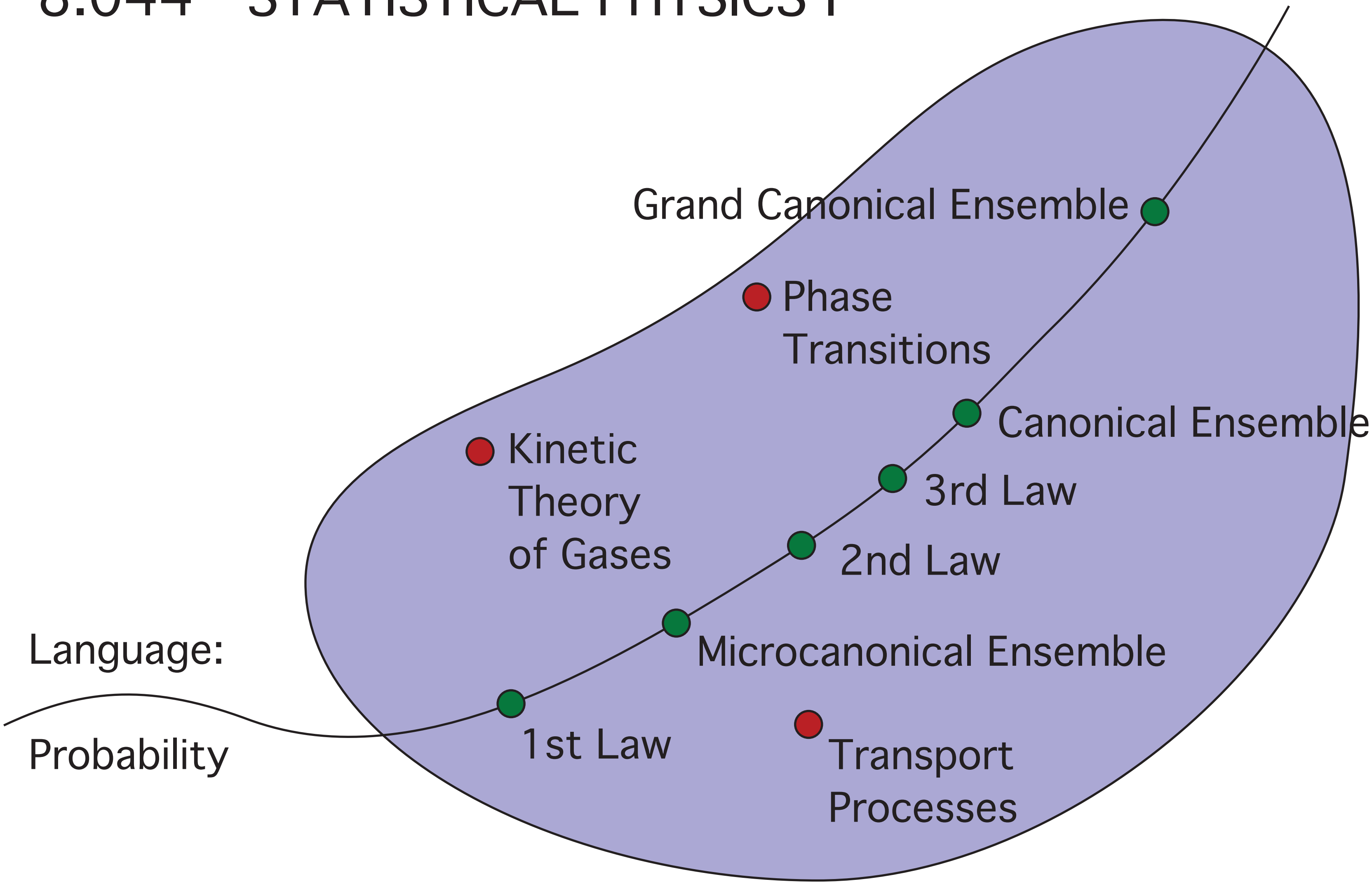
8.044 STATISTICAL PHYSICS I

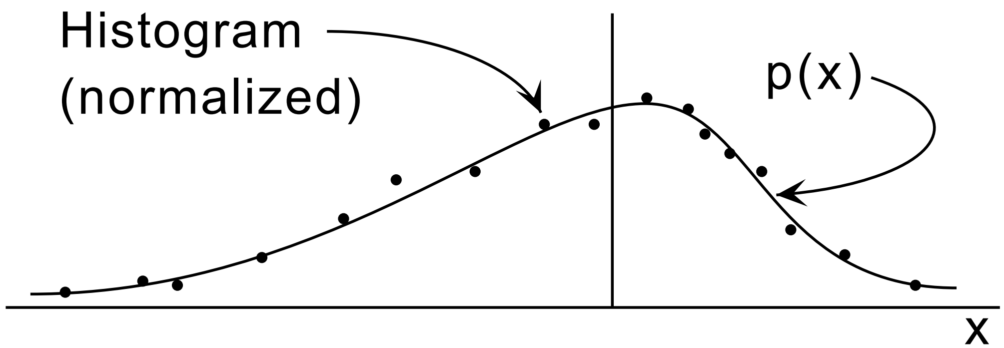


PROBABILITY

Random variable (ignorance and/or QM)

Continuous, discrete, or mixed

Probability density: $p(x) \leftrightarrow p_x(\zeta)$



