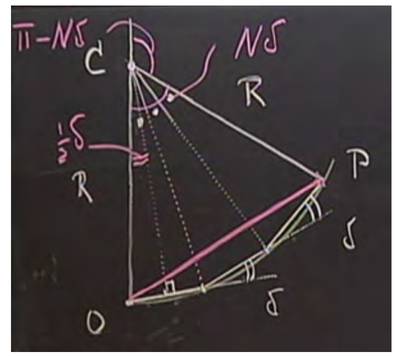
Massachusetts Institute of Technology OpenCourseWare

8.03SC **Fall 2012**

Notes for Lecture #21: Diffraction

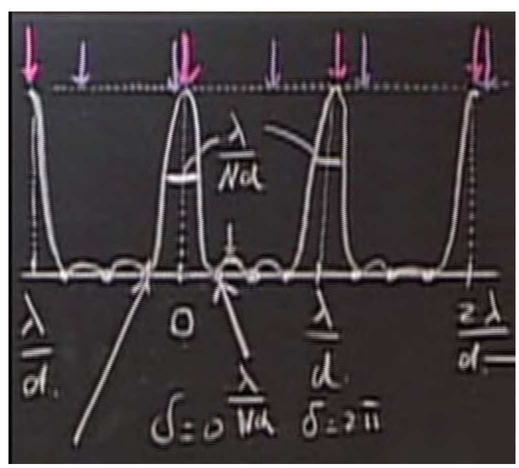
The lecture starts with a review of two-slit interference, but between two neighboring sources among many. For light of wavelength λ emitted at an angle θ from two slits separated by a distance d, a phase shift of $\delta = \frac{2\pi}{\lambda} d \sin\theta$ is present. A longer discussion of the geometrical addition of two vectors shown at (**4:00**) is given at the end of these notes.

Now, consider adding N separate $\vec{E}$ contributions, all with the same relative phase shift δ, illustrated with a sketch of 3 such vectors. The total angle is $N\delta$, so the angle to the vertical is $\pi - N\delta$. Each wedge has a radius R, half-angle $\delta/2$, and half-width $A/2$, so $\sin\delta/2 = \frac{A/2}{R}$, i.e. $R = \frac{A/2}{\sin\delta/2}$ (**8:00**). The length of the summed vectors (OP) can be found from triangle COP, which has sides R and external angle $\pi - N\delta$. That gives (see details at the end of these notes) $OP = 2R\cos((\pi - N\delta)/2)$. Substituting for R, and using a trig identity to simplify the cosine term gives: $OP = A\frac{\sin((N\delta)/2)}{\sin(\delta/2)}$.



The intensity is proportional to the square of $\vec{E}$ so $I(\delta) = I_0 \left[\frac{\sin(N\delta)/2}{\sin\delta/2}\right]^2$ where I_0 is the intensity that would arise from a single slit (**11:00**). When $\delta = 0$, the "0/0" reduces to N^2, so then $I_{max} = N^2 I_0$. For $N = 2$, $I(\delta) = I_0 \left[\frac{\sin 2(\delta/2)}{\sin\delta/2}\right]^2 = I_0 \left[\frac{2\sin\delta/2\cos\delta/2}{\sin\delta/2}\right]^2 = 4I_0\cos^2(\delta/2)$ using a double angle trig identity. Note that this same result was found already last lecture.

Using $\delta = \frac{2\pi}{\lambda} d\sin\theta$, the intensity can be plotted versus $\sin\theta$, labeled at intervals of λ/d (**14:00**). An example for $N = 4$ is shown. The overall pattern is of principal maxima which are quite large, with zeroes and much smaller secondary maxima in between. The separation of the large peaks is proportional to the wavelength (**17:00**). In addition, the width of the principle maxima is inversely related to the *number* of slits, the first minima occurs at $\sin\theta = \lambda/Nd$. As one example of how small the secondary maxima are, the first peak away from $\theta = 0$ for $N = 4$ has an intensity $I = 1.17I_0$, much smaller that the $16I_0$ of the principal peaks (**21:00**). The more slits, the better the *spectral resolution.* The



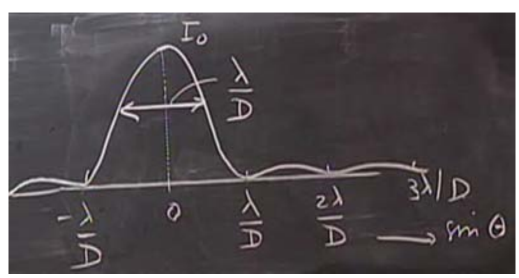
narrowing of the peaks is required by energy conservation in order to compensate for the increase in intensity at the maxima ($\propto N^2$) (**24:30**). By drawing the vector addition of the light from the 4 slits, the number and location of the minima between principle peaks is easy to illustrate (**27:30**).

A demo uses a 3 mm wide beam from a green laser with a wavelength of 532 nm and a grating with spacing 1.9×10^{-6} m, so that about 1600 gratings are illuminated. The maxima ($\sin\theta_n = n\lambda/d$) occur at 0°, 16.3°, 34°, and 57° for $n = 0$ to 3 (the highest possible order) (**31:00**). Several more demos follow, including white light shining on a reflection grating, in which the slits are ruled on a reflective surface (**37:00**). Many orders are visible, each corresponding to a peak in the diffraction pattern, and each spot (except at zero) is further subdivided into bands for the many colors in white light. Gratings can be used to separate the wavelengths in the spectra of atoms, for example He (helium). (**41:30**). Ne (neon) has many more lines due to its more complex atomic structure.

