8.03SC Physics III: Vibrations and Waves, Fall 2012
Transcript – Lecture 21: Diffraction

PROFESSOR: Last time, we discussed the interference pattern of two openings in a screen, which establishes the wave character of light. Today, I'm going to expand that to N, capital N, and we will make N thousands of openings in a screen. And when it comes to optical lights, we call those gratings.

So suppose here, each one of those dots, is a small opening in a screen. It could be a small hole, or could be a slit perpendicular to the blackboard. And imagine that, plane waves are coming in. And so, each one of those openings are going to be Huygens' sources, and they're going to radiate spherical waves. And the question now that we want to answer is, if we look in the direction theta, away from the normal to that screen, what is then the light intensity that you will see as a result of the interference of all these Huygens' sources?

Suppose that the separation between two adjacent Huygens' sources is d. In other words, here is one, and here is the other. So this is a blow up of what you see here. You see hundreds, maybe thousands. Here, I have only two. And if that separation is d, then I can calculate the phase difference between those two.

If this is theta, this is no different from what we did last time for the double slits. Then the path difference from this spherical wave, to a point far away, at an angle theta, and this spherical wave, the path difference is this. And this is d sine theta.

And so, we will introduce, just like we did before, phase angle delta. Which is the phase between this spherical wave, from this point and that point, with two neighboring points. And here there are thousands. So this, is only between the two neighboring sources.

And so, first, we want to know, how many times can we fit a wave length on this path? And for each time that we can fit a wave length on there, we have a phase difference of 2 pi. So this delta, is the phase difference. And we have the same equation last time for double split interference. No difference.

And if this delta is a multiple times 2 pi, then you will have constructive interference. My goal today is way more ambitious. I want to know what the light intensity is for any angle theta. It's going to be an extremely complicated function.

I first want to revisit some of your high school geometry. If you have a triangle, and this side is A, and this side is also A, the length of the triangle, the lengths of the sides. And if you want to know what this one is, and if this angle is delta, then this side here is, 2A times the cosine of 1/2 delta. And you can revisit that in your high school geometry, but I need that today.

And now, I'm going to do the superposition. Vectorial superposition of capital N E vectors, that come from these various sources. And the neighboring ones are off by delta. That's the phase angle between them. I will raise this later, but I will first, work above my head, so that you can see what I do.

I start with a circle. You will see shortly why I do that. The radius of the circle is unimportant. You will see that that will cancel. So this radius, arbitrarily chosen, is R. This length here, is also R. Call this point C. This point O. And this point P. And I'm going to add three vectors, which are all offset relative to each other, over an angle delta. But I will make the calculation as if they were capital N.

Capital N could be a million, there's no limit. But I'm going to make the drawing only for three. This is one vector. Let's say it has a length a, but it's really the electric vector that's going to be added vectorially to the other electric vectors. And so, here's the other, the second one. And here is the third one. And so, the angle between the second and the third one, this angle here, is delta. And this angle here, is also delta. That's the delta between the neighboring sources, which we just derived.

It follows, from the geometry of a circle, that this angle here is delta, this angle here is delta, and this angle here is also delta. So that means, if I have capital N of these vectors, that this angle here, is then N times delta. And that means, that the angle here, this angle, is then pi minus N times delta.

I will draw one more line right through the middle of this vector. I think of this as length a, but it is the electric vector. You could also think of it as E0. And I draw a line straight through there. So this angle is 90 degrees. And so, that means that I know that this angle now is 1/2 delta. And this is all the geometry that I need to calculate incredibly complicated light intensity as a function of theta.

My goal is to find the magnitude of the vector OP. Because that's the result, in this case, of these three vectors. But I will make my calculation as is if there were N.

First, look at this triangle here, which has a 1/2 delta angle here, this is 90 degrees. And so, this part here, is 1/2 A. I just give it the length A. So that means that 1/2 A divided by R, is exactly the sine of delta over 2. That's exact, it's not an approximation. It is not a small angle approximation. This is exactly the sine of that angle. In other words, my radius R, is 1/2 A, divided by the sine of delta over 2.

So I'm really interested in OP. And now, I'm going to use this knowledge. I have a triangle. I know two sides. And I know this angle. Then, I can calculate this one. I know two sides. This is R, and this is R. I want to know the third side. And I know this angle, which is pi minus N delta. And therefore, this length here, is 2R times the cosine of half that angle. Pi minus N delta divided by 2.

With pi over 2 is 90 degrees, and 90 degrees minus the angle is the sine of the angle. So I can also write for this, 2R times the sine of N delta over 2. Now, I take this R here, and I pop that in here, to eliminate my R And so now, I get that the length of that vector OP, which is really my goal, that is the vectorial sum of N E vectors. The adjacent one is off by phase angle delta. That is going to be 2 times R. So that gives me an A upstairs. And then, I get the sine of N delta divided by 2 divided by the sine of delta divided by 2.

And this is really the key of what is following. This is now the length, the magnitude of the E vector. If you think of this A as being E0, that's the magnitude of the electric field. So the light intensity, the Poynting vector, obviously goes by the square.

And so, now comes the very famous equation, that I as a function of delta, is I0 times the sine of N delta divided by 2 divided by the sine of delta divided by 2, and this whole thing squared. And you could call this the grating equation. You will see that this is a very complicated function. We will beat it to death together. This is an exact derivation, this is no approximation.

Delta can be anything from 0 to 10 million pi. There is no approximation made here. 10 pi, 20 pi, 30 pi, anything for delta is allowed. It's not an approximation.

Now, the intensity, always think in terms of watts per square meter. And the meaning of I0 is that if there were only one opening in the screen, instead of N, then this is the intensity that you would see, I0.

Now, if the upstairs here is 0. You would think that the intensity is 0. That is not always the case. Because if the upstairs is 0, and the downstairs is also 0, you get 0 divided by 0. And now you need l'Hopital's to calculate what that ratio is. And that ratio then, becomes the maximum value possible. So I'll write that here. So the maximum value possible, if you use l'Hopital's, you will find that that ratio is N squared. And so this becomes N squared times I0. So that's the case where you get 0 divided by 0, in that equation.

