8.21 The Physics of Energy
Fall 2009

8.21 Lecture 5

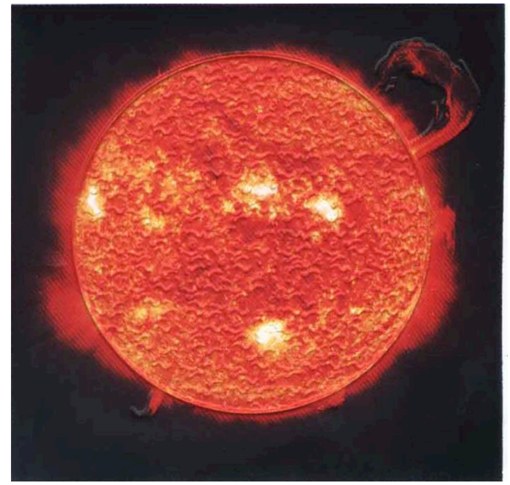
Electromagnetic Energy

September 18, 2008

Normal material

Supplementary material

Scale of EM energy use...



- All solar energy is transmitted to earth as electromagnetic waves. Total yearly solar input,

$$1.74 \times 10^{17}\ \mathrm{W} \quad \Rightarrow \quad 5.46 \times 10^{24}\ \mathrm{J/year}$$

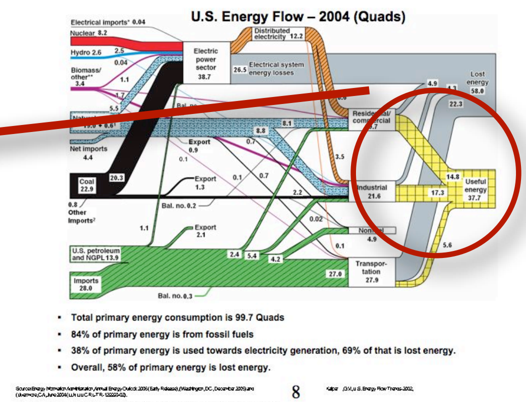
Compare total human energy consumption:

$$488\ \mathrm{EJ} = 4.88 \times 10^{20}\ \mathrm{J/year} \approx 10^{-4} \times \text{ solar input}$$

- U. S. 2004 yearly energy use

106 EJ U. S. primary energy consumption

- 41 EJ U. S. electrical power sector
  - 13 EJ distributed
  - 28 EJ losses



A closer look

Image courtesy of Energy & Environment Directorate, Lawrence Livermore National Library