Using Huygens' principle, light passing through the different sections of a single opening of width D should interfere (**44:00**). The linguistic distinction drawn between "interference" from multiple slits and "diffraction" from a single slit is artificial, both are really the same phenomenon. Consider destructive interference of light from one edge of the slit with that from the centre of the slit so that $(D/2)\sin\theta = \lambda/2$. If this is the case, then similar pairs of points can be found as one moves down the slit, so the argument applies to the whole slit and complete destructive interference occurs.

Define β to be the phase difference at an angle θ for two points separated by the half-width of the slit (**47:15**): $\beta = \dfrac{2\pi}{\lambda}\dfrac{D}{2}\sin\theta = \dfrac{\pi D}{\lambda}\sin\theta$ (**47:15**). It can be shown that the intensity is $I(\beta) = I_0\left(\sin\beta/\beta\right)^2$. When $\sin\theta$=0, then $\beta = 0$ and I is I_0, as expected since light going straight ahead from the slit should have the highest intensity. If $\sin\theta = \lambda/D$, then $\beta = \pi$, $\sin\beta = 0$, and I is 0, as already expected from the argument given above. A similar thing happens when $\sin\theta = 2\lambda/D$, and at many other points (**50:30**).

On a plot of intensity as a function of $\sin\theta$, the width of the central maximum is twice that of the other maxima, and its half-width is about λ/D. For 600 nm (red) light, with $D = 0.1$ mm, viewed at $L = 3$ m, $\lambda/D = 6 \times 10^{-3}$ radians, so at the screen the width will be ~2 cm, 200 times wider than the slit (**54:15**). In fact, the smaller the slit is, the *wider* will be the pattern. The first mini-maximum, at $\sim 1.5\lambda/D$, has $\beta = 1.5\pi$, $\sin\beta = -1$, and $I = 0.045 I_0$. The maxima farther out are even smaller.



The demo first shows the increasing width of the central maxima with increasing wavelength (**58:15**). Red laser light and a variable size slit show the increasing width of the central maxima with *decreasing* slit width. The whole image becomes fainter as the slit width is reduced since less light can get through, but the widening of the central maximum is very apparent (**1:02:00**).

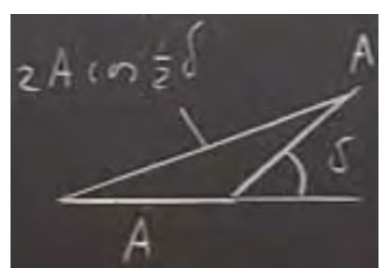
For multiple slits, the width of the individual slits also causes diffraction, with the result that all the maxima are not, in fact, the same intensity. Taking both effects into account, the variation for a single slit modulates the strength of the grating maxima: $I = I_0 \left(\frac{\sin\beta}{\beta}\right)^2 \left[\frac{\sin N\delta/2}{\sin\delta/2}\right]^2$. Note that the multi-slit maxima are closer together than the single-slit minima since the slit-slit spacing must be larger than the slit width (**1:05:00**). The effect is very clear, with a demo showing the 5^{th} order grating maximum being killed off by the one-slit modulation (**1:08:00**).

Similar considerations apply to circular apertures, although the math is slightly more complex. The angle to the first minima is larger by a factor of 1.22. A laser of $\lambda = 594$ nm illuminating a 1/8 mm opening (pinhole) shines on a screen at a distance of $L = 4$ m, giving a central maximum about 5 cm across. Even though faint, the ring pattern is clear (**1:11:15**). A bright LED also has monochromatic light and serves as a good light source for personal observation of these effects, although it could not be captured on the video.

Such diffraction ultimately limits the angular sizes we can see (**1:13:30**). In the case of stars, circular patterns arise, which overlap if close together. The so-called Rayleigh criterion says that things are perceived as double if separated by the distance to the first minimum, i.e. $1.2\lambda/D$. This limit applies if the atmosphere does not add additional blurring as is the case for the Hubble Space Telescope. With a diameter of 2.4 m, $1.2\lambda/D$ is about 1/20 of an arc second for 500 nm (green) light (**1:16:30**). Telescopes on Earth can be much bigger but due to atmospheric turbulence (astronomers call this "seeing") they typically cannot resolve better than about 0.5 arc seconds. The eye, with its much smaller aperture, should be able to resolve about 0.5 arc minutes in theory but, in practice, the number is closer to about 1 to 2 arc minutes (**1:19:10**).

Detailed geometry explanation:

Consider the addition of two vectors of the same length A with one rotated by an angle δ. The large triangle can be divided into two smaller identical ones with a common side starting at the joining point and perpendicular to the long side. From the symmetry, and since δ plus the full angle at the point of joining add to 180°, the angle at the common end of the small triangles must be $\frac{1}{2}(180° - \delta)$.



Therefore, the end angle of the small triangle is $\delta/2$, and with the hypotenuse length A, the 3^{rd} side has length $A\cos(\delta/2)$, so the length of the long side of the big triangle is $2A\cos(\delta/2)$.

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These viewing notes were written by Prof. Martin Connors in collaboration with Dr. George S.F. Stephans.