Before I will show you how dramatic this function is, I want to remind you, that for N equals 2, which is what we covered last time, you can use this equation. This holds for any capital N, holds for any value for delta. So if you substitute, in this equation that you have there, N equals 2, and you do a little bit of massaging of the algebra-- I will let you do that-- you will find-- so this is for the double slit interference-- you will find that I is 4I0 times the cosine squared of delta over 2. And for those of you who have a good memory, remember that I derived this in class. When you only have 2 vectors that you add, that the light intensity goes with the cosine squared of delta over 2. Now you also have this here, so you also know that at the maxima with two slits that you see four times more light than if there were only one opening. And you can do this of course on your own by substituting, in this equation, N equals 2.

So the first thing that I want to do now, is to make a drawing, a plot of that function. And I will do that for N equals 4. And then, we will discuss all the consequences, also for cases that N is much larger than 4. I'm going to plot this for N equals 4. So we only have four openings now. And I'll always plot only sine theta.

The reason why I like to plot always things in terms of sine theta, theta is a real geometrical angle. Here is this screen, with the openings, and theta is an actual angle in the lecture hall. So theta is something that I can immediately relate to. This is 10 degrees, this is 20 degrees, this is

30 degrees. Delta is a phase angle. Right? That's not a real angle in space. And so I always like to plot the intensity in terms of sine theta. But you can also do it if you want in terms of delta.

All right, if sine theta is 0, then you get 0 divided by 0, and you're going to get a maximum. If sine theta is lambda divided by d, you're going to get a maximum. If sine theta is lambda divided by d, you see that delta is 2 pi. That means you get a maximum. So you get a maximum here. If sine theta is 2 lambda divided by d, you get a maximum. And of course, on the other side, minus lambda over d, you also get a maximum.

And according to the equation, if you really believe that equation verbatim, then all these peaks will have the same maximum. Which would be 16I0 because this is the N squared. And I will show you, but first I will plot it-- that if there are four slits that in between the prime maxima, there are N minus 1 locations whereby you have completely destructive interference. There is zero light. N minus 1, in this case, is 3. You will see shortly why it is N minus 1. For the three locations here, whereby there is zero light-- I put them in here and then I will draw the curve. So that's the zero, and so I'm going to make an attempt now, to draw the light intensity.

So these are these prime maxima when 0 divided by 0 is N squared. Here's another one where 0 divided by 0 becomes N squared. And here's another one whereby 0 divided by 0 becomes N squared. If you wanted to know what the delta is here, well the delta here is 0 of course. And the delta here is 2 pi. And the delta here is 4 pi.

I first want to show you, or at least draw your attention to the fact, that there's a wavelength dependent, lambda. And what that means is, that if you take 650 nanometers, which is red, the red would have a maximum here. The red would have a maximum here. The red would have a maximum here. And the red would have a maximum there.

But if now you have 400 nanometers, which is violet light, it would have a maximum at different locations. Here at zero it would always have the same location as maximum. But the wavelengths is shorter for blue-- for violet, so here would be the maximum for violet. Here would be the maximum for violet. And roughly here would be the maximum for violet. And roughly here. The reason why the red and the blue there almost coincide, I mentioned that also last time for the double slit interference, is that 2 times 650 is roughly 3 times 400. So they live a life of their own.

Let's now address the issue of the N minus 1 zeros. I first want to calculate what the location is here, where you have your first 0, complete 0. Well, you would have your first 0 when the upstairs is 0 but the downstairs is not 0. Because if they're both 0, you are at what we call a prime maximum. So what is the first time that this one becomes 0 when the downstairs is not 0 that is when this is pi When it is 0 they are both 0 but when that is pi then of course I would have my first 0. So let's do that here. So I call that my first 0. That is the case when N delta divided by 2 is pi. And so that means when delta is 2 pi divided by N.

So now I go to this equation, and I put in here for delta, 2 pi divided by N. And what you see then, that sine theta becomes lambda divided by sine theta, is now lambda divided by N d. In other words, this point here, in terms of sine theta-- which of course is the same as angle theta in radians, because these angles are small. So this is an angular dimensional plot. So this here is lambda divided by N d. And then of course, the second one will be twice lambda divided by N d. And the third one will be 3 times lambda divided by N d. You will again have completely destructive interference.

You may be interested in what the magnitude is, what the light intensity, I should say is, of this little mini maxima. Well, that is very easy to calculate. If we know that the sine of theta, here, is lambda divided by N d. And we noted, here, it is twice that much. Then all I have to ask that equation, what is your light intensity when the sine of theta-- so now we go to that mini max, the first mini max, right here. So I ask the equation, what is your intensity when the sine theta is now 1.5 times lambda divided by N d. Then I'm right in between these two zeros. Now whether I'm exactly on the maximum, I don't know. I'm really not interested, but I'm surely I'm close.

So when sine theta is 1/2 lambda divided by N d, then delta is going to be 3 pi divided by N. That's easy, because you take this equation, 1/2 lambda over N d. So you put in sine theta 1/2 lambda divided by N d. Your 1 1/2 times 2 becomes 3 pi. You get an N downstairs. So you get that delta is 3 pi divided by N.

And now you revisit this equation, and you just put in there, N equals 4. You know what capital N is. You know now what delta is. So you calculate your upstairs, you calculate the downstairs. And you will find now, that I, it's approximately 1.17 I0. That's low compared to 16. This is only some 7%, 6.8%. So this height here, is only 7% of that height. So it's very low. So you have seen now the bizarre consequences that if you get 0 divided by 0, you get these maxima at 16 I0. Then you get N minus 1 point whereby you get complete 0's here. But even these mini maxima don't mean very much. They are very low in light intensity.

And even if you go to an N of 1,000-- and today we will even go beyond that with our experiments. You will have capital N, we go up to 2,000. If you make capital N 1,000, you can redo all this, and you will see that this many maxima is roughly 4 and 1/2% of this maximum. That maximum now becomes a million times I0. If you have 1,000 of these openings, they will add up at the maxima. You get a million times the intensity that one alone will do. The reason is obviously, you get 1,000 times the E vector, and they're all in phase with each other. And the Poynting vector is the square of the amplitude of the E vector, you get the million.