$$\text{PROB}(\zeta \leq x < \zeta + d\zeta) = p_x(\zeta)d\zeta$$

$\Rightarrow p_x(\zeta) \geq 0,$

$$\int_{-\infty}^{\infty} p_x(\zeta)d\zeta = 1,$$

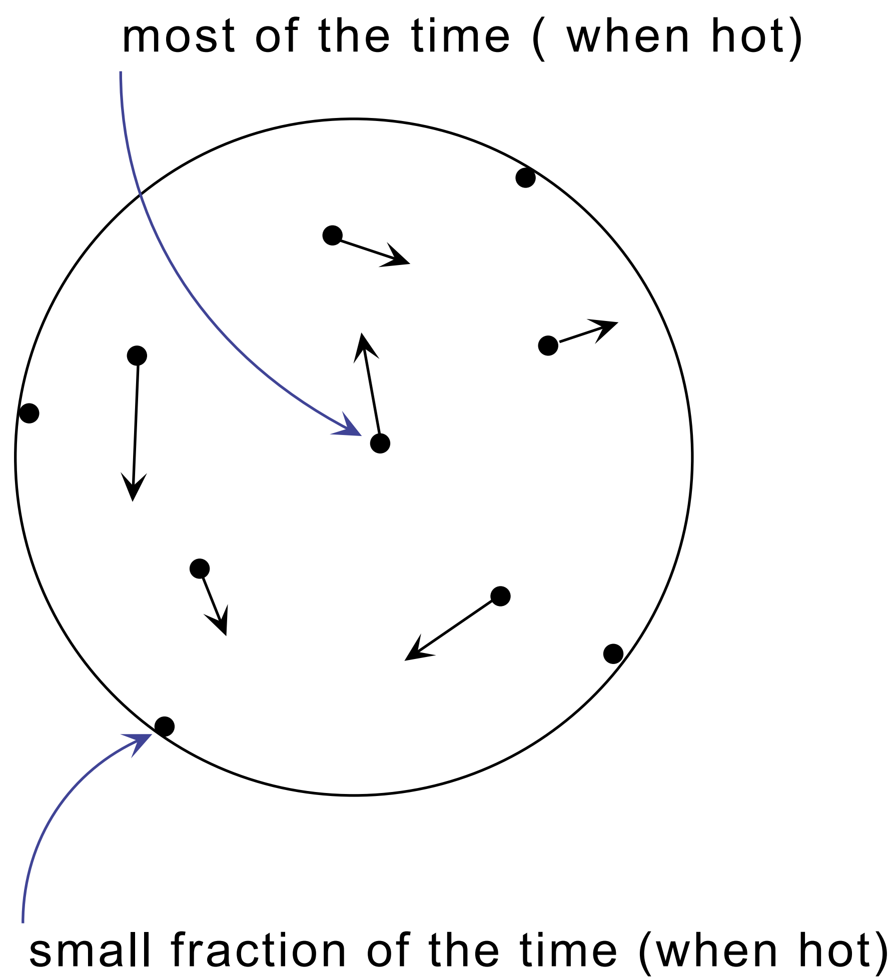
$$\text{PROB}(a \leq x < b) = \int_a^b p_x(\zeta)d\zeta$$