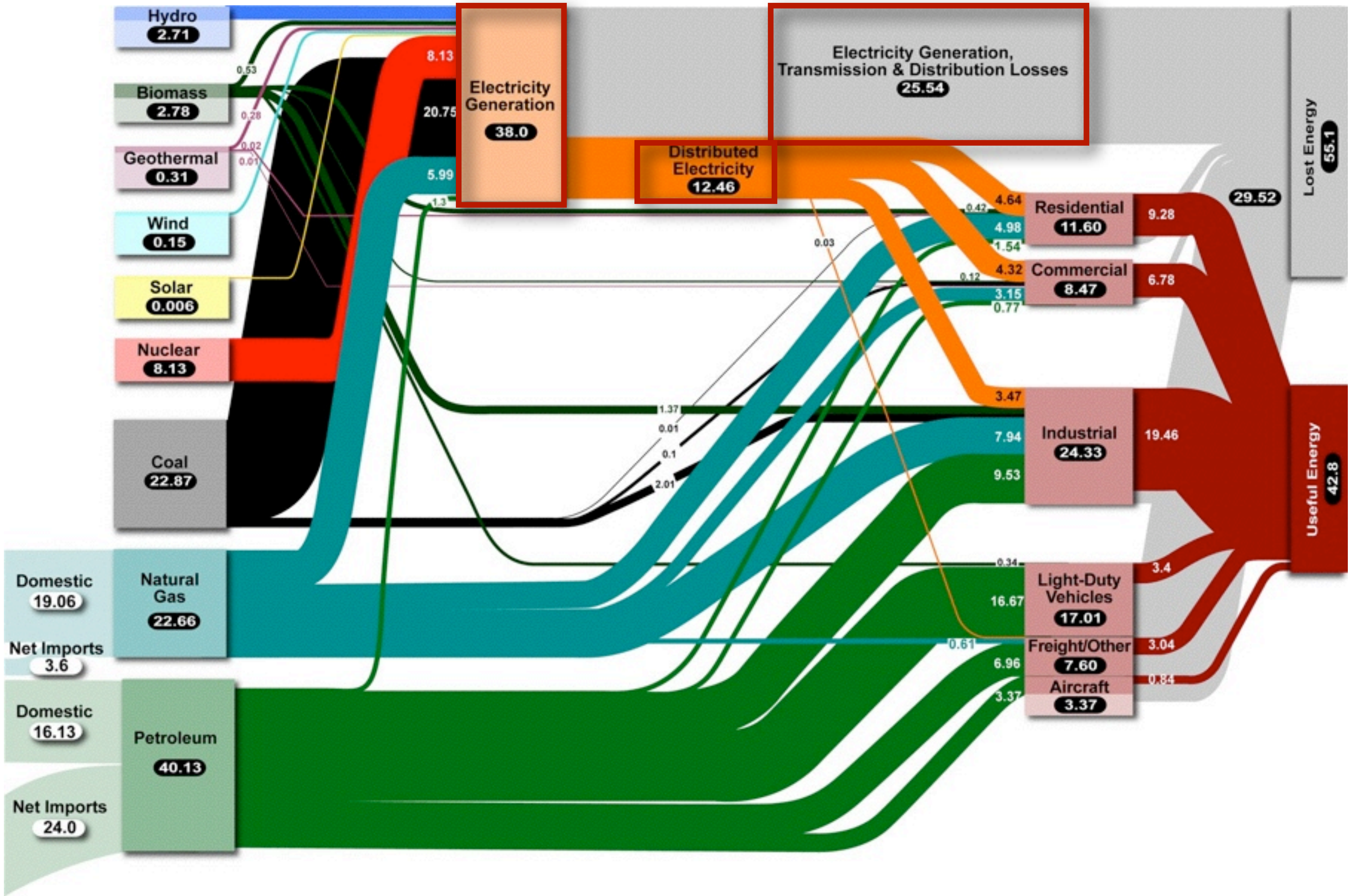
Electricity consumption 2005 (quads) *

Generation, Transmission, Distribution	Losses	25.54
Residential		4.64
Commerical		4.32
Industrial		3.47

Household electricity use (2001) ($\times 10^{15}$ J) †

Air Cond.	659
Kitchen Appl.	1098
Space Heating	418
Water Heating	375
Lighting	364
Electronics	295
(PC's & printers	83)

† http://www.eia.doe.gov/emeu/reps/enduse/er01_us_tab1.html


Hydro
2.71
Biomass
2.78
Geothermal
0.31
Wind
0.15
Solar
0.006
Nuclear
8.13
Coal
22.87
Domestic
19.06
Net Imports
3.6
Natural Gas
22.66
Domestic
16.13
Net Imports
24.0
Petroleum
40.13
Electricity Generation
38.0
8.13
20.75
5.99
Distributed Electricity
12.46
Electricity Generation, Transmission & Distribution Losses
25.54
Lost Energy
55.1
29.52
Residential
11.60
4.64
4.98
1.54
9.28
Commercial
8.47
4.32
3.15
0.77
6.78
Industrial
24.33
3.47
7.94
9.53
19.46
Light-Duty Vehicles
17.01
16.67
3.4
Freight/Other
7.60
6.96
3.04
Aircraft
3.37
3.37
0.84
Useful Energy
42.8

Outline

- The big picture
- Electric fields, forces, and work
    - Review of $\boldsymbol{E}$-field, voltage, electrical energy.
    - Capacitive energy storage
    - Ultracapacitors
- Resistive energy loss
    - Power dissipated in a resistor
    - Applications: space heating, incandescent light, transmission lines
- Transforming (to and from) electromagnetic energy
    - Motors/dynamos
    - Phases
    - Energy storage and propagation in EM fields

The big picture: The roles of electromagnetic phenomena in energy flow

- Energy "generation" ★★

  (= conversion) from fossil fuel, nuclear, hydro, *etc.*

  - Convenient, relatively easy, and ubiquitous
  - Not very efficient (Eff $\lesssim 40\%$ at best)

$$\mathcal{EMF} = -N\frac{d\Phi}{dt}$$

- Energy Transmission
  - Radiative: solar, microwave ★★

$$\boldsymbol{S} = \frac{1}{\mu_0}\boldsymbol{E} \times \boldsymbol{B}$$

  - High voltage transmission over medium and large distances ★★

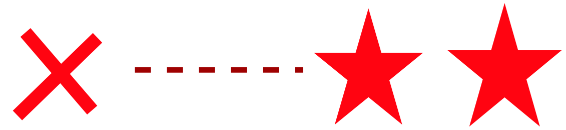
$$P = I^2 R$$

- Energy Storage ✗
  - Capacitive

$$U = \frac{1}{2}CV^2$$

  - Electrochemical (battery)