All these maxima have the same width. And if this is lambda divided by N d, then this here, on this side, in terms of angular distance, is of course the same. That is complete symmetry. And so, the width here, if I take the width roughly, without being very precise, the width of each one of those peaks must be roughly lambda divided by N d. So I take 1/2 this distance, angular distance, all these things are angles in radians because sine theta is very much smaller than theta.

Now, you can see that the larger N is, the more of these openings you have, the narrower these lines are going to be. If I think of these as being a line. And that means your ability to distinguish two neighboring frequencies from each other, two different lambdas increases and that's what we call spectral resolution. So the larger N is, the better spectral resolution you have. Your ability to separate two neighboring frequencies then increases.

There's a very easy way that I can convince you without any math, little math, why the widths of these peaks, the widths of these maxima, must be proportional to 1 over N. And that's purely an energy conservation argument, and follow me closely. If I have N of these openings in the screen, they will let N times more light through, than one opening. That's straightforward, you can tell that to your kid, probably. If you have N openings in the screen, you get N times more light through than if you had 1. That's non-negotiable, right? But if each maximum is N squared times higher, the only way that you can conserve energy, is if you make the maximum N times smaller. Then you know N times more light went through.

So the argument once more, you have N openings, that gives you N times more energy than one opening. But if each maximum gives you a light intensity which goes with N squared, the only way that you can conserve energy is if you make the maxima N times narrower. So that's a very easy way to see that the widths here must go down with increasing N. Very powerful argument. Now I will make you see in another way if we have four of these openings in the screen, why there are only three minima. And all these methods that I'm using, in a way are complementary. It's all the same thing, but I just want you to see it in different ways. It helps me enormously to look at it in different ways.

So we have N equals 4. And I started with delta equals 0. So there's no phase difference between adjacent sources. So this is the case, that this is the E vector of source number 1. This is E vector of source number 2, number 3, and number 4. All four E vectors line up in the same direction. Otherwise, delta could not be 0. Aha. Therefore, I get 16 times the light, because I square 4E and I get 16. So this is your factor of 16, and you get a maximum.

Now we go to delta equals pi over 2, 90 degrees. I don't have to look at that equation. I don't need that one. I know what 90 degrees is. I went to high school, I'm educated. This is one vector. This is 90 degree, that's the second vector. This is 90 degrees, it's the third vector. This is the fourth vector, 90 degrees. What do I end up with? 0. So if all four, relative to neighbor's 90 degree phase angle, then clearly, you have zero here.

Now I go delta equals pi. I know what pi is. One, two, three, four. What do I end up with? I add four vectors, 180 degree, flip, flip, flip, flip. What is the net result? 0. So it's dark. This one, is this one. And the second one, is this one.

Now I'm going to do this one for you. Delta equals 3/2 pi. 3/2 is 270 degrees. Well, this is one, this is 270 degrees. This is 270 degrees. And that is 270 degrees. What is the net result? 0. Well that's this one.

Now, I'm going to do 2 pi. Ah, when I do 2 pi, I'm back here. You get another maximum, And that's this one. And so, you can see, purely by playing vectors, it's very simple, high school level, you can see that there will be N minus 1 minima, exact minima, between the prime maxima.

So the first thing that I want you to see, is when I take the grating that you have, and I use my laser pointer, the first thing I want you to see-- incredible impact of using many, many lines. My laser beam has about a diameter of about 3 millimeters. So here is this laser beam. And this, I estimated, to be roughly 3 millimeters.

Your grating, believe it or not, is a super grating. That grating has 13,400 lines per inch. Imagine how anyone can put groves in your plastic, 13,400 per inch. That means that the separation, d, between 2 groves is about 1.9 times 10 to the minus 6 meters. That is only two microns. How anyone can do that beats me, but it can be done.

So I can calculate now, how many of those lines I have here in the 3 millimeters. And I end up with N is about 1,600. So when I shine my laser beam through my grating, I use effectively 1,600 of those lines. And I can calculate now what the angles are. Where on the screen, there, on the wall, the maxima will fall, those maxima. And by the way, there will be 1,599 of these zeros in between. And these minima are so small that you won't even see them, you'll only see the maxima.

So let's calculate, what the angles we would then see the first-- by the way these things have names. We call this, zero order. And we call this the first order. And we call this one, this is first order. And we call this second order. And this is also called first order, but it is on the other side, of course. So we call these orders of the spectrum.

So you can now calculate that the sine of theta of n is n times lambda divided by d. And so, when n is 0, that your zeroth order you get, of course, a maximum for the zeroth order is at theta equals 0. Now your first order, is when the sine of theta 1, is lambda divided by d. And I can take my lambda, which in my case, of my laser, I have to tell you, my lambda is 532 nanometers, is green.

And so I can calculate what theta 1 is, and I find 16.3 degrees. And then I can go to the second order, theta 2 for that color-- it's different for different colors-- and I find 34 degrees. And I can go to the third order, and I find theta 3 is then 57 degrees. And there is no fourth order. That would make the sine of theta larger than 1. So there's only zero order. And then there are first, second, and third order.

The grating that you have, I always carry that with me, no matter where I go, it's easy to put in your calendar. And so, I will show you now, by simply shining through this grating. I'll show you there on the wall. Zero order with fall right smack in the middle, so to speak. First order, 16 degrees away. If you know my distance to the wall, you can calculate how far that is. It's probably a meter and a half. And so you will see these maxima, you'll see them extremely narrow. High value for N. And well, it speaks for itself.

I have to rotate my grating to make sure that the groves are in this direction. If the groves are in this direction, the spreading is out in this direction. So here you see. So the one that you see right now, on the right side of the screen, there is my zero order. You can't tell that it is zero order, because I have only one color. And then you see, I will move it a little, you see the first order now, on the right side of the screen. And that angle should be very accurately about what I calculated, the 16 degrees. And then you see on the blackboard here, show it right here, you see the second order. And then you see here, the third order, if you have good eyes. And the fourth order doesn't exist.

