8.03SC Physics III: Vibrations and Waves, Fall 2012
Transcript – Lecture 22: Rainbows and Other Optical Phenomena

PROFESSOR: I'm going to ask you a few questions which have to do with a rainbow. All of us have looked at rainbows. But the question always comes up, have you really ever seen it? I could ask you the same question about art. All of us have looked at art, but it's very different from seeing art.

And so here I have 15 questions. Let's look at them, and ask yourself, how many of them can I answer? We've all looked at rainbows. So the first question is, what is the color sequence? Is the red on the outside or on the inside? And what is the radius of the bow? And what I mean by that is the following. Say here is the horizon, and you see a rainbow, then there's somewhere a point that you could call the center of the circle. And then you can always measure the angle in terms of degrees. Not in inches but in terms of degrees. What is that radius typically?

And then we can ask the same question about the length of the bow. The width of the bow. Their colors here. How wide is that in terms of angle, roughly? I don't expect you to be precise, but roughly. Have you ever noticed that there's an enormous difference in the light intensity outside the bow and inside the bow? And if so, where is it bright and where is it dark? Have you ever noticed that? It will be staring you in the face when you look at the rainbow.

What time of the day do you expect the rainbow? Do you expect to see the north, east, south, or west? Are there more than one bow? And yes, I already answered that for you. There really is a secondary bow. And then the question is, what was the color sequence of that second bow and where do you have to look for it? What is its radius? And what is its width?

And then comes the question that all of you can answer, because of my problem set number 10. There was a problem 10-4, which addressed the issue, are the bows polarized? And then, what is the direction of polarization? And is the polarization weak, or is the polarization strong?

All right. Professor [INAUDIBLE], how many questions can you answer?

AUDIENCE: Sheepishly, I think I would do less than 50%.

PROFESSOR: That's not bad. Mark, that is not bad. Who can answer all 15? 14? Thirteen? 12? 11? 10? Don't be ashamed. Nine? Eight? Now it becomes an area where you may have to become ashamed. Seven? Six? Who can answer six? That's not bad. Five? Four? Who can only answer four? Three? Two? One? Who can answer zero? I'm proud of you. All of you can at least answer one.

All right. I'm going to make you see a rainbow the way you have never seen it before. Because most of you have never seen a rainbow. This is the picture that you also had on your problem set number 10.

Lights from the sun, in this case, come straight from the left. We're going to give the sun an angle in the sky very shortly. We used to call this angle theta 1. I call it i, now for today, angle of incidence. And I call this r. When the light gets into the water drop, I call that the angle of refraction. I call that r.

And so with Snell's law, we would then have that n1 times the sine of i, which is the angle of incidence, of which the light strikes the water drop. n1 times sine i, equals n2 times the sine of r. And for all practical purposes, n1 is 1, that is in air. And n2 is then an average value for water, n2. Average value is about 1.336. Average over the colors.

I asked you in problem 10-4, to concentrate on the following trajectory. Light strikes the water. Some of it is reflected at A. I'm not interested in that. I want to know what goes into the water. At B, some of it comes out. I'm not interested in that. I want to how much is reflected. And then at C, some of it is reflected into the water drop. I am not interested in that. I want to know what's coming out.

And what you see now, is what comes out makes an angle phi, with this horizontal line. And this angle phi is going to play an important role in the rainbow. The first thing I ask you in problem 10-4--one of the first things--to calculate phi in terms of i and r.

Well here is i. And the same angle i shows up here. Here is an r, here is an r, here is an r, here is an r. It's only a little bit of high school algebra, to show that phi is 4r minus 2i. You've all done that because you already turned in your homework.

So what is interesting now? If you take i is 0 degrees, which is this light that strikes the rainbow head on, so to speak, then i is 0 right? Because i is the angle between the incident radiation and the normal to the surface. So that is 0. Then r is also 0. The angle of refraction. And consequently phi is 0. In fact, what the light is doing, penetrates the raindrop and it comes straight back. So this angle phi is then 0.

But now comes something that is very non-intuitive. If you increase i, then we will all accept that r goes up, and we will also all accept that phi goes up. What is not so intuitive is that there comes a certain angle of i for which phi has a maximum value. Which is roughly near 60 degrees for i. And when you hit the water drop higher, that the value for phi, which is the maximum value, will go down.

So there is no longer an angle phi larger than a certain maximum. Which I call, phi maximum. And you can see that very easily. It is a simple exercise that is no more difficult than applying Snell's law. You see Snell's law there at the top. And I remind you that phi is 4r minus 2i.

So you pick an angle of i, you calculate r with Snell's law, and you calculate phi. And you see indeed, that in the beginning, as i goes up, r goes up, and phi goes up. But when you reach i of about 60 degrees, you see that phi reaches a maximum, which is somewhere near 41.6 degrees. And look when i is 70 degrees and 80 degrees, that phi is distinctly lower. And this now, is going to be a key point in our rainbow.

The first question that I can ask you in 8.03, may be a little harder to ask high school students. Can you calculate, really, at what angle i you will get that maximum value for phi? In other words, this is sort of trial and error. Can you actually algebraically derive that? And that was also part of the homework assignment. What you now do is you say d phi di equals 0.

And then you have a long way to go. It's actually surprising. It takes a long way algebraically, to finally conclude that indeed you can calculate that angle of i for which phi is a maximum. The cosine squared of that angle is n squared minus 1 divided by 3.

