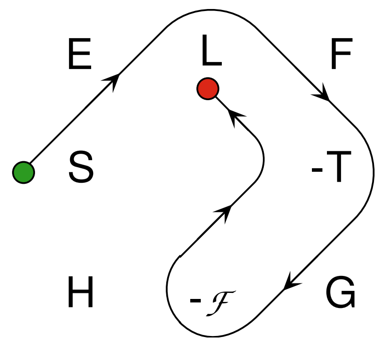
Example Elastic Rod

Given $\mathcal{F} = \underbrace{(a + bT)}_{+\ \text{for stability}} (L - L_0)$ and $C_L = e\,T^3$

Find $E(T, L)$ and $S(T, L)$.

$$dE = \underbrace{T\,dS}_{đQ} + \mathcal{F}\,dL = \underbrace{T\left(\frac{\partial S}{\partial T}\right)_L}_{C_L = eT^3} dT + \left(T\left(\frac{\partial S}{\partial L}\right)_T + \mathcal{F}\right) dL$$



$$(-1)\left(\frac{\partial S}{\partial L}\right)_T = \left(\frac{\partial \mathcal{F}}{\partial T}\right)_L = b(L - L_0)$$

$$\left(T\left(\frac{\partial S}{\partial L}\right)_T + \mathcal{F}\right) = -bT(L - L_0) + \mathcal{F} = a(L - L_0)$$

$$dE = eT^3 dT + a(L - L_0)dL$$

$$dE = eT^3 dT + a(L - L_0)dL$$

$$E = \frac{e}{4} T^4 + f(L)$$

$$f'(L) = a(L - L_0) \qquad f(L) = \frac{a}{2}(L - L_0)^2 + c_1$$

$$E(T, L) = \underline{\frac{e}{4} T^4 + \frac{a}{2}(L - L_0)^2 + c_1}$$

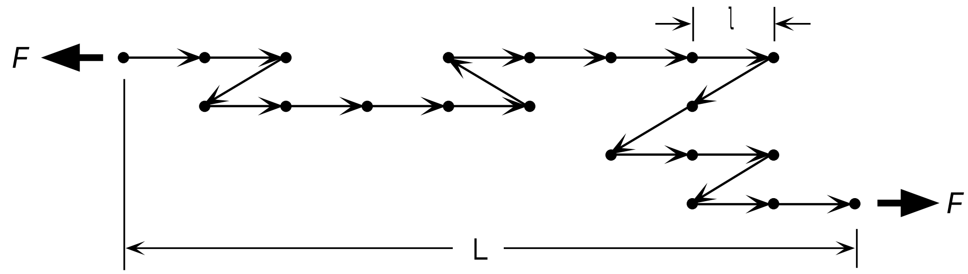
$$dS = \underbrace{\left(\frac{\partial S}{\partial T}\right)_L}_{C_V/T = eT^2} dT + \underbrace{\left(\frac{\partial S}{\partial L}\right)_T}_{-b(L-L_0)} dL$$

$$S = \frac{e}{3} T^3 + g(L)$$

$$g'(L) = -b(L - L_0) \qquad g(L) = -\frac{b}{2}(L - L_0)^2 + c_2$$

$$\underline{S(T, L) = \frac{e}{3} T^3 - \frac{b}{2}(L - L_0)^2 + c_2}$$

Homework problem 4-5, A Strange Chain



$$L = Nl\,\tanh(l\mathcal{F}/kT)$$

For small extensions

$$\alpha = -\frac{1}{T}$$

A rubber band has a negative thermal expansion coef.

$$d\mathcal{F} = (a + bT)dL + b(L - L_0)dT \quad \text{set} = 0$$

$$\Rightarrow \left(\frac{\partial L}{\partial T}\right)_{\mathcal{F}} = -\frac{b \overbrace{(L - L_0)}^{+ \text{ when extended}}}{\underbrace{(a + bT)}_{+ \text{ for stability}}} < 0 \quad \text{for rubber}$$

$$\Rightarrow b > 0$$

$$S(T,L) = \frac{e}{3}T^3 - \frac{b}{2}(L - L_0)^2 + c_2$$

Rapid expansion of rubber band $\Rightarrow \Delta S \sim 0$

Increase in $L \Rightarrow$ increase in T.