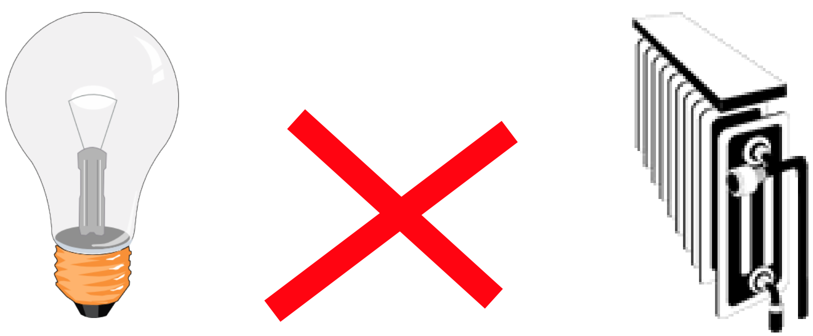
- Energy "Use"
  (= conversion) to mechanical, chemical, heat etc.
  - Convenient, relatively simple, and ubiquitou
  - Relatively efficient



$$EMF = -N\frac{d\Phi}{dt}$$

Image of Electric Motor removed due to copyright restrictions.
See http://en.wikipedia.org/wiki/Image:Electric_motor_cycle_3.png

$$P = I^2R$$



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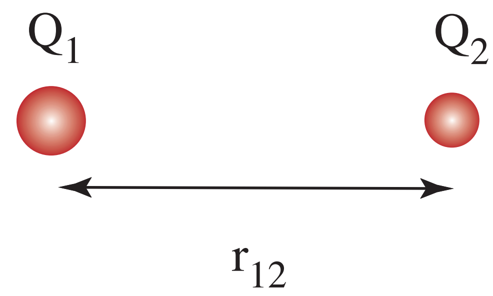
Electric fields, forces and work



Coulomb's Law — force between point charges

$$\frac{1}{4\pi\epsilon_0} = 8.99\times 10^9 \ \mathrm{Nm^2/C^2}$$

For two point charges

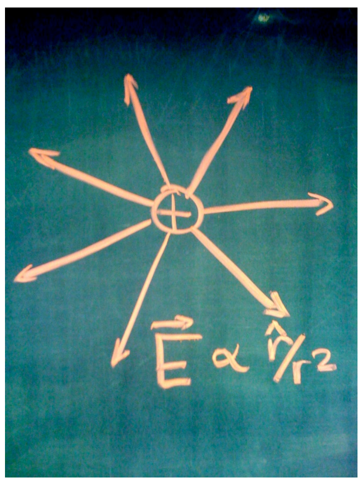
$$\boldsymbol{F}_{12} = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2 \hat{\boldsymbol{r}}_{12}}{r_{12}^2}$$

Electrostatic forces are very strong: Two Coulombs of charge separated by 1 meter repel one another with a force of ≈ 9 billion Newtons ≈ 2 billion pounds!

Electric charges produce electric fields

Point charge

$$\vec{\boldsymbol{E}} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{\boldsymbol{r}}$$



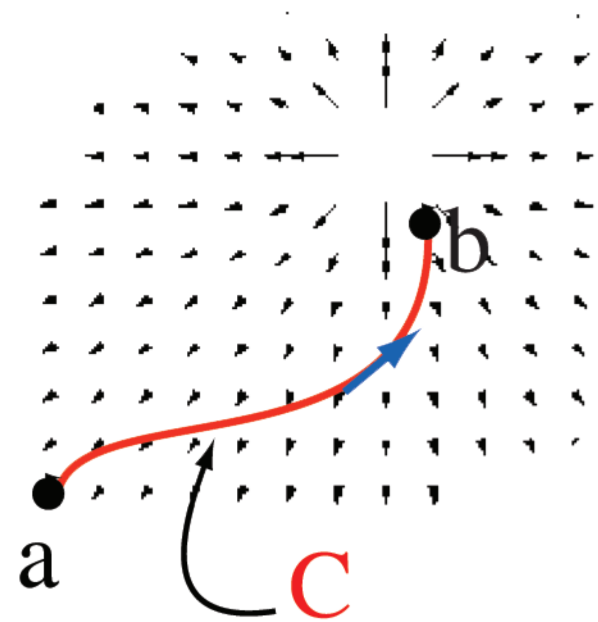
Electric fields exert forces on charges

$$\boldsymbol{F} = q\boldsymbol{E}$$

SI Units:
- Charge in Coulombs
- Current in Amperes
- Potential in Volts
- Field in Volts/meter