And imagine that, between those maxima, if I really use 1,600 lines, that would be 1,599 points with exact zeros and then all these silly mini maxima, that you don't even see. So that is the power of grading when you use many lines. If I use 1,600 lines, I use 800 times more lines then when I have to double slit interference. So the double slit interference pattern will be extremely different from this. That is why those locations are so narrow. Because if it were double slit interference, you would get the cosine square function. And so you would see the maxima would be 800 times broader than this. That's the power of using 1,600 lines.

Now, I want you to get your gratings out, and I want you to look simply at a desktop lamp. And the reason why I want you to do that is, that I want you to be disappointed. Because you may not see what you erroneously expected that the zero order is incredibly narrow. It is not, it's big, it's the lamp itself.

Of course, if your light source, in angular size, is way larger than this angular size, you cannot expect to see the light source get smaller. In other words, clearly the limiting factor of seeing zero order very narrow, is only if the light source itself is small enough in size. And so, when you look at this light now, you will see all colors at zero order. That's one thing that's important. You just see the lamp. That's your zero order maximum. That's the red, the blue, the green, the yellow, the violet, that's all of it. And then, you will see on either side, blue appear.

First the blue, that has the lowest, shortest wavelength. I don't think there's much violet in this lamp. And then, you will see the blue, the red-- make sure that you line it up so the groves are vertical, so that you get the spread in the horizontal plane. And then you see the green, you see the red, and it really looks like you have a spectrum, a continuous spectrum. Even at second order, it still looks like a reasonable spectrum. It starts in blue and it goes to red. But when you go to the third one, you already see that the blue and the red are going to interfere with each other. So then they really don't look any more, clearly, like separate spectra.

So I want you to appreciate the fact that if your light source is huge, that each little location of the light source gives you a line, which is this narrow but if you have many, many of those locations, it smears it out, of course, and you see a very broad line. If I call this a line, this zero order maximum.

We have here, a grating, that works not in transmission. Your grating works in transmission, the light goes through it, like what we discussed here. But we have here one, that is metal, whereby grooves are put on the metal. And this is called a reflection grating. And I can show you with this reflection grating, which has a large light source, just like this, it's large, I can show you the spectrum when we project it here on the screen. If Markos can give me a hand. Thank you, Markos.

So we have white light that goes on to this reflection grating, it's a big spot of white light. And you will see then, the first order, will not be narrow because the light source itself is so big. And then, you will see on either side something similar to what you saw here. You will see the white, the zero order, is always the light of the source itself. Which, is in this case, white. And then on both sides, you will see, first order, second order. And I think you see up to four or five orders.

I don't remember how many lines there are per inch. It's not important, because it's really the effect I want to show you. I was quantitative on my own demonstration with my green laser. So let us turn this on. I think that's it. And we turn all the lights off, so you can enjoy this in its full.

So here you see the size of that zero order maximum. It's not narrow at all, because the light source is not narrow. And then you see here, nicely, the blue, the green. And you see here, the red. Blue, green, red. You begin to see spectra. And then the whole thing sort of peters out. And you see the same on this side. So this is a reflection grating.

If we turn on a laser, a red laser, which is much narrower, then you get the advantage of your many, many lines. And then, you get of course, an extremely narrow zero order maximum. And that is this laser, I believe. We will know very shortly. Yep. So this is a 633 nanometer laser. And so now, I take advantage of the many lines that I cover. Now my light source is narrow enough to let me benefit from the grating the many slits that I use. And you see, as you expect, that the red zero order always falls at the same location. And then, the red here, the 633, coincides of course, with the red from the white light, and so on. And you even see here, 633-- and I don't think-- so this is first order, second, third order, there's no fourth order.

The separation in terms of angle, is only a function of d, the spacing between the grooves, and of course lambda. And so, if the spacing of a double slit interference pattern is the same as the spacing of a multiple one, the separation in angle is the same. But of course, the gain is that you make them narrower, and they go up with N squared, so that's what you gain. So now, I would like you to take full advantage-- thanks, Markos-- to take full advantage of your grating. And we have a prepared light sources, which are narrow enough so that you come very close, maybe not exactly, you come very close, to seeing the width of the light source, about lambda over N d.

If you put the grating in front of your eye, your pupil itself, has a diameter of about three millimeters. So you will only use, effectively with your eye, about 1,600 lines. Just like my laser beam because the same three millimeters. So you look through about 1,600 of these lines. And you know what d is, the separation of the grooves.

And so you can now look at a line source, which we have prepared. So we have prepared line sources, which are very narrow. Not like this one, but very narrow, which are in here. And now if you look at that light source, which is helium, and now you use your grating, then you begin to understand the idea of spectral resolution. You will now begin to see the individual atomic lines, nicely separated through your grating.

And so, we'll turn this on, and we'll make it completely dark. And then, I want you-- oh thank you-- and I want you to appreciate this and spend some time looking at these lines. It is really quite remarkable. And this of course, you could never do with two slit interference. You need these many, many lines. You look through about 1,600. This very strong yellow, in the helium, is a well known helium line. And it has a wavelength of 587 nanometers. It's the brightest, the strongest one, in helium.

And you can see them in first order, you can see them in second order. And then gradually, when you go to higher orders, the various colors begin to overlap with each other. Because they each live their own life. The angles are only dependent on lambda over d.

And now, I can show you neon, which has even more lines. And so you can look at your grating. Amazing. You know, this grating doesn't cost more than maybe $1. It's absolutely stunning. And it has an incredible spectral resolution already. But because you need a prepared line source to take advantage of that spectral resolution.

And as I said, you will probably approach-- certainly the audience all the way in the back of 6-120-- you will approach this angular resolution. The ones that are closer, may not approach it, because they see the line source, of course wider. The angle at which they see the line source may well be larger than this width. But the ones in the back of the audience, are therefore a little better off. So the angle that you see to the line source, the width of the line source becomes smaller the farther you are away. This is remarkable, isn't it? Absolutely incredible.