Heat Engine

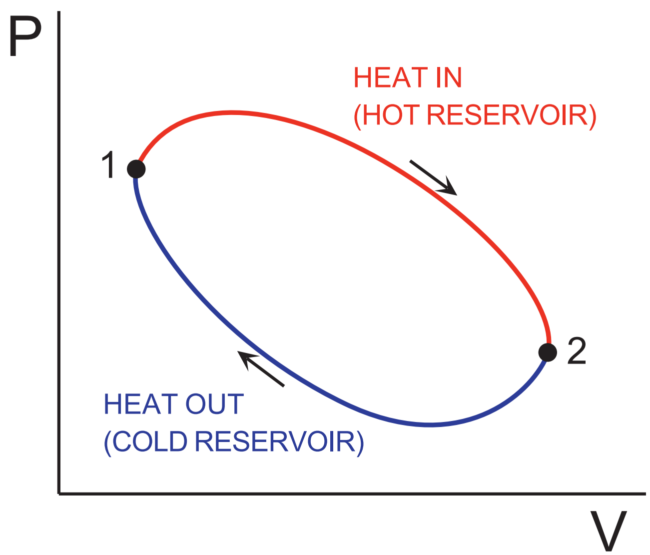
- Takes a substance around a <u>closed cycle</u>

- Heat is put into the substance and taken out

- Work is taken out

- Efficiency, $\eta \equiv$ (work out) / (heat in)

$$\text{Closed cycle} \Rightarrow \Delta U = \Delta Q + \Delta W = 0 \Rightarrow \Delta Q = -\Delta W$$



$$\Delta Q \equiv \oint đQ$$

$$= \underbrace{\int_1^2 đQ}_{\equiv |Q_H|} + \underbrace{\int_2^1 đQ}_{\equiv -|Q_C|}$$

Most General Case

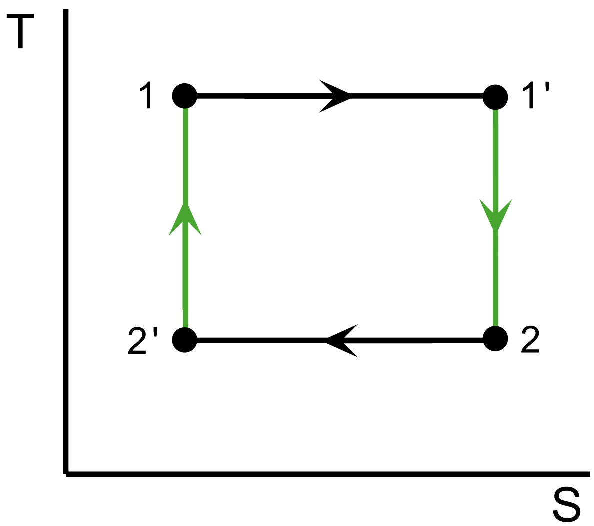
$$W_{\text{out}} = -\Delta W = \Delta Q = |Q_H| - |Q_C|$$

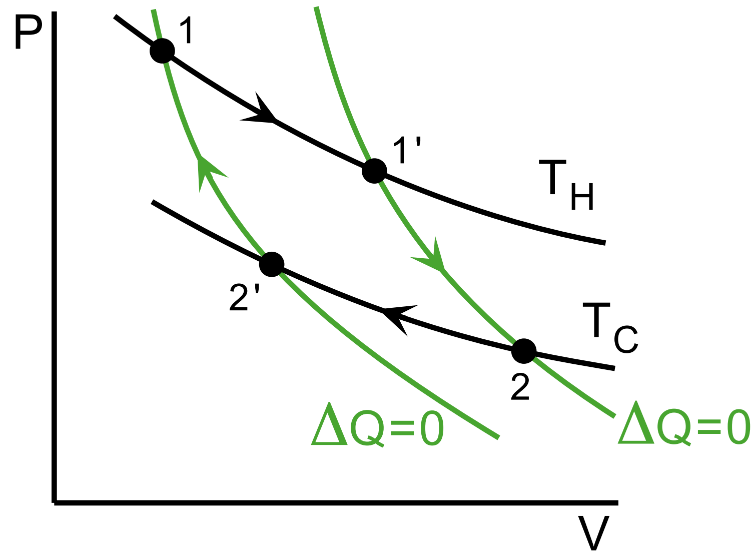
$$\eta \equiv \frac{W_{\text{out}}}{|Q_H|} = \frac{|Q_H| - |Q_C|}{|Q_H|} = 1 - \frac{|Q_C|}{|Q_H|}$$