In other words, this only gives you the angle of i, for which phi is a maximum. And so you substitute in here the value for n. In this case you take 1.336, and then you know what i is. Well once you know what i is, you can calculate what r is with Snell's law. And then you know what phi maximum is because you know that is phi is 4-- you know that phi is--I wrote it on the blackboard here--4r minus 2i.

So, let us right now on the blackboard here, what happens for the different colors. So we have here n, red, is 1.331. Notice that I am going to be rather specific about colors, because that's what of course, is behind the rainbow.

And violet, which is sort of the shortest wavelength that we can see. For violet light, n is 1.343. For clean water. Sea water it is a little different. So notice there is a 1% difference. That means the speed of light for violet blue light is about 1% lower in water than the speed of light of red light.

And so with this equation now, with these different numbers for n, we can now calculate what i is for which phi is a maximum value. You will find now 59.53. You can now calculate the value for r for which phi reaches that maximum. You do that from using Snell's law. You get 40.36 degrees. And so now you have your goal, which is your phi max value, which is 42.37. And if you want to you can round all these off of course. And if we do the same for violet light, then you get 58.83. And then for r we get 39.58. And for phi max you get 40.65.

So what does this mean now? This means that if sunlight strikes a water drop--just one water drop, that's all it takes--that a cone of light will be reflected into the direction of the sun. Because all angles for i are present. The sunlight comes from the left so all angles for i are present.

And so let me put here one raindrop. And let's still assume, for now, that the sunlight come straight from the left. It means then that the largest value for phi is going to be a cone of light because the whole problem is, of course, axial symmetric. You're dealing here with spheres. So red is really the winner. Red has the largest value for phi maximum that exists.

For that we have made a special triangle, which has here for me, this angle of 42 degrees. This was made a long time ago. And so I can now draw here an angle of 42 degrees. And 42 degrees in this direction, because the whole thing is axial symmetric. And that means a cone of light, by this angle, rounded off, is then 42.4 degrees. That is the maximum value for phi. There's no value for phi that this larger.

And then I need my violet light. Violet color, where did I leave it? Oh we have one here. And then the other colors have a smaller value for phi maximum. This is not to scale. This is then the violet light. And so inside here you have that cone for the violet light for which this angle is smaller. It's about 40.65 degrees.

At angles smaller than the maximum value for violet light, all colors are being reflected back. There's no restriction. There's only a restriction on the maximum value for phi, but not on the minimum value. That means all other colors can come back inside this cone. And therefore, since all colors come back to roughly the same intensity, it will be white light. Because where all colors overlap, your brain will tell you that it is white light.

But, if you only concentrate on this journey, refraction at A, reflection at B, and refraction at A, there is no light that can come out outside this cone. Keep that in mind.

So suppose I had a screen here. And I had a small opening through the screen so that the sunlight can fall through. And I project on that screen, I intersect this cone with that screen. What you would see then is the following. You would see the circle on the outside, which is red. And then you would see all the other colors that follow. And then the last color is the violet. Blue violet. The inside would be white light, and the outside, there would be no light.

Now there's a key question that some of you must be burning to ask. And that is, how come the violet light in a rainbow is so clear? The red is clear. It's obvious why the red is clear. Because the red is not polluted by any other colors. Only the red can come out at an angle of 42.4 degrees.

But the violet comes out at a smaller angle. At that same smaller angle of 40.65 degrees, the red can also come out. And the green could also come out. And the yellow can also come out. So why then is it that the violet is still so prominent? And that answer lies in Fresnel equations. An exercise I will not go through with you, but you are all capable of calculating. Now that comes down to light intensities.

If you calculate for the various colors, and here you plot phi, the light intensity. And remember intensity is always watts per square meter in terms of units. Then you will see that the red light is the winner. This angle is 42.4 degrees, has a maximum here. It's enhanced. It's not obvious at all. But there is an enhancement at that phi maximum.

And then the green light, which is a little bit inwards--because phi maximum for green is a little lower--would also have a maximum. And then ultimately the violet light, which is the last one, will also have a maximum.

And so it is because of the enhancements at that phi maximum, which is by no means obvious, which doesn't follow from Snell's law, but it follows from Fresnel's equations. That is the reason why the blue here, the violet here, still dominates. And here all the colors overlap. And so that's why you see the white light there.

So now comes the question, why do we see a rainbow? Well I'm going put you now somewhere, some other location. This is the ground, and here you are. And the sun is in this direction. The sunlight. And you know that because you see your shadow. This is your head, this is your body, and these are your legs. You're looking straight at the shadow at your head here. 180 degrees away from the sun. The sun is there, and you see your head there.

Let us assume now, because that's a condition for a rainbow, that it is raining, or that there is water here. And that somehow, the sunlight is not obscured. In other words, if it's also raining here, that's tough luck. Than the sunlight cannot get through and so you won't see a rainbow. So it is important that it is clear in the direction to the sun and that sunlight can strike the raindrops without any interference of clouds.

So it just so happens that it's raining. And you look in this direction of the sky. You're looking up in the sky. And you pick any raindrops that you want on this line. There are zillions of them. Let's just pick this one. So the sunlight strikes that raindrop, like so. And what does this one drop do? Like all the other zillions of drops, they reflect into the direction of the sun. A cone, which is like this. Each one does that individually.