Electric energy and potential

Work done against an electric field is stored as electrostatic potential energy



Carry charge q from a to b along a curve C. Work done is independent of the path,

$$W(a \rightarrow b) = -\int_C \vec{\boldsymbol{dl}} \cdot \vec{\boldsymbol{F}} = -q \int_C \vec{\boldsymbol{dl}} \cdot \vec{\boldsymbol{E}}$$
$$= qV(b) - qV(a)$$

V is the *electrostatic potential* $\Leftrightarrow$ the electrostatic potential energy *per unit charge*

Work per unit charge:
Electrostatic potential $\equiv$ Voltage

Units of voltage are energy per unit charge, or Joules per Coulomb.

1 Joule/Coulomb $\equiv$ 1 Volt

Point charge

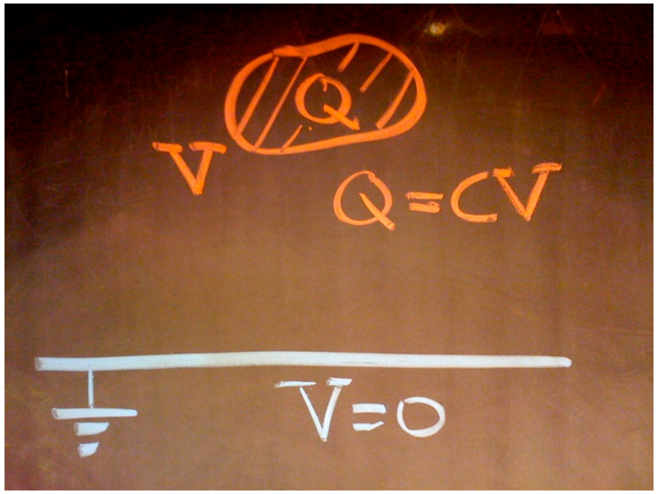
$$V(r) = \frac{Q}{4\pi\epsilon_0 r}$$

ΔV — potential difference between two locations

Electromagnetic energy storage: capacitors

General idea of a capacitor:

- Place a charge Q on a conductor
- Voltage on the conductor is proportional to Q.



- Capacitance is proportionality constant

$$Q = C\,V$$

"Capacity" of conductor to store charge.

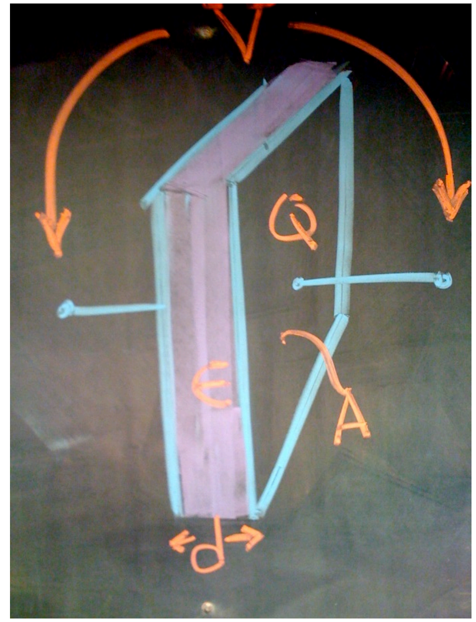
- Energy stored in a capacitor:

It takes work to move each little bit of charge through the electric field and onto the conductor.

$$U = \frac{1}{2}CV^2$$

Parallel plate capacitor

- Two plates, area A,
- Separation d,
- Filled with dielectric with dielectric constant $\epsilon = k\epsilon_0$



For parallel plates $C = \dfrac{\epsilon A}{d}$

To increase capacitance: increase Area (size limitations); decrease distance (charge leakage); or increase dielectric constant (material limitations)

For a parallel plate capacitor with $\epsilon = k\epsilon_0$

$$C = \frac{\epsilon A}{d} = \left(8.85 \times 10^{-12}\,\mathrm{F}\right) \times \frac{k\,A[\mathrm{cm}^2]}{d[\mathrm{mm}]}$$