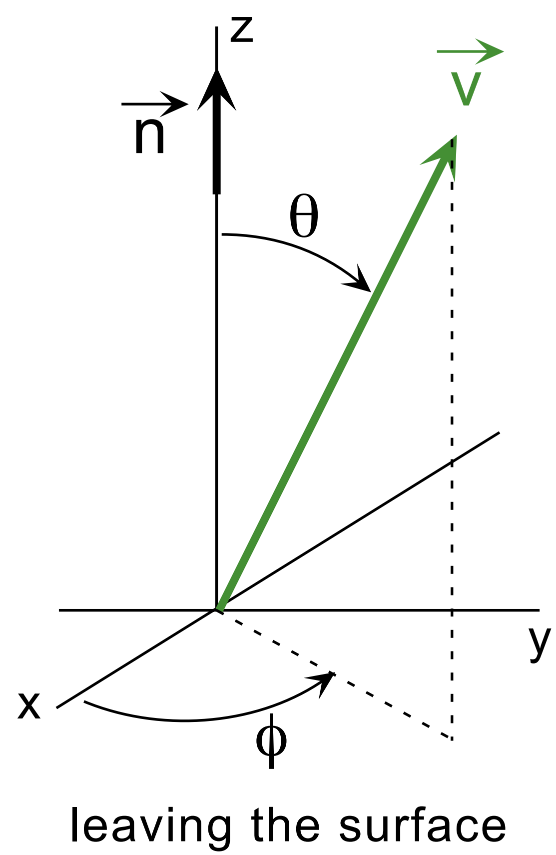
Cumulative probability:

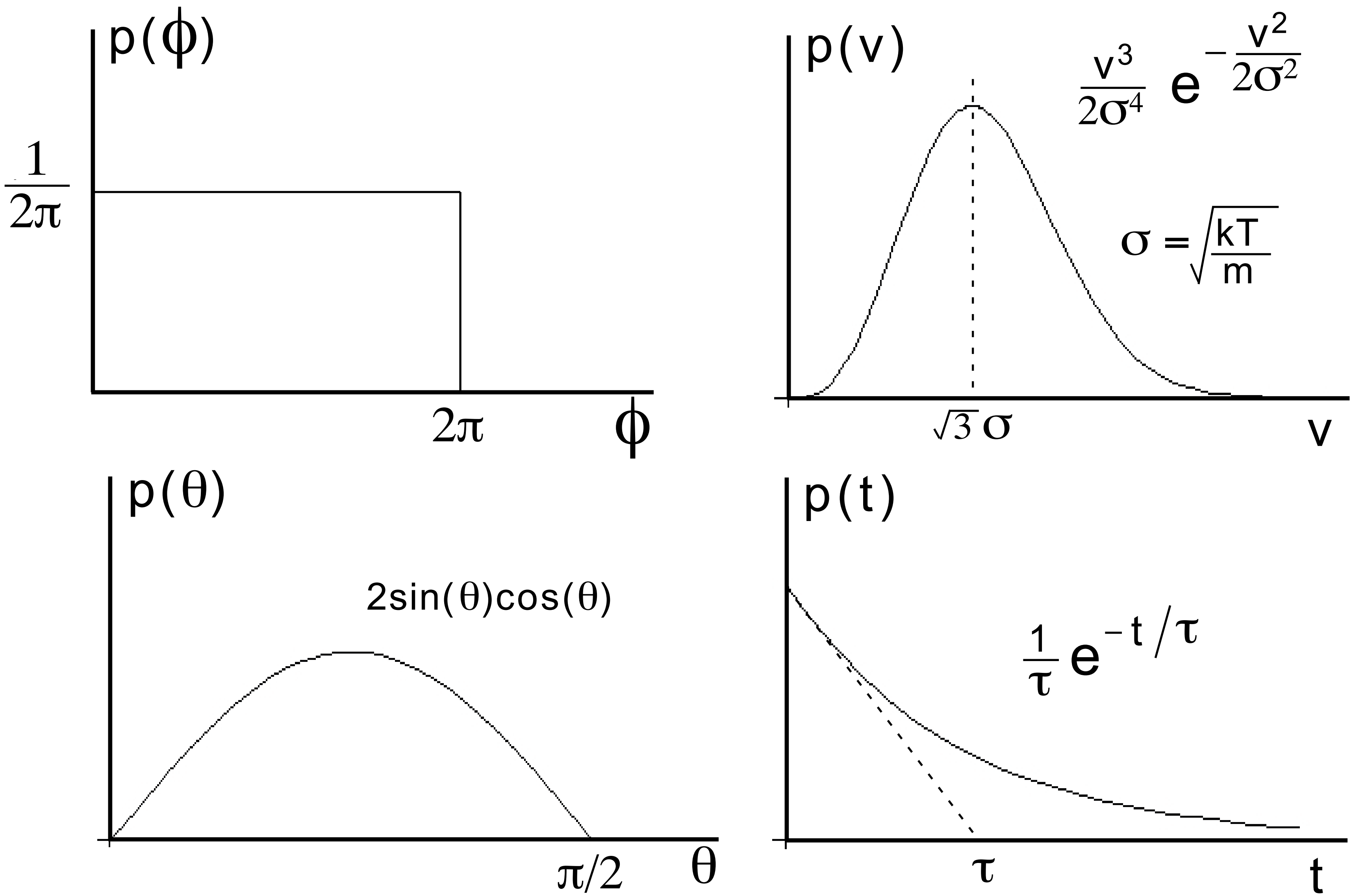
$$P_x(\zeta) \equiv \int_{-\infty}^{\zeta} p_x(\zeta')d\zeta' \;\; \Rightarrow \;\; p_x(\zeta) = \frac{d}{d\zeta}P_x(\zeta)$$