So there we go, 42 degrees. Here is that cone. Forty two degrees. Here is that cone. And I will not draw the violet one. And you look at that raindrop. And at all the others in this direction. Will you see any light? The answer is no, if you accept this picture. That no light can be further out than the maximum value of the cone angle. So you don't see any light reflected from the sun. So you see the dark sky. Because when it's raining, in general, the sky is dark.

Now, you look in this direction. You pick any raindrops on this direction. There are zillions of them. Let's pick this one. Oh, I'll pick this one. So here comes the sun. And this one raindrop, like zillions of others, is going to reflect into the direction of the sun. A cone of light, or by this angle, 42 degrees. And this angle is 42 degrees. And there it is. What will you see? Do we see light?

AUDIENCE: Yes.

PROFESSOR: Then what color?

AUDIENCE: White.

PROFESSOR: White. You're looking straight into that inner cone. Because look how small the angle is. You're looking right into this area somewhere here. And so you see white light. If you look high in the sky, you see the dark sky, the background. You look lower, and then you say, ouch, white lights from the sun. And you will see in pictures, it's enormous.

But now, you look in a very special direction. You're going to look into this direction. The direction which is 42 degrees away from this line. So this angle now, is the famous 42, if you want to make it 42.4 that's fine, degrees, away from this line. Away from your shadow, 42.4 degrees away from your shadow. And here is water. Zillions of water drops along the line. And here is the sun. And the sun light comes in like this.

What is this water drop going to do? It's going to reflect back into the direction of the sun. A cone. This happens to be the direction of the 42 degrees. And then of course, this is the direction of the 42 degrees. So here is that cone. What do you see? What will you see?

AUDIENCE: Red light.

PROFESSOR: You will see red light. Pure red light. You will see the light. This light. So you look high in the sky. No light is reflected from the water. You look at the 42.4 degrees and you see only red. And then you are way lower and you see white light. And we now understand why you're going to see the other colors. Because we understand the enhancement, due to Fresnel equations, of the green and of the violet.

And so, if here is the horizon. This is away from you now. Since the whole thing is axial symmetric, you can turn this around that line because a sphere is axial symmetric, right? All the raindrops have axial symmetry. So what you can do in this direction, 42 degrees, you can also do in this direction, 42 degrees. The sphere doesn't know the difference between up and down and left and right.

And so what you're going to see now is you're going to see an outside bow, which has a radius of about 42 and 1/2 degrees. Here would be your shadow on the ground. And so this angle would be, very roughly then, the 42 degrees, if I round that off.

And then you would see the other colors follow. And then ultimately you would see the violet. And you know which angle that would be. That would be at the 40.65 there. I draw the bow specifically a little lower than your horizon because there's nothing wrong with the rain falling, of course, nearby. If the rain is very close, you could actually extend this even further. So you often see that the rainbow extends below the horizon, of course.

And so this angle then, indicates the elevation of the sun. How high the sun is above the horizon. And if the sun is very low above the horizon, this point will be here, and you will see a much larger bow. And in fact the bow at sunrise and sunset, this angle is 90 degrees. But if the sun rises in the sky, then this point goes down, and with it to the bow goes down. And therefore, midday you almost never see rainbows. That's the reason. Because the whole bow is then below the horizon, unless you spray water around you.

Nature produces two bows. One is called the primary, which is that one. And the other is called a secondary. And the secondary bow is the result of light that makes one extra reflection inside the water. And so I have not shown here every time, when the light comes in, that some of it goes back into the air. I've only shown the trajectory of the key light that I need to make the secondary.

Here the light strikes high up on the water drop. Here it strikes low on the water drop. In fact, this angle is around 72 degrees. It refracts into the rain. Reflection number one, reflection number two, and here it comes out. And this is then that angle phi. Compare that with this angle phi. Right? The radiation comes out like this, and I define this as the angle is the horizontal. So that's really this angle. Sorry that it's a little bit on the edge of the blackboard.

If you do your homework the same way that I have done for the primary, you will see that now, there is not a maximum value. That there is now a minimum value for phi. So now we have phi minimum. And the value for phi minimum for the red light is 50.37 degrees, and for the violet light it is 53.47 degrees.

So this means that the secondary bow is higher in the sky then the primary. Because look, the radii are larger. But it also means that the colors are reversed. The red is inside the violet bow. So if I make an attempt, this would be outside of the bow, and this would be inside of the bow. And then very roughly, the radius, if I round it off a little bit, the radius is about 52 degrees.

Now the secondary bow is much fainter. And that's why many of you who have been staring at rainbows, were so impressed by the primary, which is so dominant, that you don't pay attention to the secondary. Is much fainter. The reason why it's fainter is that, you people know, because you worked with Fresnel equations, you have an extra reflection inside the water drop. That means, of course, the light intensity goes down dramatically. The secondary is substantially fainter.

So now, I'd like to look at some slides. To see whether, what we have derived here, we can see that also in our slide. So we'll turn this off. Make sure that I have light enough. Yes, it's OK to start with the first slide. Yeah, if you would turn the light off. Thank you very much, Markos. Oh, I have to do that myself, right? I've been promoted to advancer of the slides. So here you see a situation that to all of us- oh, yeah, OK that's fine.