So typical scale for a capacitive circuit element is pico farads:
1 picofarad $= 1 \times 10^{-12}$

Electric field and energy storage in a parallel plate capacitor

Charge Q, area A

Charge density

$$\sigma = \frac{Q}{A}$$

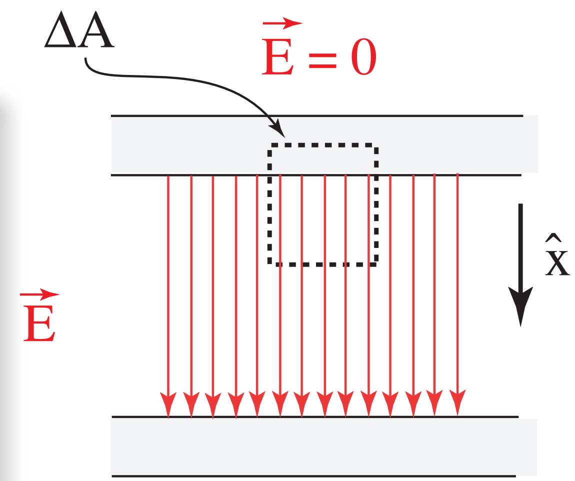
Gauss's Law:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

Integrate over pillbox:

$$\iiint_V \vec{\nabla} \cdot \vec{E}\, dV = \iiint_V \frac{\rho}{\epsilon_0} dV = \frac{1}{\epsilon_0}\sigma \Delta A$$

$$\underset{\text{Gauss's Law}}{=} \iint_S \hat{\mathbf{n}} \cdot \vec{E}\, dS = |\vec{E}|\Delta A$$



$$\vec{E} = \hat{x}\frac{\sigma}{\epsilon_0}$$

Energy storage: charge capacitor bit-by-bit. When charge is q, what is work needed to move dq from bottom to top plate?

$$dW = \int_0^d dx \vec{E} \cdot \vec{dl} = \frac{\sigma d}{\epsilon_0} dq = \frac{d}{\epsilon_0 A} q dq$$

Now integrate from $q = 0$ to $q = Q$,

$$W(Q) = \int_0^Q dq \frac{d}{\epsilon_0 A} q = \frac{d}{\epsilon_0 A}\frac{Q^2}{2}$$

$$= \frac{Q^2}{2C} = \frac{1}{2}CV^2 \equiv U = \text{ energy stored}$$

Capacitor summary:

Capacitor basics

$$Q = CV$$

$$U = \frac{1}{2}CV^2 = \frac{Q^2}{2C} = \frac{1}{2}QV$$

$$I = \frac{dQ}{dt} = C\frac{dV}{dt}$$

Capacitance units

$$[C] = \frac{[Q]}{[V]} = \frac{\text{Coulombs}}{\text{Volts}}$$

$$1\ \frac{\text{Coulomb}}{\text{Volt}} = 1\ \text{Farad}$$

Increase capacitance:

- Increase effective surface area: gels, nanostructures
- Increase dielectric constant (polarizable but non-conducting materials)
- Decrease effective separation

Electrical energy storage?

- Batteries are expensive, heavy, involve relatively rare and unusual materials (eg. lithium, mercury, cadmium,...), toxic.
- Storing electrical energy in capacitors is not a new idea, but using novel materials to make "ultra" capacitors is!

"Super" or "Ultra" Capacitors

Using sophisticated materials technology, capable of tens or even thousands of farad capacitors in relatively modest volumes.