Either $p_x(\zeta)$ or $P_x(\zeta)$ completely specifies the RV x.

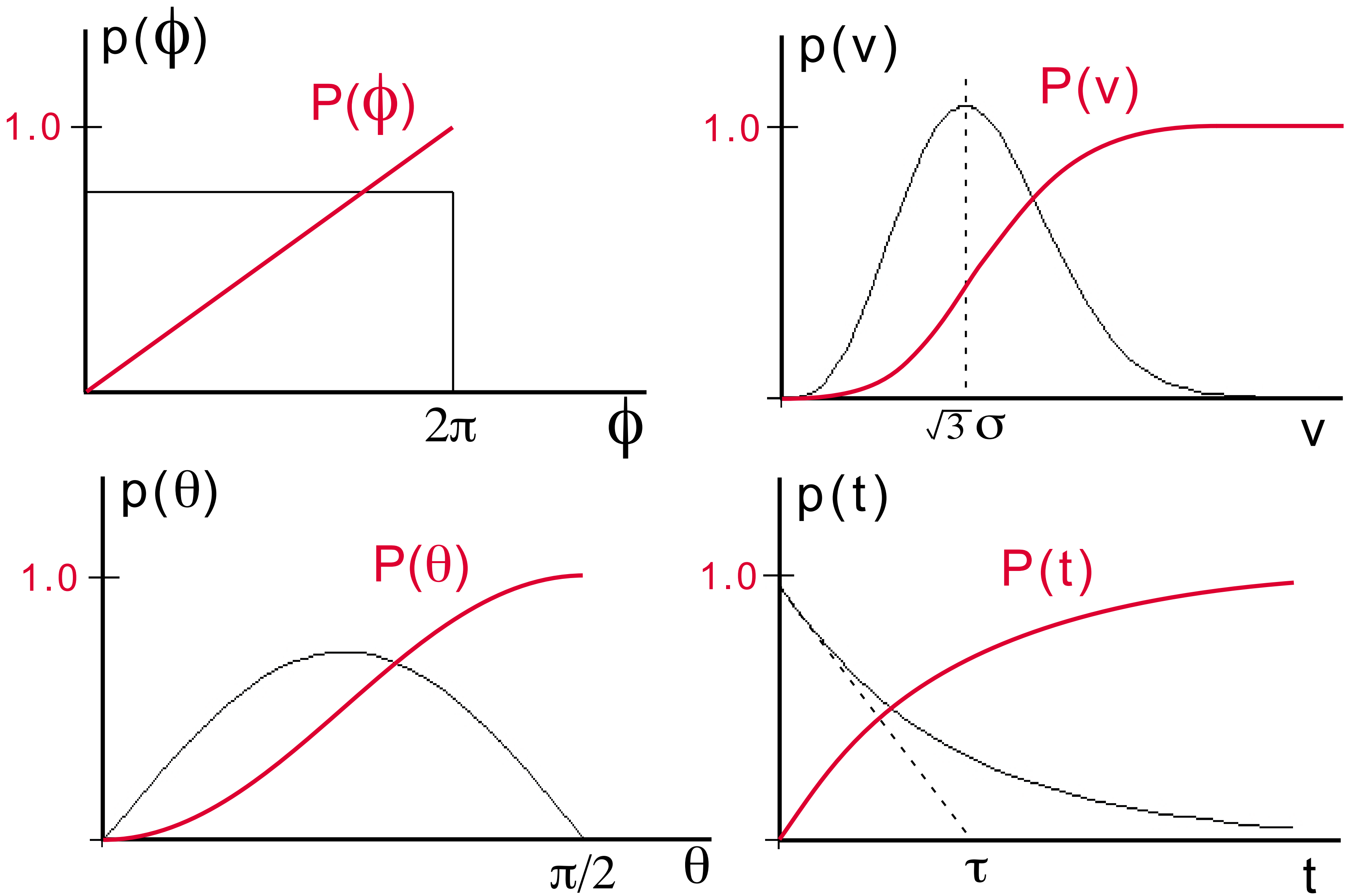
<u>Example</u> Physical adsorption of a gas







