds to a phase angle of 2 pi. And so here you get 2 pi. And it's this pi that kills you, that gives you the darkness. Then n equals 3, you'll find that d is approximately 267 nanometers. So now 2d translates into two wavelengths.

Now I'm going to choose this thickness. And with that thickness now, I'm going to work and I'm going to radiate white light onto that thickness. So I select that one, for which I already know that in blue light, I kill it. Destructive interference.

We now have white light. And we have these 133 nanometers. So that is non-negotiable. d is 133 nanometers. I'm going to write here lambda air in nanometers. I'm going to write here delta, and

I'm going to write here the cosine squared of delta over 2. That is a measure for the light intensity, it's proportional to that. We discussed that and I showed that, I derived it for you.

400 nanometers, blue light. Delta is 3 pi. This is 0. We already calculated that that's the destructive interference if you make it that layer, if you make it 133 nanometers. Now I need green. Now I'm going to also shine on it green light, 500 nanometers. I know what d is, that's not negotiable. And now my lambda air is now 500 nanometers. So I can calculate what delta is, untouched by human hands, very simple. I find 2.6 pi. If I take cosine square of delta over 2, I find 0.35.

Now I go to red and I do 650 nanometers, and now I calculate what delta is and I find 2.2 pi. I get 0.90. Now I am very close to a maximum. So this is very convincing, that if you make your film this thin, that it will look distinctly red. Maybe not perfect red, but it will look distinctly red. The red color dominates and the blue practically absent at all, apart from the imbalance that you may have between the 4.0 and the 3.7.

The question that I've asked over the years at the final exam for 8.03 is whether thin film interference is the result of the difference between the index of refraction between the different colors. And the answer is, absolutely not. The index of refraction I have taken the same for all colors, for all practical purposes. This does not explain the colors. The color is explained by the different path lengths divided by lambda. That is why you see colors. It has nothing to do with the index of refraction.

You can see thin film interference in soap, and I will try to demonstrate that. You can see it in oil spills on the road. I will show you an example. Only thin films give you colors. Thick films do not give you colors. If you go through the exercise, which I want you to do, to make d 0.1 millimeter, which is by practical standards still very thin, you will not see any colors.

And the reason is that if you go through this equation and you calculate for which colors you will see constructive and destructive interference, there are so many colors in the visible spectrum for which you get constructive interference, so many you will see, that your brain will tell you you see white light. There's not one color that dominates. The end values that you will need, by the way, are going to be very high. They're in the range 500 to 700. Huge values of n are necessary in order to get the destructive and the constructive interference. Go through this exercise on your own, and you will see very quickly that so many colors constructively interfere that the film will look white.

The first thing that I want to try, it's not so easy, to make you see these colors with soap. Which of course, all of you must have seen in. In fact, if you just take a shower and you soap yourself, then the bubbles themselves in reflection already have these colors. I'll try to make one that is slightly larger in size, but we do not always succeed. But let's try that. So the idea is I want you to see colors. Well maybe you did, but that was a little quick.

Now you see colors. Did you? That's all I wanted you to see. Did you see them? If you had said no, I would have done it again, but now I won't. You want me to do it once more? It's not easy though. Now you saw it, right? Thank you.

So soap bubbles give these colors because they are exceedingly thin. It also gives you a feeling of how thin they have to be. Really, they can't be much thicker than a few times the wavelengths of light. If they become much thicker, you lose the colors for the reason that I just mentioned to you, that there are too many colors that constructively interfere and then you don't see it anymore.

Now I have a demonstration which is very tricky. It works most of the time, but not all of the time. I'm going to make a soap film on a metal frame. It's going to be done in this box. We have a metal frame, and we dip it in soap, and so you get a soap film there. Then gravity will make the film thinner at the top and thicker at the bottom. So as you make a cross section through here, then the film-- this is then the soap.