Compare with conventional rechargeable (*e.g.* NiMH) battery

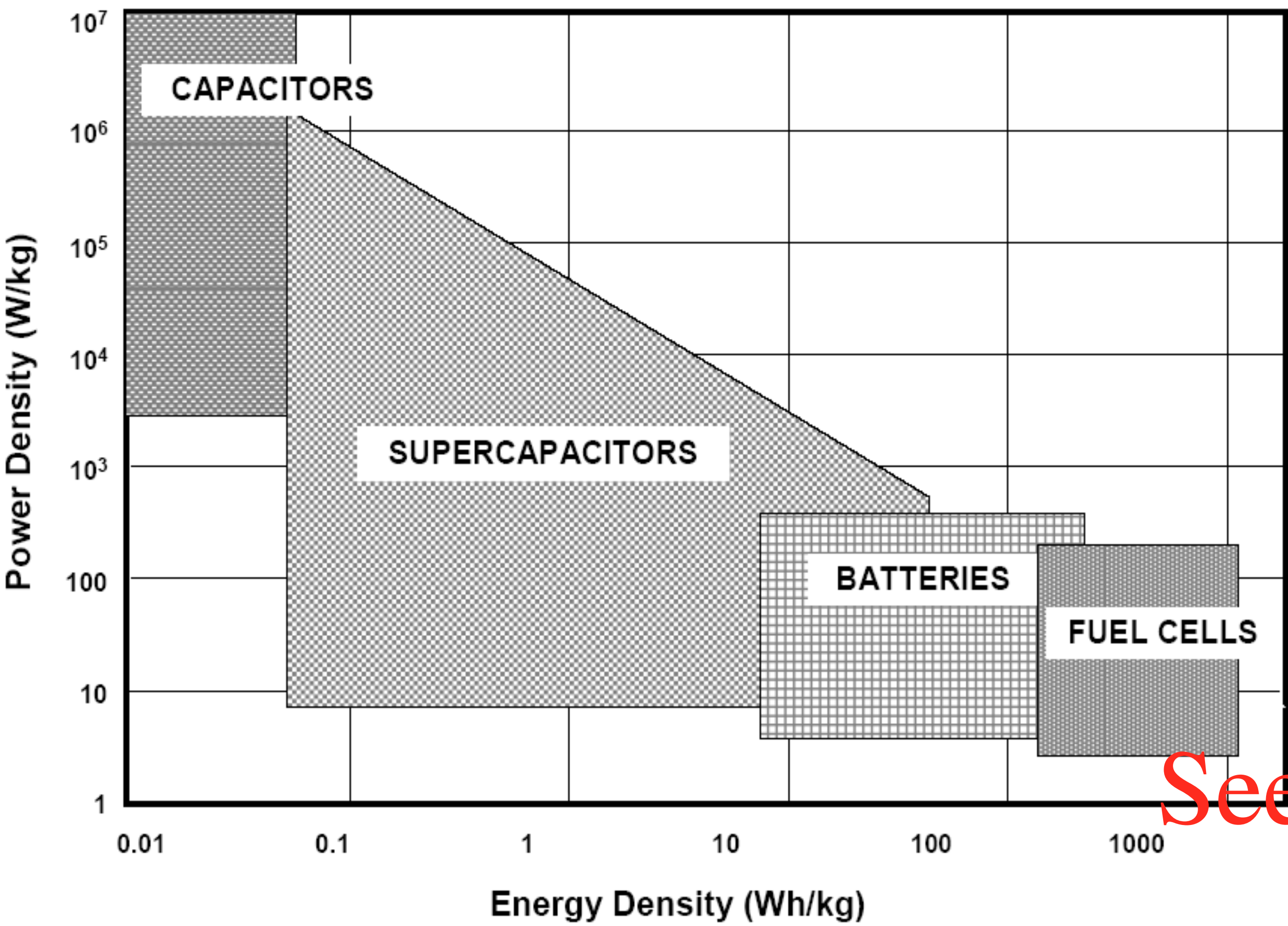
Advantages

- Discharge and charge rapidly $\Rightarrow$ high power
- Vastly greater number of power cycles
- No environmental disposal issues
- Very low internal resistance (low heating)

Disadvantages

- Low total energy
- Intrinsically low voltage cell
- Voltage drops linearly with discharge
- Leakage times ranging from hours (electrolytic) to months

Introducing the *Ragone* Plot



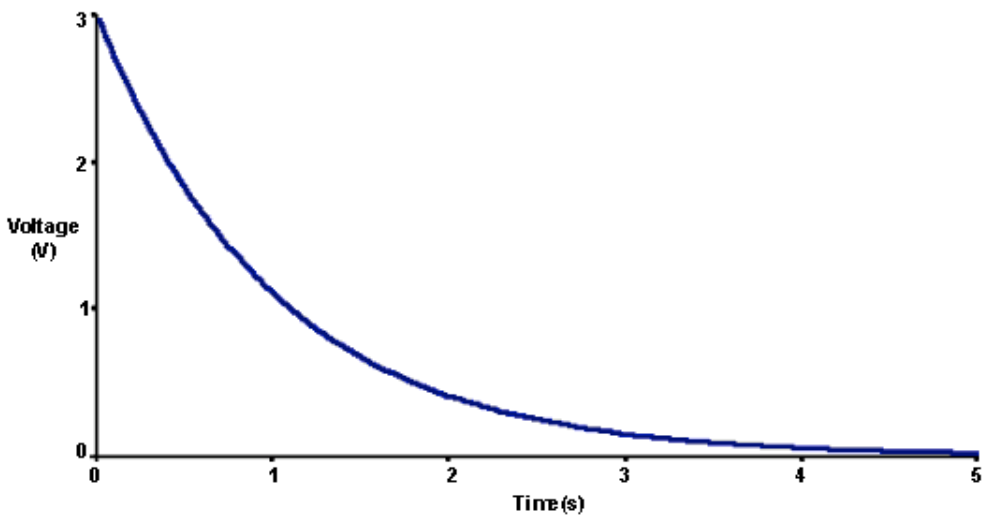
See Problem 3 on Assignment 2!