p(ϕ)
1/2π
2π
ϕ
p(v)
$\frac{v^3}{2\sigma^4} e^{-\frac{v^2}{2\sigma^2}}$
$\sigma = \sqrt{\frac{kT}{m}}$
$\sqrt{3}\,\sigma$
v
p(θ)
2sin(θ)cos(θ)
π/2
θ
p(t)
$\frac{1}{\tau} e^{-t/\tau}$
τ
t



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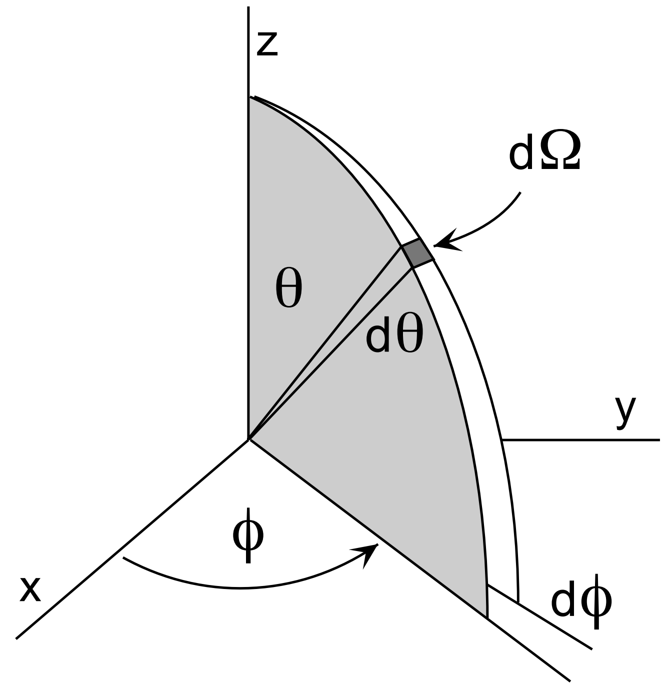
$$\text{PROB} = p(\theta)d\theta\, p(\phi)d\phi$$

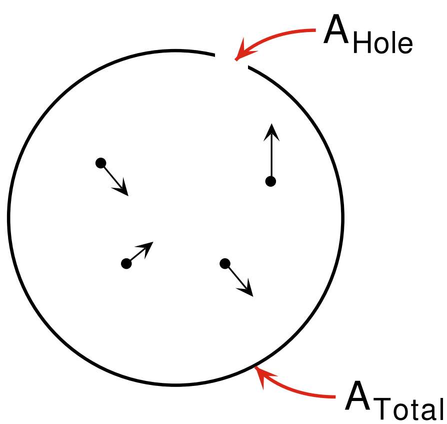
$$= 2\sin(\theta)\cos(\theta)d\theta(1/2\pi)d\phi$$

$$d\Omega = \sin(\theta)d\theta d\phi$$



$$\text{PROB}/d\Omega = (1/\pi)\cos(\theta)$$

Example Atom escaping from a cavity



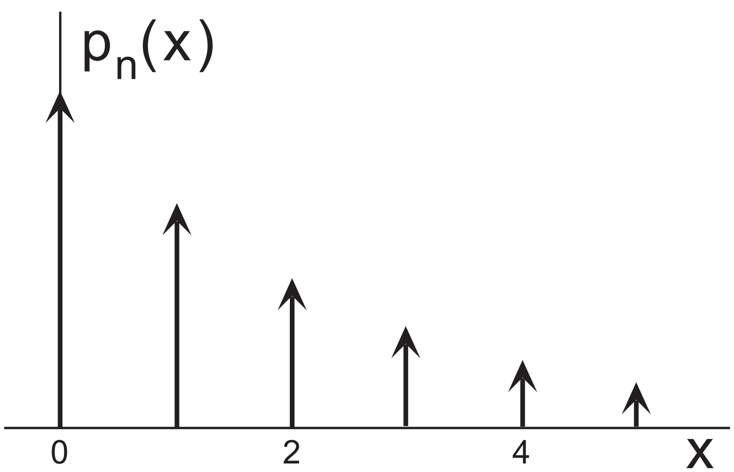
Atom escapes after the nth wall encounter