So I think this is a great moment to rest and to digest this wonderful experience, and to have a four minute break. So thank you very much.

So what I want to discuss now, is the logical consequence of this whole concept of Huygens sources, where spherical waves come from each point in the aperture. And we're now going to extend it to a single opening. Not multiples, but one single opening. And the opening is now D. So this separation is now this opening, is D.

Think of them as being a slit, which has a width D. It's open, single slit. And we have plain waves coming in, like this. And now the question is, if I look in various directions, and that's my famous angle theta, what will I see now on a screen which I place very far away?

Well, each point in this aperture, can now be considered, according to the Huygens-Fresnel principal, as a source of spherical waves. And they're going to interfere with each other, strangely enough, for reasons beyond me, we call this diffraction. This is exactly the same phenomenon as interference. But we draw a strange distinction in physics, between interference, which was the grating and the double slits-- we say double slit interference, no one would ever say double slit diffraction, but it is the same thing. Somehow, when we deal with individual openings, we call that diffraction. So it is the same, you can use it any way you want to. You can call it interference.

But the individual Huygens sources, there they are, they're all going to do their own thing. And I pick this one at the top, number 1. And I pick this one, number 2, right in the middle. Why don't I do that, right? You'll see why I do that. So now, I can calculate what the path difference is between the Huygens source right in the middle, and the Huygens source right in the top. Well, this path difference here, is clearly 1/2 D times the sine of theta. We did that before, we had a little d here.

If I make that 1/2 lambda, I claim that in that direction, it will be darkness. Why will there be darkness? Because if source number 1 can kill number 2 because they are 180 degrees out of phase. Then the source just below 1, can kill this one, below 2. And the one below there can kill this one. So I could always identify two pairs which kill each other. That means darkness. So this must be a criterion for destructive interference. Now it may not be the only angle for which there is destructive interference. But I want to convince you that there's at least one for which you see no light.

I will now introduce the phase angle beta, which is the phase difference between source 1 and source 2. So if this is the slit, one side of the slit and the middle of the slit. It's not the phase angle between two neighboring sources, because the neighboring sources touch each other. There's an infinite number of Huygens sources. It's a continuous Huygens source. So it is the phase angle between the edge of the slit and the center of the slit. That's the way I define beta. And so my beta now, becomes 2 pi divided by lambda times this, times 1/2 D times the sine of theta. And so you see, the 1/2 eats up the 2, so you get pi D divided by lambda times the sine of theta.

And now I will not derive for you as I did. So precisely for the gratings, I will not derive for you what the light intensity is a function of angle. Again, it is a matter of adding vectors. But you can look it up in Bekefi and Barrett It's done in section 8-7. So I will only give you the answer. I did it for the grating, you do this one.

And you can show now, that the intensity of the light, as a function of that phase angle beta, is I0 times the sine of beta divided by beta. And of course no surprise that you get a square there, because that has to do with the Poynting vector. And this function is very different from a gradient function. And before I plot it, let me make a few calculations. I'll do it here on the center board, so that you could still compare with this equation.

So I'm going to write down here what the sine of theta is, and then here in my column comes beta, and then here comes the sine of beta, and here comes the intensity, I. Let's first take the sine of theta, is 0. Well if the sine of theta is 0, it's immediately obvious that beta is 0, and it's also obvious that the sine of beta is 0. So you get 0 divided by 0. You use l'Hopital's and this ratio becomes 1.

And now, the light intensity is I0. Well, that's not so surprised that you have there a lot of light. Because of course, if you have an opening here, and you shine light through it, you expect to see right on the wall there, in the middle of the slit, you expect to see a lot light. So that's not so surprising.

This, by the way, is the way we define I0. So I0 is defined as that maximum that you will see when you, so to speak, look straightforward at an angle theta 0. That's the way we define I0.

So now let's take sine theta is lambda divided by D. So now beta is pi. We put in here, sine theta is lambda divided by D. Here it is, so you see that beta is pi. So the sine of beta is now 0. And therefore, the intensity 0. Because the upstairs is 0, but the downstairs is not 0. Ah, that's this. Because look, you take this 1/2 away, you take this 1/2 away, you get that the sine of theta is lambda divided by D. I already predicted that you would have destructive interference. That's exactly this case, of course.

And now I have 2 lambda divided by D. That gives me a beta now of 2 pi. That gives me again a 0 here. And there's another 0. So there are more locations in space where there is complete darkness, not just one. In fact there is an infinite number of them, you can go on like this.

I will first plot the curve, and then we'll discuss it in a little bit more detail. And I will only plot curves in terms of sine theta, because that is an angle that I can relate to. I can tell my mother about theta, I can't tell my mother about phase angles, but I can tell her about theta. Mom, this is theta. 10 degrees, 20 degrees, 30 degrees. That's a real angle in my laboratory. That is that theta. So that I can relate to. I always plot things in terms of sine theta.

And so here is my 0, and let this be lambda divided by that capital D. That is the slit width. This is 2 lambda divided by D. And here are 3 lambda divided by D. And the other side, minus lambda divided by D, and so on. I'll put one more in.

And so now, the curve that you see, not so obvious, but when you plot it you will see that you get here this maximum, which we defined to be I0 is a central maximum. And then you get here, your first 0. You get here some kind of a mini maximum. And then you get another 0 here, and you get an infinite number of 0's. Every time you get 3 lambda over D, 4 lambda over D, 5 lambda over D, and on this side here, and so on.

So the central maximum, the width in terms of angles. Think of this as angles, right? Sine theta as opposed to theta in terms of radians. So this is an angular size. So the linear size depends on how far you are away from the screen, which you show it. Then you have to multiply these by L, where L is the distance to the screen. This is the angular size. So this width here, very crudely, is about half this. And so that width is about lambda divided by D.

Let's take an example, which is, I think the demonstration that I have lined up for you anyhow. We'll take a laser light, which is about 600 nanometers. The fact that it is 633, of course is not so important, I give you easy numbers. And suppose we have D which is about 0.1 millimeters. So we have a slit that has a width and opening of only 0.1 millimeters. And we'll put a screen there, L at a distance of about 3 meters.