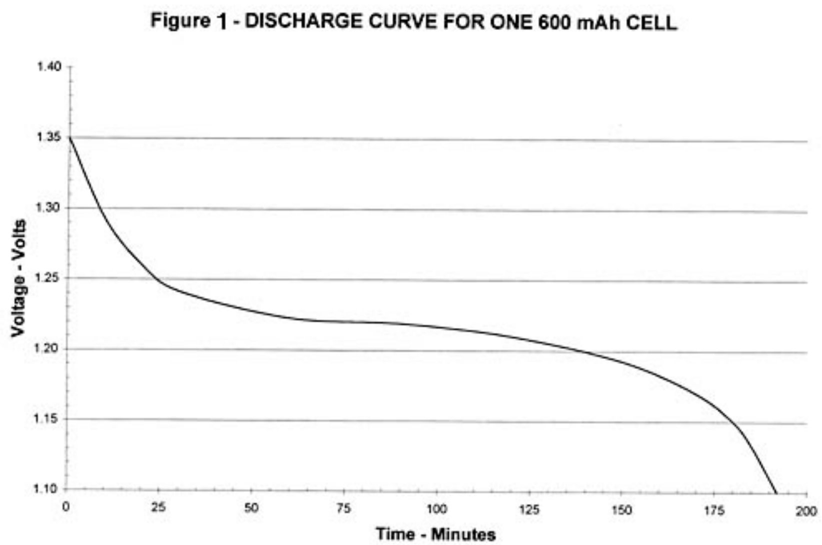
Comparison of Maxwell Technologies BCAP0120 P250 (C-cell size super capacitor) and C-cell NiMH battery

- Max. energy density 12,900 J/kg
- Max. power density 21,500 W/kg

- Max. energy density 252,000 J/kg
- Max. power density 250 – 1000 W/kg

Voltage profiles during discharge





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Resistive energy loss

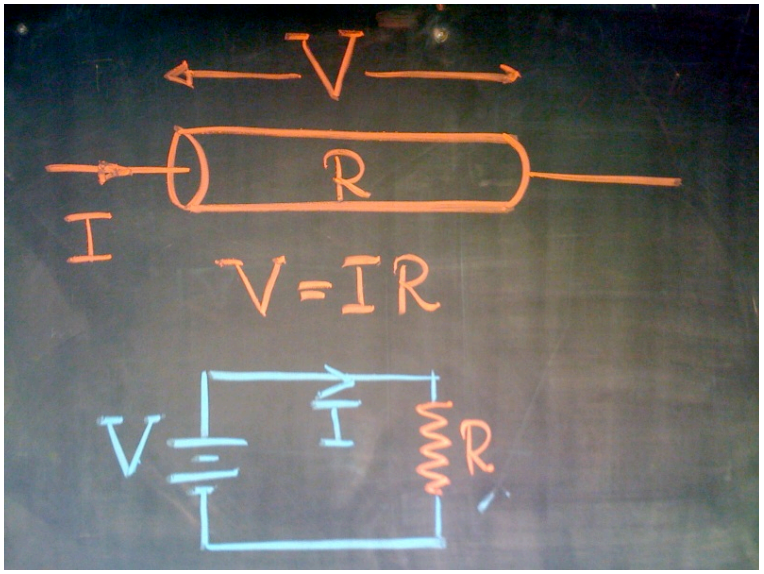
Electric current passing through a resistance generates heat.

- Resistive energy loss in transmission of electric power is a major impediment to long distance energy transmission.
- Resistance converts electrical energy to heat at nearly 100% efficiency, but that's not the whole story.

Electric space heating — GOOD??