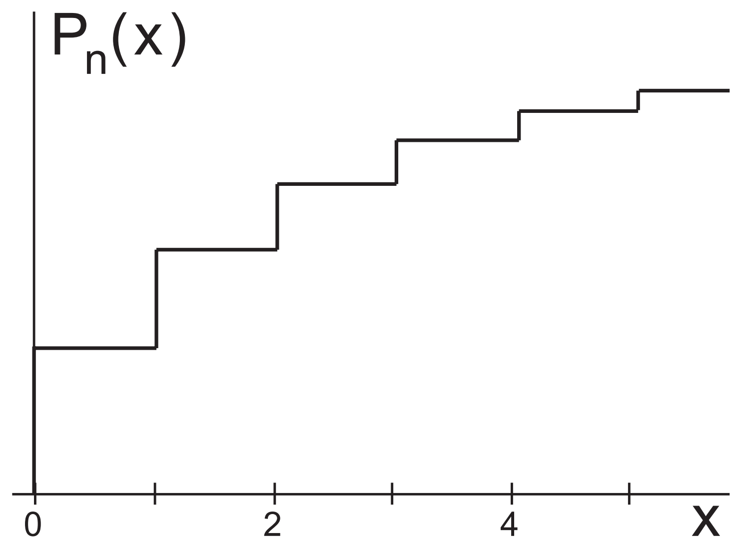
$$p(n) = (\frac{A_H}{A_T})(1 - \frac{A_H}{A_T})^n$$

$$n = 0, 1, 2, \cdots$$

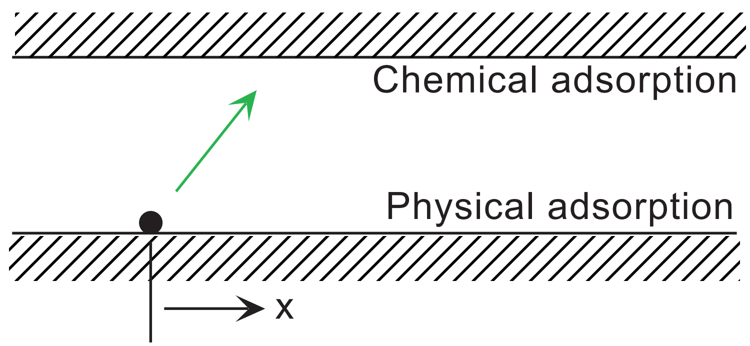
$$p_n(x) = \sum_{n=0}^{\infty} (\frac{A_H}{A_T})(1 - \frac{A_H}{A_T})^n \delta(x - n)$$

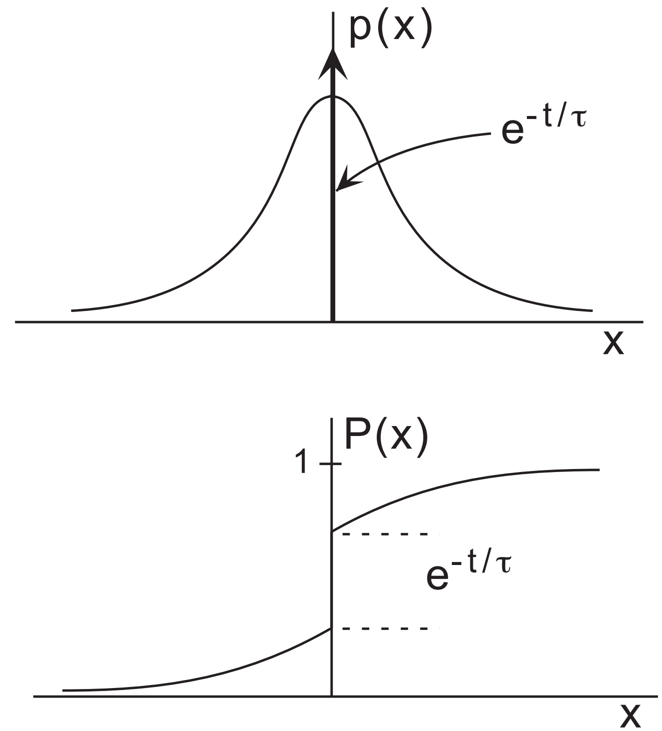
Called a geometric or a Bose-Einstein density