I can make this so thin that it's completely dark in reflection. That is what I promised you that d is not 0, but d divided by lambda is so small-- may be 1/20 or 1/30 of the wavelength of light-- that it will turn completely black. And as this proceeds in the beginning, the top will show you colors. It will go through these phases of colors, and then it gets thinner and thinner and thinner, and then it turns completely black. I will show it to you upside down. Don't get confused. It has to do with the way that we project it. So the thinnest part of the film will be low on the screen and then the part of the screen that is the thickest of the film will be the upper part.

As you look at it, appreciate the fact that when we go into the dark phase, if we succeed, that you are talking about the thickness of the film, which is many times smaller than the wavelength of light, which is typically half a micron. Let me first get the film up. Actually, let's not get the film up. Let's get the light up. Let's turn it off here. It has to be completely dark. Didn't we agree it has to be completely dark?

So I'm going to dip it in here. So the bottom is the thinnest. You see the colors. You see them? You see that the thickness changes over the film, and that's why there are areas where the red dominates and then other areas where the red dominates again. And so the bottom gets thinner and thinner and thinner. The white light that you see now here this is the result of Markos's trick. In order to avoid that the film burst, he has to put glycerine in the soap. And what you see here is a reflection of the layer of glycerine which forms on top of the soap. So we'll have to wait a little bit for that glycerine to also go down.

And then you will see the pure soap, and that is then the area-- and boy, look at this. Black. That area that you're looking here now is way thinner than the wavelength of light. And so the reflection is-- well, when I say 0, of course it depends on the asymmetry that you have, that we discussed here on the blackboard, between the 4.0 and the 3.7. It's a little different for water than it is for oil. But you see? Black. We can just look at it for a while. It may burst. You will also see some very interesting thin film interference in the glycerine itself, which is the oil layer which is on top of the soap. It is not uncommon, you may remember when you see oil spills on the road, that you sometimes see these very nice structures. Brownish, reddish, bluish. But here, look at this. Isn't that incredible? Isn't that fabulous? The film is holding up, Markos. We can't complain.

You see these nice oil structures here in the glycerine? The nightmare that we have always is that it pops too fast, and then you have to do it again and again and again. And our experience is if somehow the soap is not right, then in general, no matter how often you try it, it keeps popping. Look at how beautiful. Look how incredibly nice the thin film interference in the glycerine, and then here the total darkness because the thickness of the film, of the soap, is many times smaller than the wavelength of the light in the soap. I never get bored when I look at this. Isn't it gorgeous? Think about the physics that is going on in there.

All right. Let's look at some other phenomenon of interference. Try to remember this. This is a gorgeous picture. So there's a whole family of ways that you can see film interference. In your problem sets, we give you two problems to work on that quantitatively. And there's also this take-home experiment which supports the problems. I want to concentrate on those, too, because they are very nice to demonstrate. Because I can demonstrate them to you in monochromatic light.

Whenever you have monochromatic light, that means you don't have white light. Of course, the fringes-- we call these fringes, the dark and the bright areas, we call them fringes-- show up so much better than when you have white light, because then you get the superposition of all the colors. And the maxima and minima depends on the color. Remember, for one of the same thickness, you may have destructive interference for one color, but you have constructive for the other. So if you do it in monochromatic light, you can see this very dramatically.

And then one of the classic examples is that you can do that at home, or you should do that at home, is the two microscope slides. So you have a microscope slide, which itself is way too thick to give you thin film interference. Don't even think about the fact that it is glass there, it will do nothing. Way too thick. Probably 1/10 of a millimeter. It is ridiculous. And then there's another microscope glass here. And I exaggerate highly this in between, because they are lying flat on top of each other, but there is an air gap between them. And this is that air gap.

It is this air gap now that acts like a thin film. So you're going to get interference now of light that bounces off this layer and bounces off this layer. And so it is this thickness, d, of the air layer that determines the darkness and the brightness. The destructive and the constructive interference. And you're being asked in your problem set to calculate where those maxima and minima are. And you could also do that, then, with the take-home experiment. And I will demonstrate it very shortly. There's another one, which is a classic, which I will also demonstrate and which is also part of your problem set.