This is the maestro himself. His name was Isaac Newton. He understood the rainbow quite well. In his book, "Optics," you see this picture. And you see here, the viewer, which is you, this is then that direction away from the sun. So this is where your shadow would fall. And here you see the inner bow, which is the primary. And the light from the sun has one reflection in the back. And then here, you see the secondary bow, the light hits the water drop below, and then comes in this way. So the secondary bow outside the primary bow.

And then here is a sketch that I made that indicates that even if the sun is very high in the sky, if you want an ego trip, and if you spray your lawn, than you can have a rainbow encircling your legs. Which gives you a feeling of great power. Believe me. I've done it many times.

[AUDIENCE LAUGHS]

This is a painting from the 8th Century, Turkey. And I've always been intrigued by this painting. It undoubtedly has a connection with the Bible because you see a hand here. And I think in the Bible it says, I do set my bow in the clouds. Now you wonder, the color sequence is not correct. But then again there's only blue and there's only red.

So I think in this case we have a few options. One option is that the artist really didn't look carefully at a rainbow. That's a possibility. Another possibility is that physics was different in the 8th Century.

[AUDIENCE LAUGHS]

I think we can dismiss that possibility. And then the third one, which I think is the most likely, is that this was a conscious decision on the part of the artist. He purposely, or she purposely, reversed the colors. It was his or her opinion, impression, of a rainbow. And that's what art is all about. And then two colors is perfectly fine to get the idea across.

Anything in this country is for sale. So also rainbows. And they're dirt cheap for 12 bucks you can get a rainbow.

[AUDIENCE LAUGHS]

But as always you get what you pay for. This is a fake. Who owns a fake? Because look, the colors are in the wrong sequence. So let's not even look at this.

[AUDIENCE LAUGHS]

Rainbow maker, yeah. Sure. Ha! When I started lecturing 8.03, that's a long time ago. It was in 1973. And I wanted so badly, pictures of rainbows that I had made myself, using a water hose and spraying water around. And I managed to do that, in fact my daughter did. And so water is coming from the left. And what you see here, is clearly the primary, the red on the outside. You see the blue violet on the inside. You see all this white light. You understand now why there is this white light. But look outside the bow. You don't see the white light reflected. So that's why you see the sky's so dark.

My poor daughter, Emma. She suffered so much with a father who was a physicist.

[AUDIENCE LAUGHS]

It was January. Because my lecture started in February. And it really was freezing cold. And she was crying. But look, you have to sacrifice sometimes for the sake of science. Poor Emma.

I also wanted to make an attempt to get a picture of a secondary. That is very difficult because the secondary is so faint. And so I did that over my driveway. I lived in Winchester at the time. And so you clearly see the primary. I don't have the point where the primary is. And this is the secondary. It is further out, and you see that the colors are reversed and it is much fainter.

A friend of mine, Michael Sorgi, sent me this slide. Very nice. It was a waterfall somewhere in Austria. You see very dramatic the white lights from inside the bow. That the reflected light from inside the bow. And then it terminates right here at the red. And of course that is water here as well. But it doesn't come back in your direction. That's why you see the dark forest behind it.

Now comes here a picture that was given to me by Doug Johnson. This picture was taken in Socorro, where there's the famous VLA, the Very Large Array radio telescope. And now you see truly all the features that you want to see. The primary bow, color sequence as predicted. And notice how much brighter the sky is inside the bow than it is outside the bow. And then you see the secondary. You see that the colors are reversed.

Now, keep in mind, the secondary only has a phi minimum. So that means light can go out at a larger angle. Because phi minimum means only a lower limit. That's why the sky here becomes a little brighter again than here. Because white light can make it through two reflections. So you see here quite dramatically why it is so white here, why it is so dark here, and why some of the light returns. Because of the fact that the secondary has the phi minimum.

All this is the result of what we call geometric optics, Snell's law, and Fresnel. Nothing else. However, this is all very nice and dandy, as long as the water drops have a diameter of a few millimeters. When the water drops become smaller, and this effect begins to be noticeable around one or two millimeters, then refraction starts to play a role. And reflection has the ability for constructive and destructive interference, as we have seen in 8.03.

And without going through the math, which I cannot do this time--I do have some lectures that I do go through the math--I'm just telling you the result that you can see now, sometimes dark bands inside the bow. It's always on the inside of the primary. And then of course other areas are a little brighter. The dark bands are then destructive interference. And when you look carefully at this one, we call by the way, supernumerary bows. And when you look at this one you can actually see such a dark band here. There's a little bit of imagination.

Diffraction becomes very important if the size of the water drops get down to 100 microns, a tenth of a millimeter, and 50 microns. And when it is even smaller than 50 microns, diffraction ruins all the colors. All the colors begin to overlap. And the bow also becomes much wider. And then when the colors go away, you see white light. And so you see a white rainbow, it's also called a fog bow.

I have never seen a white rainbow. But I was so fortunate, that when I lectured in 1973 for the first time, 8.03, that there was a student in my audience who that summer after the lectures, went on an expedition to the North Pole. And he was at Fletcher Island. And he sent me a picture of a white rainbow which you will see very shortly. That he took 340 miles from the North Pole at Fletcher Island. He took it at 2:00AM in the morning, at midnight, in July when the sun is above the horizon.

And there is a striking example of a white rainbow. And whenever you see a white rainbow, which there must still be water, it's not ice. Ice doesn't give you rainbows. You need spherical symmetry. You need spherical objects to get rainbows. You always see supernumerary bows because of the fact that it is diffraction that is doing it. And so you see here this dark bend here. And then it starts white again. So this is not so uncommon that in white rainbows, you see supernumerary bows. I've never seen this one.