Example Mixed, t dependent RV



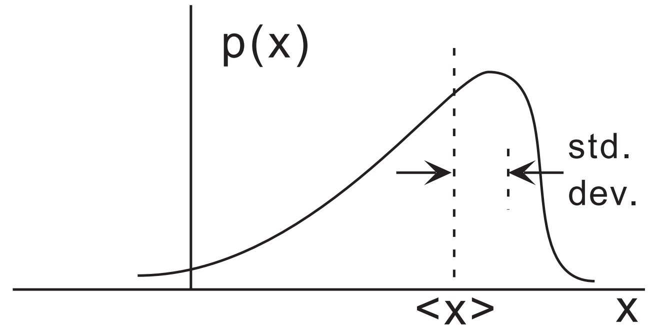


Given: atom on bottom at $t = 0$

$$p(x) = e^{-t/\tau}\,\delta(x) + (1 - e^{-t/\tau})\,f(x)$$

Averages

$$< f(x) > \equiv \int_{-\infty}^{\infty} f(x) p(x)\, dx$$

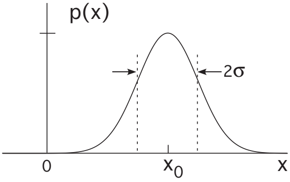


$< x >$ is the **mean**

$< x^2 >$ is the **mean square**

$$\begin{aligned} < (x - < x >)^2 > &= < (x^2 - 2x < x > + < x >^2) > \\ &= < x^2 > - 2 < x >^2 + < x >^2 \\ &= < x^2 > - < x >^2 \text{ is the } \textbf{variance} \\ &\equiv (\textbf{standard deviation})^2 \end{aligned}$$

Gaussian



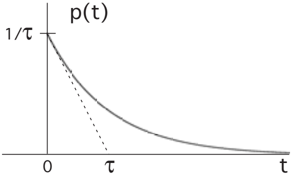
$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-(x-x_0)^2/2\sigma^2}$$

$< x > = x_0$

$\text{Var}(x) = \sigma^2$

Controlled separately

Exponential



$$p(t) = \frac{1}{\tau}\, e^{-t/\tau} \quad t \geq 0$$

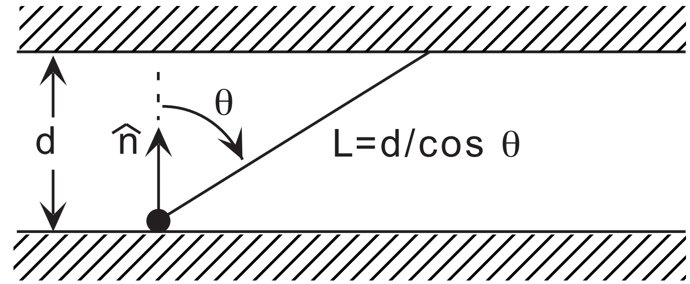
$$= 0 \qquad t < 0$$

$< t > = \tau$

$\text{Var}(t) = \tau^2$

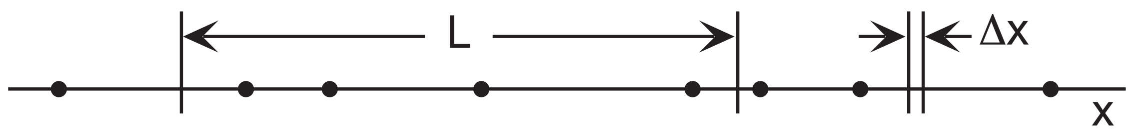
Determined by same parameter

Example Mean free path



$$
\begin{aligned}
< L > &= \int_0^{\pi/2} (d/\cos\theta) p(\theta)\, d\theta \\
&= \int_0^{\pi/2} (d/\cos\theta)\, 2\sin\theta\cos\theta\, d\theta = 2d\int_0^{\pi/2} \sin\theta\, d\theta \\
&= 2d\, \Big[_0^{\pi/2} (-\cos\theta) = 2d
\end{aligned}
$$