Very Special Case   Example: Carnot Cycle

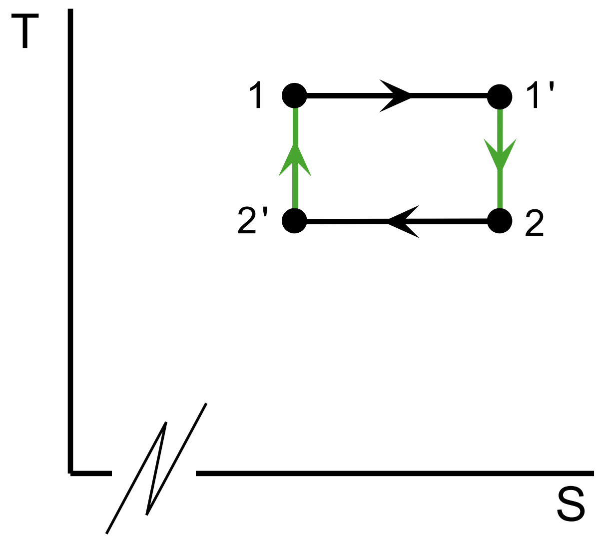
- Any substance

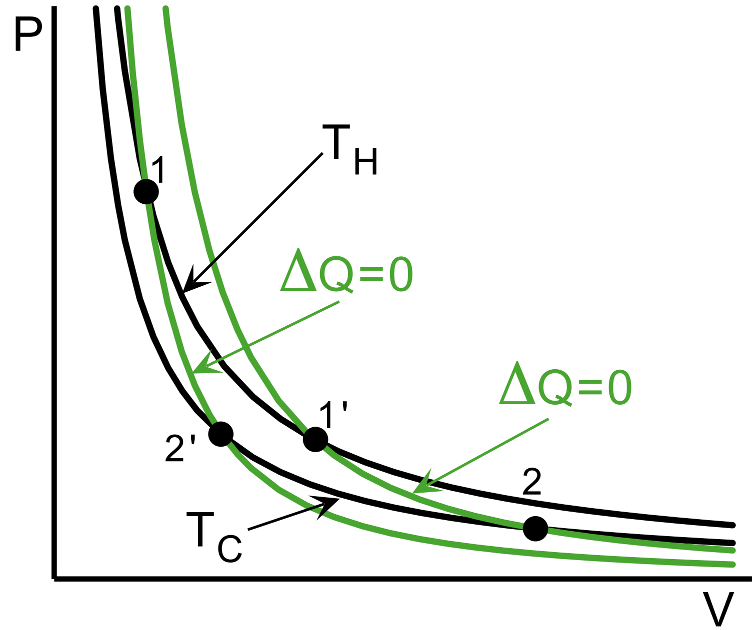
- Isothermal and adiabatic changes





Use the second law: $đQ \leq TdS$





DRAWN TO SCALE FOR AN IDEAL GAS: PV=NkT

$T_H = 1.5\ T_C \qquad S_{HIGH} - S_{LOW} = (3/2)\ Nk\ \ln 2$

$$|Q_H| \;\leq\; T_H \int_1^{1'} dS$$

$$-|Q_C| \;\leq\; T_C \int_2^{2'} dS, \quad \text{use} \quad \int_2^{2'} dS = -\int_1^{1'} dS$$

$$\leq\; -T_C \int_1^{1'} dS \Rightarrow |Q_C| \geq T_C \int_1^{1'} dS \quad \text{and} \quad \frac{|Q_C|}{|Q_H|} \geq \frac{T_C}{T_H}$$

$$\eta = 1 - \frac{|Q_C|}{|Q_H|} \;\; \underline{\leq 1 - \frac{T_C}{T_H}}$$

Arbitrary Engine Cycle

$đQ \leq TdS$ for each element along the path.

$$\underbrace{\int_1^2 đQ}_{|Q_H|} \;\leq\; \int_1^2 TdS \;\leq\; T_{\text{max}} \underbrace{\int_1^2 dS}_{\text{positive}}$$

$$\int_2^1 đQ \;\leq\; \int_2^1 T\,dS, \quad \text{both sides are negative}$$

$$|Q_C| \;\geq\; \left|\int_2^1 T\,dS\right| \geq T_{\text{min}} \left|\int_2^1 dS\right|$$

$$\geq\; T_{\text{min}} \left|\int_1^2 dS\right| \;\text{ since } \oint dS = 0$$

$$\frac{|Q_C|}{|Q_H|} \;\geq\; \frac{T_{\text{min}}}{T_{\text{max}}}$$

$$\eta = 1 - \frac{|Q_C|}{|Q_H|} \;\underline{\leq 1 - \frac{T_{\text{min}}}{T_{\text{max}}}}$$

Carnot cycle in a pure thermodynamic approach

- Used to define temp. $\eta = 1 - \frac{|Q_C|}{|Q_H|} \equiv 1 - \frac{T_C}{T_H}$

- Used to define the entropy

$$\oint \frac{đQ}{T} \leq 0 \Rightarrow \frac{đQ}{T} \text{ is an exact differential}$$

8.044 Statistical Physics I
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