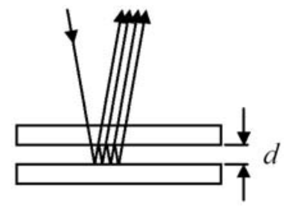
Massachusetts Institute of Technology OpenCourseWare

8.03SC **Fall 2012**

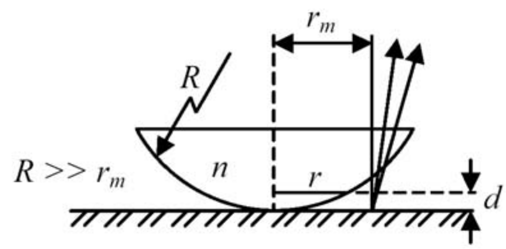
Problem Set #9

Problem 9.1 (Bekefi & Barrett 8.1)[1] – Thin film interference



White light is incident *normally* (i.e. *not* at the angle shown in the figure) on an air film of thickness d formed between two glass plates. What must be the smallest film thickness d if only blue light of wavelength 4000 $\mathring{A}$ ($= 4 \times 10^{-7}$ m) is to be reflected strongly?

Problem 9.2 (Bekefi & Barrett 8.4) – Newton rings



A plano-convex piece of glass (index of refraction n) rests on a plane parallel piece of glass as shown. The radius of the spherical surface is R and it is much greater than r_m. Light of wavelength λ is incident normally and reflected at the spherical glass-air interface and at the air-glass interface of the glass plate. The two reflected beams then interfere to produce a series of alternately bright and dark concentric circles when viewed from above.

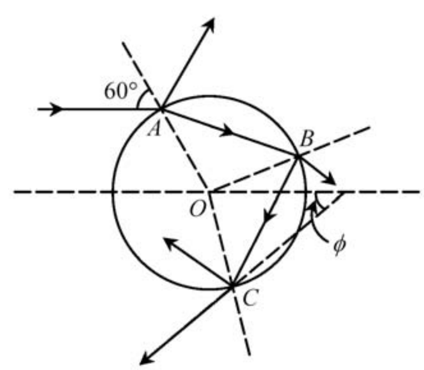
a) Find the radial distance, r, from the point of contact at which the separation between the spherical surface and the plate upon which it rests is d, i.e. find the relation between r and d.

b) Derive an expression for the radial distances, r_m, at which bright rings will be observed.

c) Same as (b) for dark rings. Let $R = 2$ m and $\lambda = 640$ nm.

d) What then is the spacing (difference in radii) between the first 2 dark rings and what is it between dark rings #25 and #26?

Problem 9.3 – Rainbows



A very narrow beam of unpolarized red light of intensity I_0 is incident (at A) on a spherical water drop. The angle of incidence is 60°. At A, some of the light is reflected and some enters the water drop. The refracted light reaches the surface of the drop at B where some of the light is reflected back into the water, and some emerges into the air. The light that is reflected back into the water reaches the surface of the drop at C where some of the light is reflected back into the drop, and some emerges into the air. The index of refraction, n, of water for the red light is 1.331.

a) What are the intensity and degree of polarization of the light that refracts into the drop at A?

b) What are the intensity and degree of polarization of the light that reflects at B?

[1] The notation "Bekefi & Barrett" indicates where this problem is located in one of the textbooks used in 8.03 in 2004: Bekefi, George, and Alan H. Barrett *Electromagnetic Vibrations, Waves, and Radiation*. Cambridge, MA: MIT Press, 1977. ISBN: 9780262520478.

c) What are the intensity and degree of polarization of the light that emerges into the air at C?

d) Let the angles of incidence and refraction at A be θ_1 (60° in figure) and θ_2, respectively. Find the change in direction of the incoming/outgoing light ϕ (see figure) in terms of only θ_1 and θ_2.

e) Calculate the angle ϕ if θ_1 is 60°. Do this for both red and blue light (the index of refraction for blue light is 1.343). The speed of blue light in water is about 1% slower than that of red light.

f) For a given wavelength, there is one and only one value of θ_1 for which ϕ is a maximum (ϕ_{max}). Prove that this occurs when $(\cos\theta_1)^2 = (n^2-1)/3$, where n is the index of refraction.

g) Using the equation in (f), calculate the values of θ_1 for both red and blue light that give rise to maximum values for ϕ. Using your result from (d), calculate the maximum values of ϕ (each wavelength will have its own set of values for θ_1 and associated ϕ_{max}).

h) In a world far-far away, rain comes down as small drops of glass (with index of refraction of about 1.5). The living souls there talk about a "glass bow". What is the maximum value of ϕ for these glass bows? Compare this with our rainbows.

Formation of Rainbows: The 60° angle of incidence shown in the figure is very close to the values you found in (g). Thus, the value of ϕ as shown in the figure is also very close to your ϕ_{max} values in (g). The fact that ϕ_{max} is different for the red light than for the blue is key to the formation of the rainbow. The geometry shown in the figure will play a central role in the lecture on rainbows (Lecture 22). A rainbow will be made in the classroom. It is advisable to bring an umbrella.