So we can calculate now what lambda divided by D is. So that would give you the angle in radians. That is 6 times 10 to the minus 3 radians. Sine theta is very close to theta. And so now you can calculate the linear size of this central maximum, as I'm going to show you there on the screen. And the linear size is now L. So the linear size, the linear dimension, is L times lambda over D. So that's how wide that central maximum will be. And that, in this case, will then be about 2 centimeters.

Now think about it. Think about the absurdity. We have a slit that has an opening of 1/10 of a millimeter. And because of Mr. Huygens, it will show up there with a width of 2 centimeters. 200 times broader than the actual opening.

Whereas, if you would think high school, then you would say, well if you had light going through 1/10 of a millimeter, you look and what you see on the wall, you see 1/10 of a millimeter. No, you see 2 centimeters. And that's the result of the diffraction. That's the result of the fact that each one of those sources in this aperture is going to radiate spherical waves. They're going to interfere with each other, and they then call these huge broad center.

And the smaller you make D, the more you tighten the nuts on the slit, the wider it's going to be. Because look, if you make this a smaller D, then this angle will become larger. Very non-intuitive.

Before I show you this, I want to know roughly what that maximum is here, that mini maximum. Well, that's easy, you could do that now on your own because all you have to do is, you have to substitute in this equation, sine theta, which is sort of halfway in between. So if I do that, halfway in between, that is 1.5 times lambda divided by D. Right? That's right in between these two. So that gives me then a beta of 1 and 1/2 pi, 1.5 pi I can go to this equation, and I put in for sine theta, 1 and 1/2 lambda over D. The lambda and the D cancel, you get 1 and 1/2 pi. Straightforward, just turning the crank.

So now that you have a beta, you can calculate what the sine of beta is, which is minus 1. And so now, you know what the sine of beta divided by beta is, and you'll find 0.045 times I0. So this mini maximum is 4.5% of the central maximum. And when you go further out, these maxima are even smaller. But when I show you this phenomena, which I will, you will see distinctly these zeros. You will see actually very nice, dark locations, and you will see this central maximum, and then a little bit of light, but not very much light on this side.

And of course, again, this is wavelength dependent. So whether you do this in red light, or in blue light, you will see something very different. If you do it in red light, you will see these wide. And if you do it in blue light, you will see it narrower. And also those locations, of course, then, are further in. I have a slide which shows you the idea in different colors. Maybe we can make it a little darker, if you use the TV button, thank you, Markos.

So here, you see it in three colors, red, green, and blue. So notice how different this is from a grating. You see some broad, central maximum, that is that central maximum that you see here, on the blackboard. And then you see the sharp, black locations, where there's almost no light, here. And you see the red farther apart than for green. And for green farther apart than for blue. And then when you do it with white light, of course, then it becomes always more difficult, to see the sharp dark areas, because the colors overlap.

All right, so now I want to demonstrate this to you. And the way that we are going to do this, is with a slit that we can vary in size. So I'm going to do this with 600, and where was my calculation? I made a calculation here. So I'm going to do it with a 633 nanometer laser light. And the slit, so that the beam of that laser is about 3 millimeters and then we have a slit here so this is the opening so this is the D we can make the D smaller.

And so we already made a prediction, that the linear dimension then, if the opening is only a 1/10 of a millimeter, you would expect that the central maximum on that screen which is about 3 meters away-- that's why I chose the 3 meters. The central maximum, will then be 2 centimeters wide. But I could make it way wider, because I can make D way smaller than 1/10 of a millimeter. This is it. All right, so what you see now, is that the slit is very large, very open. I don't know, maybe a millimeter or so. And I'm going to tighten it. And I'll stop at one point here, so that you can-- this is a nice moment to stop.

So here you see that central maximum. See how powerful and overwhelming that is in terms of its brightness. We understand now why, because of this crazy function sine beta divided by beta squared. Already, now, it is here, oh I would say 5 centimeters. So already, now the slit width must be less 1/10 of a millimeter, because it will be 2 centimeters if it were 1/10 of a millimeter.

And I hope you can see distinctly, those dark locations. And you see there a lot of them. But keep in mind, that this mini maxima, next to the broad maximum, is only 4.5% and it gets smaller, and smaller, lower, and lower, as you go further away. So now, I go way beyond 1/10 of a millimeter, way smaller, way smaller. Now, keep in mind, when I make the slit width smaller, less light will go through. I can't help that. So the whole image will become fainter. That's the price I pay for letting less light through. But what I gain, is to show you the absurdity that the center maximum gets wider, and wider, and wider, as I make the slit smaller, and smaller, and smaller. So this is now narrower, and narrower, and narrower, and narrower. And the central maximum that you see there is almost a foot.

So the center, the opening of my sit must be something like maybe only 10 microns or so, extremely, so this is a very highly, highly, accurate device. Whereby we have the optional making this slit width indeed as small as 10 microns. You see this is the result that you get. So I will now make the slit open, and open, and open, and open, and open. And here we have the point-- if I make the center point about 2 centimeters, then this slit width which is about now is about 1/10 of a millimeter.

All right, I now have to make an important confession about the grating equation. Some of you who were very observant may have noticed that the maxima of the grating that showed, also when I did the experiment with my own laser, they were not all exactly the same brightness. There was a difference, and no one asked me about it, and I was hoping that no one would ask.

And the reason for that is, that each one of those grooves has a finite size opening. And each one of those openings acts this way. They cause diffraction, we call that diffraction. That's just semantics. And so superimposed on the grating equation, this causes diffraction, and the net result then is that you get the product of the two.

So if I amend here now, and I'm removing this beta now, because this is now for grating. But the beta is defined this way, this D, is the opening of each groove in your grating. And little d, is the separation between the grating. Then I can write down now here, times the sine of N delta over 2 divided by the sine of delta over 2 and now I put here the square. And now, I have the real grating equation. And so what you see now is, that since little d is always larger than capital D, you're going to see that the maxima, which come from this equation, are being modulated, by this one.