And there's another kind of rainbow that I've never seen that I would like to see. And that is a rainbow that you can see very shortly very close to sunset. Very shortly before sunset, or very shortly after sunrise. We all know because we have covered Rayleigh scattering in 8.03.

Why then the sun only lets red light penetrate to you? The whole sky becomes red. The clouds become red. There's only red light. There's no white light anymore. There's no blue light. There's no green light anymore. So what is nature going to do when it produces a rainbow everything turns red? And that is what you see then here. This is a red rainbow. And I wish I would ever see that.

Notice that all that white light from inside the bow is red now. But also notice that the bow is still there. That clearly is an enhancement of red light at that angle 42.4 degrees. That's because of Mr. Fresnel. That enhancement is still there at phi maximum. That is not undone by the red light. And so it's not surprising then that the bow itself still stands out very clearly. And then that the inside of the bow becomes now also red.

You've now reached the point that you should be able to answer all the questions. And to prepare you for that, I think this is a good moment to have our four minute break, our traditional break. So we will reconvene in four minutes. And then I expect all of you to be able to answer these questions. So we will start again in four minutes.

All right. So here is your chance, most of them, of course, are trivial for you now. Clearly red is on the outside. The radius is about 42 degrees. You work through that quantitatively. And the length of the bow, that depends on where it's raining. If it's only raining there, you would only see a small portion of the rainbow, of course. And what is important is the elevation of the sun. The higher the elevation of the sun, the less rainbow you see. We just discussed that.

And then the width of the bow. That's a good one. That's a good one. The width of the bow. You would think that the width of the bow is the difference between the two angles phi maximum. And that means that the width of the bow would then be 1.8 degrees. That's not true. If there any astronomers in my audience, you can probably tell me why you have to add to the 1.8 degrees, why you have to add half a degree. The actual width is closer to 2.3 degrees. Why is that? I see one hand, I want to see a few more hands. Any astronomers? Why is it?

AUDIENCE: I'm thinking maybe this is just the beginning of the violet light.

PROFESSOR: You tried. The others didn't. I said is there an astronomer in my audience. I gave you a hint. The sun is half a degree in the sky. That means each point of the sun makes a rainbow. And since in this direction you have half a degree, and in this direction you have half a degree, imagine that when you see these rainbows there, that you obviously get the spread in this direction of half a degree, and also in this direction. And so you have to add half a degree and that's actually quite noticeable. The finite size of the sun. Half a degree is the diameter of our sun.

And so also the width of the secondary is not just the difference between these numbers, which would give you about 3.1 degrees, but it's closer to 3.6 degrees. And so we know there is a secondary. The time of the day. Yeah, Well when does it rain? And when is the sun low in the sky? Midday, you're not likely to see rainbows because the sun is too high in the sky. Depends of course on the season, whether it's summer or winter.

I've been told that it rains more often in the afternoon then in the morning. If that's true, then the afternoon is a more likely time then the morning. And that is when the bow would obviously appear then in the east when the sun is in the west. And certainly in Boston, the sun during midday would never be in the north. You have to go for that to the southern hemisphere. So you're not likely to see a rainbow in the south. But you might see rainbows in the north in the winter when the sun is low in the horizon. So that covers the idea of time of the day and in what direction.

So the secondary is there. The color sequence is reversed. We just discussed the width of the bow and the radius of the bow is about 52 degrees. And now comes the question that all of you have worked out. Problems set number 10, is the bow polarized? And the answer is yes. And is it weakly polarized? No. It is enormously polarized. And you all came up, I hope with the right answer. Problem 10, 4h I think it was, 91%. The linear degree of the linear polarization is 91%. And I will of course demonstrate that today.

As far as the direction of the polarization is concerned, well think about Fresnel's equations. And think about the Brewster angle. Because the Brewster angle is really responsible for the fact that the rainbow is so highly polarized. Remember the tangent of the Brewster angle in this case would be 1 divided by 1.336, if I take this as the average index of refraction for light. That leads then, to a Brewster angle of about roughly 36.8 degrees.

But, remember that r, this angle of reflection, when you make the rainbow, that r is 40 degrees. That's where the colors come from. Well that is only a few degrees away from the Brewster angle. So as this light comes in unpolarized, here it is still perhaps not completely unpolarized, but not very strongly polarized.

But it is here that action starts. This angle is within a few degrees of the Brewster angle. So this light that comes back is polarized perpendicular to the plane of incidence. And so whatever comes out here is of course also perpendicular to the plane of incidence. So the E vector is oscillating like this. Remember the plane of incidence is defined as the direction of the light and the normal to the surface. So, that's in this case the blackboard.

And so the radiation is polarized. You have calculated it for 91%, linearly polarized. And because of the actual symmetry of the problem, that means it is polarized like this, in this direction. And here it's polarized like this, and here, it's polarized like that. Nearly 100%.

When I see a rainbow, I always check whether it's really polarized. I always worry about it. Maybe someday it may not be polarized. And so when I go to the beach, which I often do, Plum Islands, one hour north of here. Wonderful reservation land. In the afternoon the sun is there. The ocean, the water is there. Then the waves come in and the water splashes up. And then, 42 degrees away from my shadow, when the water splashes up, rainbow. Rainbow here, rainbow there. The water splashes up high. Beautiful rainbow just for me. Beautiful.