Problem 9.4 (Bekefi & Barrett 8.5) – Superposition of N oscillators

We desire to superpose the oscillations of several simple harmonic oscillators having the same frequency ω and amplitude A, but differing from one another by constant phase increments α:

$$E(t) = A\cos\omega t + A\cos(\omega t+\alpha) + A\cos(\omega t + 2\alpha) + A\cos(\omega t + 3\alpha) + \cdots$$

a) Using graphical phasor addition, find $E(t)$; that is, writing $E(t) = A_0\cos(\omega t + \phi)$, find A_0 and ϕ for the case when there are five oscillators with $A = 3$ units and $\alpha = \pi/9$ rad.

b) Study the polygon you obtained in part (a) and, using purely geometrical considerations, show that for N oscillators $E(t) = (NA)\dfrac{\sin(N\alpha/2)}{N\sin(\alpha/2)}\cos\left[\omega t + \left(\dfrac{N-1}{2}\right)\alpha\right]$.

c) Sketch the amplitude of $E(t)$ as a function of α. (The above calculation is the basis of finding radiation from antenna arrays and diffraction gratings.)

Problem 9.5 – Think big

a) The criterion for Fraunhofer diffraction is $z \geq \dfrac{2D^2}{\lambda}$. What are the meanings of z, D, and λ?

b) The first zero in the case of single slit Fraunhofer diffraction is found when $\sin\phi = \lambda/D$. Use this result to derive, in a single line that, z should be larger than a few times D^2/λ. This is a very easy way to appreciate the Fraunhofer criterion.

Gedanken Experiment. Suppose we want to observe Fraunhofer's single slit diffraction from a star 10 light years away from us. As far as the above condition is concerned, we could make the slit about 12 meters wide without losing our Fraunhofer diffraction pattern that we want to observe on a screen. Take 500 nm as the wavelength.

c) What should the minimum distance be between the slit and screen (photographic plate)?

d) How wide would the central "bright" maximum be on our screen? Predict first, then calculate!

Now leave the screen and slit in place but narrow down the slit width to 2 meters.

e) First try to predict, then calculate the approximate width of the central "bright" maximum on the screen. Don't be too hasty with your prediction!

f) What would be the approximate width of our central "bright" maximum due to diffraction if we made the "slit" 96 meters wide, without changing the distance between the gap (on Earth) and the screen (on the Moon)?

g) List your reasons why the above "gedanken-experiment" would never give satisfying results.

Problem 9.6 (Bekefi & Barrett 8.7 amended) – Angular resolution

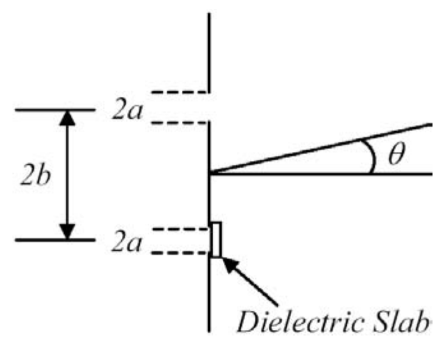
Two bright lights are 1 ft apart and 10 miles away. They are observed by a telescope, the lens of which has a diameter of 5 cm. A slit is placed in front of the lens and oriented so that **its width is parallel to the line that connects the two lights**. The slit width is variable, and it is narrowed until the two lights are just barely resolved. Find its width, assuming the effective wavelength to be 6000 $\AA$.

Problem 9.7 (Bekefi & Barrett 8.8) – Pinhole camera

A pinhole camera for visible light is made from a cubical box (length of one side = L) by drilling a small circular aperture (diameter = d) in one side and using the opposite inside wall as the screen where the film is placed. Approximately what value of d will provide the sharpest image on the film? (*Hint: Calculate the full size of the geometrical plus diffraction image of a distant source.*)

Problem 9.8 (Bekefi & Barrett 8.9) – Double slit interference

A plane electromagnetic wave of wavelength λ_0 is incident on two long, narrow slits, each having width $2a$ and separated by a distance $2b$, with $b \gg a$. One of the slits is covered by a thin dielectric slab of thickness d, and dielectric coefficient κ, with d chosen so that $(\sqrt{\kappa}-1)d/\lambda_0 = 5/2$. The interference pattern due to the slits is observed in a plane a distance L from the slits, where L is large enough so that the far field approximations may be used, that is the pattern depends only on the angle θ from the normal to the slits, as shown.



a) Consider effects due to interference only. What is the condition for a maximum in the pattern? Sketch the interference pattern.

b) Now include effects due to both interference and diffraction. How is the intensity distribution modified from that obtained in (a)? For a ratio of $b/a = 10$, sketch the resulting interference-diffraction pattern. (Assume that all angles involved are small enough so that $\cos\theta \simeq 1$, and hence that the optical path through the dielectric is independent of angle.)