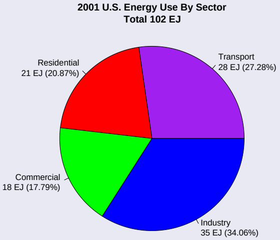
8.21 The Physics of Energy
Fall 2009

8.21 Lecture 3

Mechanical Energy

September 14, 2009

Mechanical Energy



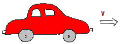
Transport:

$\sim$ 28% of U.S. energy use

$\sim$ 30 EJ/year

$\sim$ 1/3 of U.S. CO_2 output

Elementary mechanics is relevant for understanding transport:

- Kinetic Energy 
- Potential Energy 
- Friction + air resistance 

1. Kinetic Energy

Kinetic Energy of a mass m moving at speed v: $E_{\rm kin} = \frac{1}{2}mv^2$

baseball @ 100 mph

$$\begin{aligned} E_{\rm kin} &= \frac{1}{2}(5\ \text{oz})\,(100\ \text{mph})^2 \cong \frac{1}{2}(150\ \text{g})(160\ \text{km}/3600\ \text{s})^2 \\ &\cong \frac{1}{2}(0.15\ \text{kg})(44\ \text{m/s})^2 \cong 150\ \text{J} \end{aligned}$$

100 pitches $\cong$ 15 kJ $\ll$ 10 MJ daily human food energy

Camry w/4 passengers

$$\begin{aligned} E_{\rm kin} &= \frac{1}{2}(4000\ \text{lb})\,(60\ \text{mph})^2 \\ &\cong \frac{1}{2}(1800\ \text{kg})(27\ \text{m/s})^2 \cong 700\ \text{kJ} \end{aligned}$$

Transport Energy Example: ROAD TRIP!

Take Camry + 4 passengers, Boston New York

Compute energy used: Distance = 210 miles

$$30 \text{ miles/gallon} \Rightarrow 7 \text{ gallons} \times 120 \text{ MJ/gallon} \cong 840 \text{ MJ}$$

Camry w/4 passengers

Where does the energy go?

 $\overset{v}{\Longrightarrow}$ $E_{\text{kin}} = \frac{1}{2}mv^2 \cong 0.7 \text{ MJ}$

Engine efficiency? (25%: $840 \rightarrow 210$ MJ)

Hills? 

Friction + air resistance? 

Question: how much energy expenditure is really necessary?

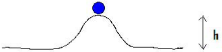
2. Potential Energy

Fundamental Principle: Energy is CONSERVED

Potential Energy: energy stored in a configuration of objects interacting through forces

Motion against a force (*e.g.* roll ball uphill)

kinetic energy $\longrightarrow$ potential energy



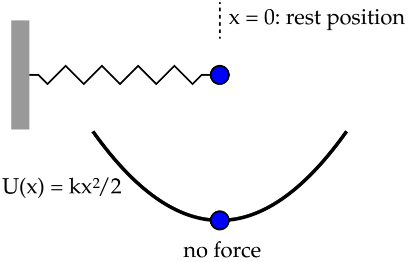
$U = mgh$ ($g = 9.8$ m/s^2)

- Force points in direction of decreasing potential energy
- Motion in direction of force: potential E $\longrightarrow$ kinetic E

Example: spring  $U = \frac{1}{2}kx^2$

Potential Energy, Forces, and Work

Consider mass m subject to force $F = -kx$ from potential $U(x)$



Newton: $F = ma = m\ddot{x}$ (Transfers KE ↔ PE)

Force over distance does work

$W = F \times d \;\Rightarrow\; W = \int F(x)\, dx$

Unpack: $\frac{dE_k}{dt} = m\dot{x}\ddot{x} = F\frac{dx}{dt}$

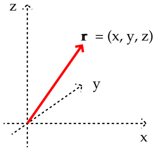
$E_k + U = \text{const.} \Rightarrow \frac{dU}{dt} = -F\frac{dx}{dt}$

So conservation of $E \Rightarrow$ $U = -\int F\, dx$, or $F = -dU/dx$

Potential Energy and Vectors

For motion in a line, $F = m\ddot{x}$, potential $U \Rightarrow F = -dU(x)/dx$.

In 2D/3D, use vectors



Vector force: $$\mathbf{F} = m\mathbf{a} = m\frac{d^2\mathbf{x}}{dt} = (m\ddot{x}, m\ddot{y}, m\ddot{z})$$

Potential $U(x, y, z) \Rightarrow$

$$\mathbf{F} = -\nabla U = (-\frac{\partial U}{\partial x}, -\frac{\partial U}{\partial y}, -\frac{\partial U}{\partial z})$$

Example: gravity

$$U = -\frac{GMm}{\mathbf{r}}$$

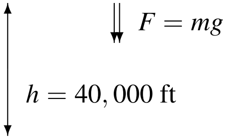
$$\mathbf{F} = -\nabla U = -\frac{GMm}{r^2}\hat{r}$$

Potential Energy: Applications

- Airplane at altitude

747 at 900 km/h has $E_{\text{kin}} \cong \frac{1}{2}(350,000\text{ kg})(250\text{ m/s})^2 \cong 11\text{ GJ}$

How about potential energy?



$$U = mgh \cong (350,000\text{ kg})(9.8\text{ m/s}^2)(12,000\text{ m}) \cong 41\text{ GJ}$$

Other examples of potential energy applications:

- Pump water uphill for storage
- Elevators, cranes, etc.