And I remember a few years ago, I took a friend to Plum Island. His name is Bill Predorski. He's also a physicist. And we were looking at it and splashing in the water. And I said, Bill, look at the rainbow. Bill looks. What rainbow? It's not raining he says. I said, look in the water. Look at the waves. Bill looks. Nothing. I said well let's just wait for really good one. And boy, there was a real wave coming in the water splashed up. A gorgeous rainbow. Bill did not see the rainbow.

[AUDIENCE LAUGHS]

PROFESSOR: So I got a little annoyed. And then I looked at Bill. I looked at Bill, and he look like this. And then I knew, and you know too. Because polarized sunglasses are designed in such a way, which was covered in 8.03, that the direction of polarization is like this. But the bow is like that. And so, you kill the bow.

So I said, Bill, would you please take your sunglasses off. He did. And I was waiting for when there came a wave so fantastic. And Bill looked and he says, nice rainbow, Walter. Very low key. He comes from the south, you know. Nice rainbow, Walter. That was a nice experience.

So now I want to demonstrate to you a rainbow. I asked all of you to bring an umbrella. But most of you didn't. Who brought an umbrella? See, the problem is that the majority did not, and we were afraid of that. So therefore, we had to change the demonstration from real rain to something that's a little bit of rain. In fact it is so little rain that it's only one raindrop. Not very much rain, is it?

Here it is. One raindrop. Can I make you see a rainbow with one raindrop? No, because you see here why you need millions of them. Can I make you see this with one raindrop? Yes I can. Here is my light gun. I'm going to blind you.

And then out of this one drop comes this cone. Red on the outside, blue on the inside, and then white. That's what I can show you. And I can show you that this light is highly polarized. I asked you to bring your polarizers, but not for this demonstration. Because when the light reflects off the screen, the polarization is lost. It's only on the way to the screen that it is polarized.

One water drop. Don't expect too much. It's only one water drop. So the rainbow will be faint, if we call it a rainbow for now. And of course you don't want to be blinded, so we also going to absorb the sunlight so that you will actually be able to see the bow there. The light is so faint that your eyes have to adjust first to the darkness. So if we turn the lights off, I'll give you 30 seconds to get used to the darkness. And in the meantime, it is so properly timed that the screen is coming down.

So let your eyes adjust. And there it is. If that's not red on outside what is? It that's not blue the inside what is? And you see the white light here? This is the primary bow. This is not the secondary. This is some weird reflection because of the glass. And this is highly polarized. I have a polarimeter here. And I will hold that in the beam. This is the way that the light can go through. Can you see that? It go through?

And now I've rotated it 90 degrees and I can kill that light. It is nearly 100% polarized. If I do it here, the angle is different. The polarization angle is like this. So this is the way that I can let the light through. Of course, there's always absorption, right? With a polarized always 50% is lost anyhow. And then there is in addition some absorption. But it's clear that the light goes through there.

And now I rotate it 90 degrees and then you kill it. But of course the white light, which is very close to the bow, is also the result of reflection at that point B in the back of the water drop. So that white light, certainly the white light here and there is of course also very close to the Brewster angle. So that is also highly polarized. And so you can see that the white light go through here, and if I rotate it 90 degrees that white light also goes away. And of course, when you go further in than the degree of linear polarization becomes less.

So this is then what you can do--yeah, we have lights again--this is what you can do then with one water drop. It has all the ingredients, all the physics, that you need to explain and to understand the rainbow. But it's really not the rainbow itself.

There are other phenomenon in the sky which are very common. And they are also remarkable. And many of you may never have seen them, and yet they are so common that I see them every week. And so I want you to become alert to them without going into the details of the physics. Ice crystals high up in the atmosphere, can cause stunning halos.

The most famous one is a 22 degree halo that you can see around the sun and you can see around the moon. You see it in summer and winter because high up in the atmosphere, the temperature is way below freezing. Red is on the inside. Colors are never truly spectacular. The 22 degree halo is so common that, it's fair to say, I see it almost every week. And the reason why you probably don't, who wants to look in the direction of the sun? That's a terrible thing to do.

Well what I do, I first hold up my fist and I block out the sun and then I look. And that's why I see them so often. The moon is, of course, easier because you don't have to block out the light from the moon. So if you look at the moon very often, several times a month you see this gorgeous 22 degree halo.

And then there are many other phenomenon, which I will show you also a picture, which are all the results of ice crystals. They have names. They're called sun dogs. That's a 46 degree halo. 24 individual names are identified in books for these various arcs, tangent, that you see from the ice crystals.

So if I can have the next slides on this, and make it dark? Then I think we're coming up to the-that's the 22 degree halo. See the red is on the inside. It's not as colourful as a rainbow but it's very distinct. There's no question when you see it, you're looking at the 22 degree halo.

And this is then, phenomenon that it's very rare, I've never seen it so complete. This is the 22 degrees halo. And then there is here one, that you don't see so well, which is a 46 degree halo. And these things are called sun dogs. And this arc has a special name. The sun dogs I see often, actually here in Boston. They are not so uncommon at all. This picture was taken near the South Pole station.