This is a lens, a glass lens. The curvature is highly exaggerated. And then here is a glass plate. And so here is now an air gap. And now, if you shine monochromatic light on this, then seen from above-- there's symmetry above this line-- you will see rings of darkness and rings of darkness. These rings of darkness and brightness will be very far apart here, and they will be very close together there. This is part of the homework assignment, to show that. That it has to do with the fact that the change here in thickness is very little because of the curvature, and the change here is much faster. This is called the Newton rings.

And then when you just walk out on the street, preferably on a day when it is not sunny but it is cloudy, you look on the ground and you see oil spills, and they have phenomenal colors. I'd like to show you one of my own pictures that I took some time ago of an oil spill. Let me first turn it on and then we'll make it a little darker. I'll make it a little darker so that you can see it better. I photograph countless oil spills. And actually, the reason why I've photographed so many is that always I try to reconstruct the thickness of the oil on the road, and I was never very successful.

For one thing, you can only get a spot like this, you would think, that a car loses some oil. So there's one drop of oil that goes. And then it starts to run out. So you would always think that it is thickest here and then gradually tapers off near the edges. I've always wondered why, then, here do you see so much white light? You would expect that perhaps it would be much darker. So many attempts I have made to convince myself that it's really thicker than there, many have failed. But there's no question what you are looking at, of course, is a striking example of thin film interference. And not only that, but you know that this film has to be extremely thin, clearly very close to that range of 100 to 200 nanometers. Otherwise, you wouldn't see over the entire area such beautifully equal blue. And here you see this reddish-- which is not pure, it never is-- very well-defined. So clearly, it has a spherical symmetry. So it is probably a drop that then spread out.

So now I want to demonstrate the microscope. So the microscopes we have here, microscope glass. Turn on the CD, TV there if everything works. And we do it with neo-monochromatic light, this light from a mercury lamp, which is largely in the green. So you see very well-established maxima and minima. That's the beauty if you do it with monochromatic light. And so these two microscope slides are just on top of each other. But yeah, there's always a little bit of air in between. So you can sort of make up your mind that they may be touching each other, really touching where they have practically no air-- for reasons that are not so clear-- which must be somewhere here. and that's where the separation between the dark lines is the largest. And then somehow here, you see these fringes, the areas of darkness and maximum constructive interference.

And when I push on one side of these slides, I change this very subtle structure of light in between these microscope, and then you will see an immediate change in the separation of the lines. I'm just pushing on it now. Changing the configuration. This is the point of contact, more or less. You can really see that. Where I'm looking at right now, I see it just as well as you can see it there. So you should really do that experiment at home because it's part of your take-home experiments.

So I mentioned in the write-up that it's not necessary that you turn in a written summary of your experiment, so it won't count for your problem set as such, but I will hold you responsible on the final. So I advise you to do the experiment, because I may ask a few questions which you can only answer if you actually did this experiment.

Here I have two flats, like the microscopes, but thicker so they're a little sturdier. This is the way that opticians actually test their optics. So there is this air gap that grows in size. And you push on it, now I make them flat. Now I see almost no fringes. Now I pushed so hard that I open them again like this. So I make one go up like this, and now you see many fringes. Look. You get a huge angle now. And then I will just show you this, which is called the Newton rings. We have it arranged in such a way that with some set screws we can actually push down on this, so we can also make the air gap larger and smaller. And that we're going to show you there.

I can actually make it even darker, that may be nicer for you. You already see there, the center portion of the contact-- where the contact is here, the air gap doesn't change very much. Therefore, you will see that the spacing of the rings is large. But when you go further out, then the spacing gets smaller. And it becomes difficult to see, perhaps. But if you're close here, I can really see the rings here much closer than the rings here. I can put some pressure on there, in other words change the geometry a little bit.

I squeeze a little harder. I hope I don't break it. And when you squeeze harder, they touch each other over a larger surface in the center. You see, there, it opens up. So over this very large area now is the gap the same. And when I loosen it up, then you expect those rings to become smaller. This is something I believe that it also part of your take-home experiment. All right, I can't wait to see the result of the mini-quiz. I'll be proud of you if you did well, let me tell you. I see you then, I hope on Thursday.

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