And so if this were a grating, whereby capital D was the opening of each individual groove. And if the grating wanted-- of course the zero order maximum will always here. And if the first zero order maximum of the grating would fall here, and the second order would fall here, and the third here. Then this is the price you pay for the fact that these grooves have an opening.

And so you see a modulation in the strength of your first, second, third, fourth order, and so on. And so when you look carefully at the grating lights, and I will demonstrate that to you. They are not all 16 times I0, if you have N equals 4. And if you have N equal 1,000, they're not all a million times I0. But they have this overall envelope, which modulates it, and that's the result of the finite opening of the grooves.

So to make sure you understand the difference between the two d's. This is migrating, and this is the open area, so this is, say, where the light cannot go through. Then the definition of D is this, and the definition of this is d. And that d shows up in here. And this capital D shows up in there. So this is the single slit diffraction, and this is the multiple split interference. There you see again, we make the distinction in wording, but it has no meaning, because it's all diffraction, of course.

If somehow D were approximately d divided by 5, and it just so happens we have a grating, d for which that is the case. Then the fifth order maximum of the grating is going to be killed. Because that is when this function becomes 0. So you would see then zero order, one, two, three, four, five would be killed here, and then they would build up, a little, again, and then ultimately, of course, they will peter out.

And so if I show you a spectrum of a grating, you can actually roughly estimate what the ratio capital D over little d is by seeing that sort of modulation pattern. And so that's what I want you to see now. It's not so exciting. Many of you who were observant may have seen it anyhow. Because it was every time there, even when I showed my own gratings. And we going to make it quite a dark for this. Or no, this is the wrong switch. So many switches here. There we go.

So this is a grating that I purposely offset. I purposely offset it so that here is the zero order. Make sure, that this is a zero order. I think it's that one actually. Easy to test, where my zero order is. OK, this is a zero order. So we aim exactly here. And so this is the first order, second order, third order, fourth order. Look at this sucker, it's almost gone. That is the result of the fact, the single slit interference.

And then it comes up again here. And the reason why it comes up again, because now you enter this little mini maximum in the single slit interference. So you see it here quite well. Sometimes with gratings, you can see it's remarkably well. Other times, it is harder to see. It depends, of course, on how many maximums you have. Here you see quite well the modulation, you see it comes up here again, and here, this little point here, would then be somewhere, here, in this maximum. So this is then the complete equation that combines single slit diffraction with multiple slit interference.

If we change the single opening from a slit to a circle-- your eye is a circle, your pupil is a circle-- then very little changes, except, of course, if you have a circular opening. Everything is now axial symmetric. And so you get circles, these things become circles, and then, which is not so obvious, that is that this minimum doesn't fall at lambda over 2 in terms of angular dimension but 1.22 times lambda over 2. And if you want to use 1.2 as an approximation, that is fine enough. So it's a little larger for a circular opening, than it is for a slit.

I have here, one of those pin holes. So this is now a circular opening, for which this relation has to be used, now. So this central, is then, a little wider. And we are about 4 meters away from the screen, is this one, 4 meters away from the screen. And I'm going to do this with a wavelength lambda of 594 nanometers. So it is a circular opening, call is a pin hole. And lambda is about 594 nanometer, it's also a laser. And the distance to the screen, L, is about 4 meters.

And what you're going to see, is a ring, which is this ring, and there you see this light inside, which is very difficult for me. It's just a very high maximum. And then you will see more rings outside. This ring is quite well defined. You're going to see that very sharply defined. And if this ring, which I measured, is about 5 centimeters in diameter, you should be able to tell me what the diameter of the opening is. Of course, because you know now what the angle is, so you can calculate what D is.

And when I did that, I came up with something like 1/8 of a millimeter, or so, but you can confirm that. So a very small opening, of 1/8 of a millimeter, if I did that correctly, would then give you a central maximum, which is from zero to zero, 5 centimeters wide. So let's take a look at that. These single pinhole diffraction's are always very difficult because the pin holes have to be so small to see it, and that means very little light will go through. And so here, you see it. For those of you who are sitting close, you clearly see that central circular maximum. And then, you clearly see the first ring, the dark ring, my pinky is right on it. And this is about 5 centimeters across. I see a second dark ring, but if you're far away in the audience, you may not see that so well. This is a nice example of circular single slit diffraction.

To make you see it even better, we handed out cards. And those cards have a small pin hole in one location, and they have a double slit in the other location. And so I'm going to aim at you, very slowly, I'm going to scan this over the audience, a light emitting diode. Bright light. And as it passes you, only get one shot at it. And you look through it, maybe we could make it darker. If you look for the pin hole, you will really see beautifully, this circular ring structure with the dark lines, and the center maximum. But if you look through the other opening, you will see the beautiful double slit interference.

And notice that the width of the dark lines and the width of the bright lines is about equal, because you only have two slits. Remember you get this cosine square function. So it's not as dramatic as a grating. And I'm going to rotate this through the class, so each one of you get the chance to look through both openings, one at a time. If I go too slowly, let me know.

You can keep these cards, but you need a very bright light, and you need a very small light too, because the light is too large in size then of course your dark and your bright areas are going to merge with each other, so you wash it out. So your light source always has to be carefully thought through in terms of its dimension, in terms of angular dimension. If the angular dimension of your light source is too large, you kill all the phenomenon.

I'm going to rotate it back. Who has not seen it? Who has not seen it? You have not seen it, Amanda, how could I do that to you? Now you can see it even longer than others.

This single slit diffraction, or single opening diffraction, has major consequences even for our daily experiences. Because it ultimately determines our ability to separate two light sources in the sky. If you have a telescope, and the telescope has a lens or it has a mirror which has a diameter D then there is a limitation to which it can separate two stars in the sky.

Let's assume there's two stars at roughly equal strength. So here is star number 1. And here is another star, which is star number 2. And the angle between them is delta theta. Then somewhere here, on a photographic plate, or in your case, on your retina, there will be an image, and that image will be like this, for one star. And there will be another image a little bit displaced from the other star. And if those two blurs are too close together, you don't see two stars anymore, but you see only one star.