So this business with ice crystals is quite common. There is another phenomenon which probably all of you have seen, which is called a glory. And a glory is not the result of refraction and reflection. But the glory is the result of diffraction. You only see glories when you have extremely fine water drops. For instance, as you have them in clouds, sometimes.

And most common, you see it when you fly over clouds and you have a seat away from the sun. In fact, that's the reason why I always, when I make reservations, I always want a seat away from the sun. Then you look in the direction where the shadow of your plane would be. And if you fly very low, you can actually see the shadow of your plane. And then you see, around the shadow of your plane, these beautifully colorful circles. They are complete circles.

Their diameter is a strong function of the size of the water drops. Like with all diffraction, the smaller the water drops, the larger the angle. And the larger the water drops, the smaller the angle. If you fly over various clouds, you may see that the glory changes in size. I've seen that many times.

And so let me show you then, a picture that I took several years ago of a glory. And many of you have seen this. In fact, I get countless pictures by email from students who took my lectures, and people who didn't take my lectures, who send me this kind of stuff and say, Professor Lewin, we see a complete rainbow. Well this has nothing to do with a rainbow.

Of course, you can see the angle is also way smaller. But that depends again on the size of water drops. The angle can be very large. And you can also know from this picture where my seat was. You're always at the center of course, where your camera is, at the center of the glory. So I was sitting just behind the wing. Which is another reason why I always not only want to sit on the side away from the sun, but I also want to have a clear view of the ground. Which is a little bit more complicated sometimes.

I always dreamed of sainthood.

[AUDIENCE LAUGHS]

PROFESSOR: Chances are small, but never zero. And so I decided, if I can somehow create a glory around my own head, it can't do any harm. Many years ago, I visited Georgia, the Caucasus, were still part of the Soviet Union. And my host took me to the 6-meter telescope there, which was at the time, the largest optical telescope on Earth. And I was welcomed very gracefully, very nice food.

And then I talked to the local astronomers and they said, you know that this telescope is a joke? It just cannot produce any science. And I said, well, what do you mean by that? They said well, it was put at the wrong location. Every evening just before sunset, the fog comes up from the valley, and you can't observe. You're in the clouds all night. This is the most ridiculous thing, here, to have a telescope.

I said to myself, this is my chance. Because if the fog comes up from the valley, then I can photograph my shadow on this wall of fog, because they said it really comes up like a wall, then I will get a glory around my head. And I thought they were joking. I said, well that's a nice story but you must be joking. And they said well just come out at 5:30 we will show you. I came out at 5:30 and fair enough, like a wall, it came up and in a matter of one minute, we were in the clouds. And we stayed in the clouds the whole night.

[AUDIENCE LAUGHS]

PROFESSOR: The next day, I had my camera. And the sun was there. And the fog came, perfectly on time. The timing was very critical because the fog comes very quickly. Let me first show you that the fog indeed comes like a wall. There it is. That's the 6-meter telescope in Georgia. And then this wall comes and in a matter of one minute, it's over you. But I was quick.

[AUDIENCE LAUGHS]

PROFESSOR: Saint Walter, after all. Isn't that a nice example of a glory around my head? OK, can I have the next slide? Oh, I think I have to do that. And then I would like a little bit more light. Can you put the light on TV?

I suppose that you recognize this slide. It is the mystery picture of 8.03, of the web page. It was the astronomy picture of the day on September 13. I received about 3000 responses from viewers all over the world. And I answered each one of them. Took me two months. I did about 50 per day. About 50 people had the right idea about what causes this phenomena. Only two from MIT.

But of those 50, there were really only about five who had a complete understanding of the physics. About 400 of the 3000 believed that is a glory. Well clearly, you know now enough, that this is not a glory. Many explanations were very interesting. Some believed it was an atomic explosion.

[AUDIENCE LAUGHS]

More than one. Others suggested that I was photographing a total solar eclipse. Imagine, the day was given. It was June 20. It never occurred to them that there was no total solar eclipse on June 20. But that's a detail. Some people who knew that I was an astronomer said, ah, you were photographing a supernova explosion.

[AUDIENCE LAUGHS]

PROFESSOR: There were three people, independently, who said the rings were caused by the vibration of a jack hammer.

[AUDIENCE LAUGHS]

PROFESSOR: That would make a very nice problem for the final. How do you get this from a jack hammer?

[AUDIENCE LAUGHS]

PROFESSOR: And then there was one who says, it's the sun shining through a peephole like those you find in sex parlors.

[AUDIENCE LAUGHS]

PROFESSOR: And then he said between parentheses, (I know).

[AUDIENCE LAUGHS]

PROFESSOR: The very best solution came from a five-year-old girl, who wrote me a letter, handwriting, and I really want to quote her verbatim. She said, Professor Lewin, it's very simple. You painted the picture with crayons on the ground and you photographed it.

[AUDIENCE LAUGHS]

PROFESSOR: Isn't that wonderful? These five year old kids. I mean, that's the easiest solution. You just throw it on the ground and take a photograph. So I sent her a very nice letter back. And I said that she was very close. [INAUDIBLE]

All right, let's look at this picture. Red is on the outside, violet is on the inside, and white light comes from inside the bow. This can only be produced by spherical transparent beads. There's no other way. The radius is very small, it's about 20 degrees. It cannot possibly be due to water. For one thing, there is no water. Also, the width of the bow, the width which you can easily measure with a ruler. The width of the bow is about 16% of the radius, whereas with water bows, that's only about 5%.