Poisson density   Events occur randomly along a line at a rate r per unit length

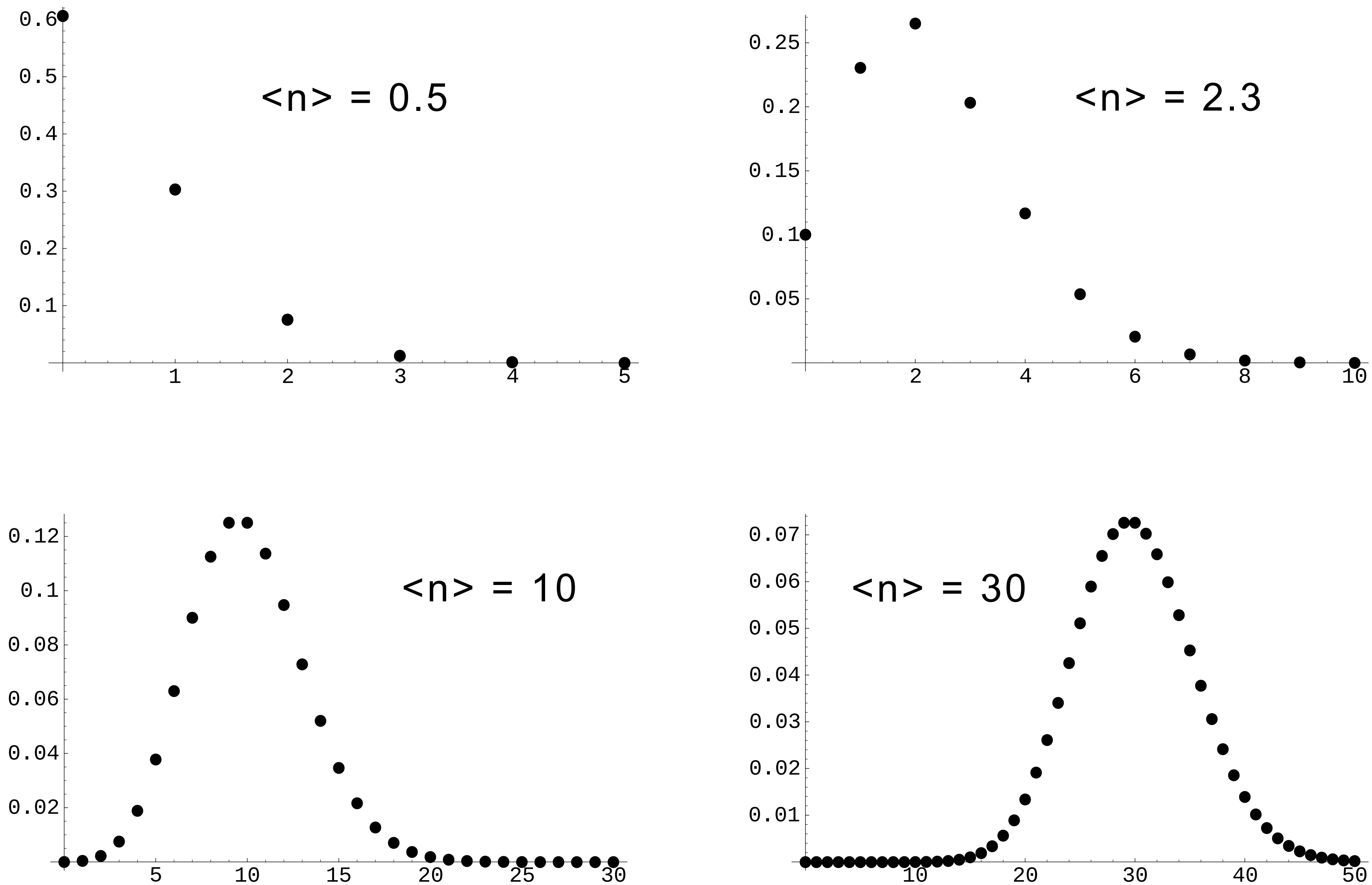


$p(1) \rightarrow r\Delta x$ as $\Delta x \rightarrow 0$

Events are statistically independent

$$p(n) = \frac{1}{n!}(rL)^n e^{-rL} = \frac{1}{n!} < n >^n e^{-<n>}$$

Examples of Poisson probability densities



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