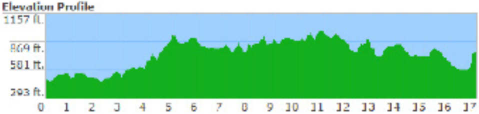
Back to the road trip!

- 4000 lb car at 60 mph → 0.7 MJ
- Using 840 MJ of gasoline energy

Boston and New York are both basically at sea level

Is potential energy relevant? Yes!

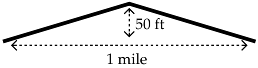
http://www.usatf.org/routes/map/



Recapture some lost energy on downhill–but not all!

Estimate effects of hills

Assume:
- Constant speed $v = 60$ mph $\cong$ 100 km/h
- 50 ft of elevation gain per mile
- Lose 1/2 of energy used going up on downhill braking



Energy needed/hill = mgh = (1800 kg) (9.8 m/s^2) (15 m) $\cong$ 260 kJ

$$\frac{1}{2} \times 260\ \text{kJ}/\text{mile} \times 210\ \text{miles} \cong 27\ \text{MJ}$$

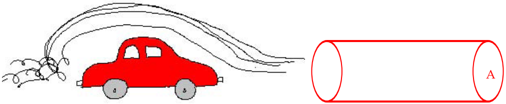
So:

0.7	MJ to get started	Still $\ll$ 210 MJ engine output
27	MJ for hills	(assuming 25% efficiency)

What's left?

3. Air Resistance and Friction

How much energy is lost to air resistance?



Car collides with air molecules, sweeps into wake

Details complicated–But basic idea is simple:

Car sweeps out tube of area A, accelerates air to $\sim v$

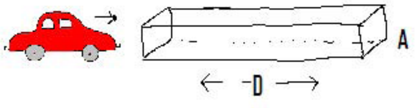
$$\Delta E_{\text{air}} \cong \frac{1}{2} c_d (\Delta m_{\text{air}}) v^2$$

c_d = drag coefficient
(typical car: $c_d \sim 1/3$)

$$\frac{dE_{\text{air}}}{dt} \cong \frac{1}{2} c_d \left[(d(\text{vol})/dt) \times (\text{mass density } \rho) \right] v^2$$

$$\cong \frac{1}{2} c_d (Av) \rho v^2 \cong \frac{1}{2} c_d A \rho v^3$$

So total energy lost to air resistance in distance D is



$$W_{\text{air}} = \frac{1}{2} c_d (AD) \rho v^2$$

For the Toyota Camry going Boston → New York

$$\frac{1}{2}(0.33)(2.66\ \text{m}^2 \times 330\ \text{km})(1.2\ \text{kg/m}^3)(27.7\ \text{m/s})^2 \cong 133\ \text{MJ}!$$

- Note: traveling at 80 mph $\Rightarrow (\times(4/3)^2) \Rightarrow$ 236 MJ
- Rolling resistance $\sim 1\%$ grade $\Rightarrow$ 54 MJ

Final energy accounting: Road trip to New York

2 MJ	kinetic energy (including 12 stoplights)
+27 MJ	potential energy of hills
+54 MJ	rolling resistance
+133 MJ	air resistance
216 MJ	total

Energy in gasoline: 840 MJ, energy efficiency $\sim 25\%$

Discuss internal combustion engine efficiency in Lecture 10

Note: City driving very different, dominated by acceleration/rolling resistance (& hills in SF)

U.S. uses 30 EJ/year for transport. How to reduce?

Simple physics → ideas for reducing transport energy cost

- $W_{\text{air}} \sim v^2$: Drive 60 not 80!
- $W_{\text{air}} \sim c_d A$: Streamline! Mass transit!
- Inflate your tires. (Decreases rolling resistance)
- More efficient engines (e.g. Toyota hybrid: Atkinson [L10])
- Regenerative brakes (capture hill, stoplight energy)
- $W_{\text{air}} \sim \rho$: vacuum tunnels? space?

In principle, with regenerative braking, and $\rho \to 0$,

$$E_{\text{transport}} \to 0!$$

SUMMARY

- Kinetic Energy = $\frac{1}{2}mv^2$
- Potential Energy: $U = mgh, \quad \mathbf{F} = -\nabla U$
- Air Resistance $\frac{dW_{\text{air}}}{dt} = \frac{1}{2}c_d A \rho v^3$
- Auto transport:

	–car engines $\sim 25\%$ efficient	840/4 = 210 MJ
	–air resistance	$\sim$ 135 MJ
	–rolling resistance	$\sim$ 50 MJ
	–hills	$\sim$ 25 MJ

Next: HEAT (please review lecture notes)