So now comes the question, how close, how small can ithis angle be, so that you still say, yeah, there are two light sources? A car comes to you with two headlights, how close does the car have to be, but you still say yeah there are two, not one. Well, that is a criteria which is a little bit arbitrary. It's called the Rayleigh criterion for angular resolution. And that is, we want the angle between the two lights, larger or equal to this angle. So that the maximum of the second light, would fall here. And so you would clearly see then, that this thing is broadened. And you may even see a little dip in that curve.

So on your photographic plate, you would really be able to say, yeah, yeah, there are two sources, and there's not just one source. And so the angular resolution would then be in terms of angle 1.2 times lambda divided by D. So delta theta, we have to be larger than 1.2 times lambda over D for you to be able to say yes, there are two stars. Which is the ultimate limit of angular resolution for you, for me, but also for optical telescopes.

Suppose we take the Hubble Space Telescope. Hubble Space Telescope, HST, has a mirror D, which has a diameter of 2.4 meters. And it is prepared so carefully, that the claim is made that it is really diffraction limited. And so that means, if I take an average wave length, the optical spectrum of about 500 nanometers, I realize it all the way goes from 400 to 650. But if I take this as a representative wavelength, then 1.2 times lambda divided by D, translates into about 1/20 of 1 arc second. That is an incredible resolution. 1/20 of an arc second.

If two stars of equal brightness are 1/20 of an arc second apart, Hubble Space Telescope can see it as two stars. The same telescope on earth will do no better than 1/2 an arc second. This is an arc second. To maybe even 2 arc seconds.

Why is it so much worse for a telescope on the ground than it is for Hubble Space Telescope? Anyone of you know that? Atmosphere. The Earth's atmosphere is in turbulence, is always in motion, thermal motion. And it is that thermal motion that is the problem. That makes your image on your photographic plate, or on your CCD move around. They work like a little lenses.

And so it broadens it, but it broadens it in an incredible way. If you see, we call this the seeing, one arc second. Then Hubble telescope, which is above the atmosphere, has an angular resolution which is 20 times better linearly, that means over a surface, it has 400 times more resolution elements, because it's two dimensional. So it's highly superior, in terms of angular resolution than any ground based observatory.

And now comes your human eye. Your human eye, the opening of human eye depends on the time of the day. At night, when it's dark, your pupil opens. During the day, it goes down a little. If we take about 4 millimeters, a reasonable number. And we take again 500 nanometers as our representative wavelength, then we can calculate what 1.2 times lambda over D is and that translates to 1/2 an arc minute. That is 30 arc seconds. That is 600 times worse than Hubble Space Telescope.

You cannot do better than this. This is Mother Nature, you cannot beat the fraction. On your retina, when you look at a light source, the image on you retina will look like this. Will look exactly what you saw there, that is what you retina will see. And if those two lights are too close together, your brain will say, sorry, I don't see two lights.

Now in practice, Mother Nature did not design our eyes, at least most it, was not really down to the fraction limitation. In practice, I think it is more like one to two arc minutes. This is my symbol for arc minutes. So the angular resolution of your eyes is not quite as good as it could be, but it's close to that. And I'm going to test it with you.

I have here a screen, and this screen has holes in it. And I'll give you the code of the holes. So there is that screen, two holes, two holes, two holes, two holes, and repeat them three times. This is 1 millimeter apart, 2 millimeter apart, 3 millimeter apart, 4 millimeters apart. Students who are 2 meters away, very few of you are. But if you were 2 meters away, so we take here a student, who is at a distance of 2 meter. If the student looks at the 1 millimeter separation of these slides, then the angular separation is 1.7 arc minutes. So you should be able to see them as two lights. If you look at the 2 millimeter separation, and obviously, it's about 3.4 arc minutes, you should have no problems. In other words, the students who are close, should be able to see this as two lights, this as two lights, and this as two lights.

But let's now go to the students who are 5 meters away. So this is the students who are 5 meters away in the audience. If we go to 1 millimeter separation, there's no hope on earth that you will see that, when you're sitting there where Christine is sitting. You will not be able to see the upper two as two light source because the separation is only 0.7 arc minutes. And I don't think that any one of you can see lights that 0.7 arc minutes apart. The 2 millimeter slots, would be 1.4 arc minutes, and the 3 millimeter slots would be about 2 arc minutes.

And so, I'm going to make it dark now in the room and I want each of you to just look at the pin holes. And I'm going to rotate this so that all of you get a chance. And then, I want you to raise your hand if you can see the top two as separate ones. And only those who will raise their hands will be those very close to me. And then we will slowly go farther into the audience. So look closely at the upper one, and then also try to see the one below there which are the 2 millimeters, which are here. The 3 millimeters and the 4 millimeters.

Can you see the upper one, Amanda, as two? Even the upper one, because you're only 3 meters away. Now I will just rotate this. So the upper one again is the 1 millimeter separation. You see the repeat three times. And then 2 millimeters, and then below that is 3 millimeters and then is 4 millimeters. So raise your hand if you can see the upper one as two light sources? No one. Oh boy. Well, if you really can, then your resolution is very close to 0.6 arc minutes because the 2 meters was 0.7 arc minutes. So that's really remarkable, but it's possible.

Who can see the second row clearly as two distinct sources? Now people are coming in, so even the ones that are close can only do it. Do you see that? No one in the audience there in the back is raising his hand. So now we're talking about a resolution of 2 millimeters, that is 3.4 arc minutes for the ones that are two meters. So you're talking about 2 and 1/2 arc minutes.

Who can see the third row as separate? Now the hands go up. Who can see a fourth row as separate? I think the whole class. The ones in the back there, can you not see? Really? Well you've got to go to an eye doctor. Really, you cannot see the bottom? I was there this morning, where you were, I could see the bottom one distinctly as two light sources. And I could kid myself that I even saw this one as two, but I was really kidding myself, because I knew it I think. But this one, I could clearly see. So, really none of you can see the bottom one as two different? So that shows that your angular resolution is no better, you shouldn't be ashamed of that, it's not your fault, than about two arc minutes. So with that idea in mind, have a good weekend.