On June 20, I visited to the Dakota Farm Museum in Lincoln. Sculpture garden. With my son, and my significant other, who is in the audience. It was about 1:00PM. And we walked by an area where new construction was going on, because the Dakota Farm was building a new visiting center. And my son, Chuck, all of a sudden said, Dad look! And we all looked at it, and this is what we saw.

I had never seen anything like that before in my life. But I knew immediately, within seconds, that it can only be made by spherical transparent beads. That's the only way it could be made. So I immediately thought of maybe glass, maybe plastic, whatever. But it had to be spherical transparent beads.

So I wondered, why would there be so many of these beads here in this construction site? And was not until days later, when I actually discussed it with Markos, who is also in the audience, who said that these glass beads, about a quarter millimeter diameter, are being used was for sand blasting. And indeed, a lot of sand blasting had been going on, you could see, in that area. And so they had spilled a lot on the ground, luckily.

In problem 10-4, I made an effort to give it away to you. So that you could score your extra credit for 8.03. I really tried to give it away. But only one person got the message. I wrote in problem 10-4, in the last question, and I quote myself verbatim, "in a world far, far away, rain comes down as small drops of glass." And then after you had done all the work for the rainbow problem, all I asked you, what is now the radius of a glass bow? And what was the answer? 22.8 degrees. But none of you made the connection with this picture, except for one person, who did.

So let's look at the physics. It doesn't take very much. If I can have all the lights. So if we take the index of refraction for glass, 1.5, and I use my--where's my cosine squared? I use this equation, then I can calculate what the maximum angle is of phi. And I'm not going to do it for different colors. I just take the mean value of 1.5. So I simply substitute 1.5 in this equation. And that gives me then an i of phi for which the maximum angle is reached, that is 49.8 degrees.

That then translates with Snell's law, to phi maximum 40. Angle of refraction of 30.6 degrees. So that's simply a matter of Snell's law. And then, phi is 4r minus 2i, and so we now get phi max is about 22.8 degrees. You can calculate, for my picture, with a ruler, the linear size of the bow. Of course we always think in terms of angular size. But you can actually calculate the linear size. If you could give me TV.

Because you know my head. It's about 20 centimeters. And so you can calculate now, what the radius of that bow is in terms of linear size. That comes out to be about 65 centimeters. Now the angle is about 20 degrees. So I will bend over a little so my head was about five feet from the ground. And I took the picture. And that exactly gives you then, the angle of about 20 degrees.

Now, I made, on the spot, a very quick calculation in my head about the Brewster angle. I remember, since I've taken 8.03, that the tangent of the Brewster angle is 1 over the index of refraction. So that it is 1 over 1.5. Now I didn't have a calculator on me, but I came roughly, that the Brewster angle was probably around 34 degrees.

And so I concluded that that was probably very close to the Brewster-- to the r value. In other words, to the value here, which makes the rainbow. So the theta Brewster for the transition from glass into air, is about 33.7 degrees. And it is within 3 degrees of this value. And even though I couldn't be so precise there on the spot, I concluded that it had to be highly polarized.

I always carry on me, as you will from now on, always a linear polarizer on me. So I took my linear polarizer, and I impressed my significant other. You can ask her. And I said look through the linear polarizer. And indeed, when we looked at the bow, highly polarized.

I took some beads home. Scooped them up. There were lots of them. Because I immediately realized I was going to use this for 8.03. We ordered a few kilograms. It is dirt cheap. Only cost a few dollars. And Markos glued them on black paper. Here is the black paper, and here are the glass beads on the black paper.

The sun is in the back of 6-120. And we hired someone, an expert, to turn on the sun. And we hired someone to turn off the lights in the lecture hall. And here I'm standing in front of the glass beads. On June 20, the sun was high in the sky, and the glass beads where there.

Today December 7, the sun is there, and the glass beads are here. And you won't believe what I'm seeing. I'm seeing that. I see a beautiful glass bow, I see white from the inside. I have my linear polarizer with me, and boy is it polarized.

And I want all of you to see this. I don't think this has ever been demonstrated in any lecture halls. When you see this you will just never forget it. Not only will you never forget this, but whenever in your life you see a rainbow, you will think of me. I want you to be very careful as you come down the stairs. Bring your linear polarizers. And take your time.

We have at least 10 minutes left in this lecture. I planned it that way. Be very careful. Don't run into this stuff. I hope to see all of you on Thursday, when I will give my last farewell lecture. And then, you're on your own, if you think you can handle it.

So walk by. The closer you are, the nicer the bow is. And if you close one eye, you even see a better bow. And this is a question for you. Why would it help if you close one eye? Which I did not have to do on June 20. So take your time. Don't rush. And use your linear polarizers. Don't rush.

PROFESSOR: Nine more minutes and then in nine minutes you can slowly get out here. And all of you should have plenty of time to stand there for at least 10 seconds. Go very close. Close one eye. When you're close, you see it even more dramatic, because the angle will always be 23 degrees. No matter what your distance is. But if you're very close, then the linear size of the bow will change. Not the angle. And then convince yourself about the degree of polarization.

You have a polarimeter, Jefferey?

AUDIENCE: I always carry it.

PROFESSOR: Oh, so do I. You're a physicist, of course. I didn't mean to insult you.

The angular size is independent of your distance, but at the linear size of the bow, of course, becomes smaller if you're closer.