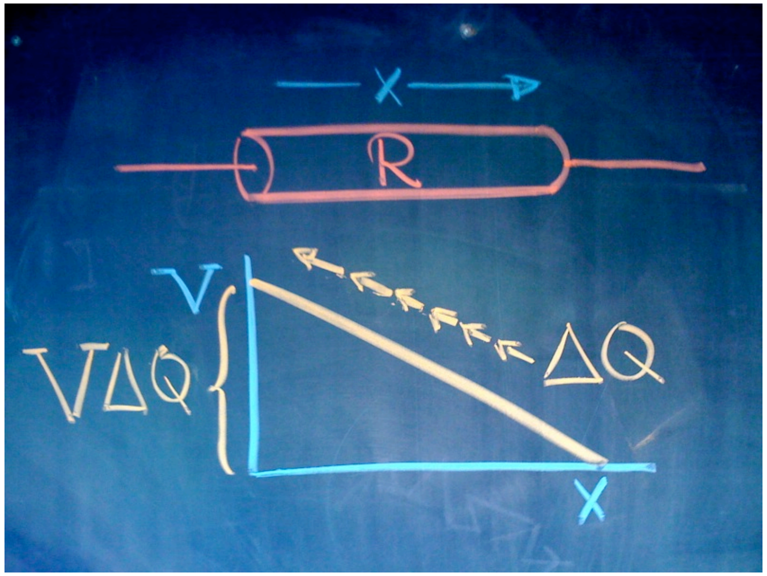
Transmission losses — BAD

Resistor properties



Current and voltage in a resistor:
$V = IR$ (Ohm's Law)

Power in a resistor:



Carry a charge ΔQ "uphill" through a voltage V, requires work $\Delta W = V\Delta Q$, so

$$\frac{dW}{dt} \equiv P = V\frac{dQ}{dt} = VI = I^2R = V^2/R$$

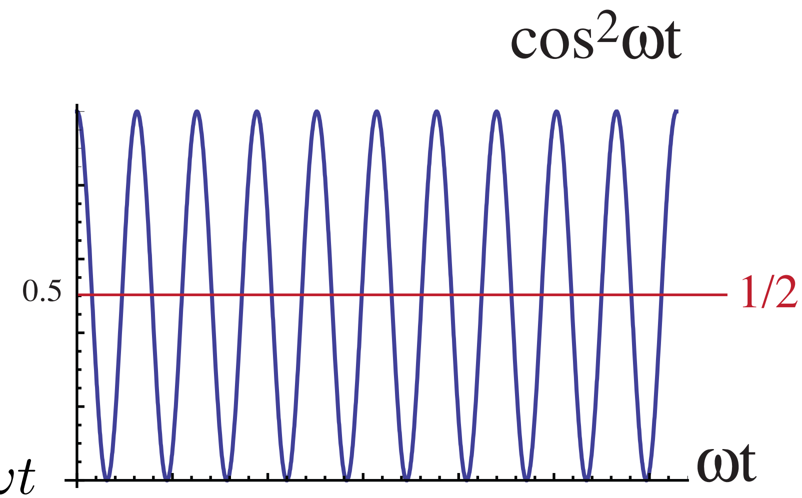
Power dissipated in a resistor:

$(DC) \quad V \& I$ constant

$$P = VI = \frac{V^2}{R} = I^2 R$$

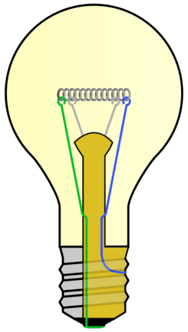
Power dissipated in a resistor:

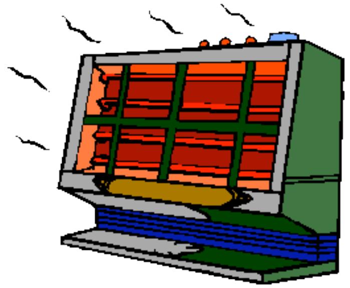
$(AC) \quad V \& I$ oscillatory

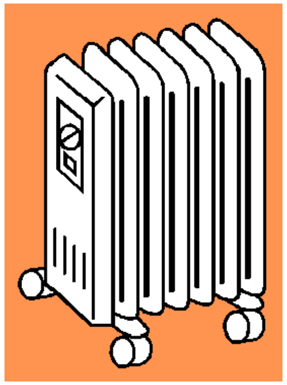


$$\begin{aligned} V(t) &= V_0 \cos \omega t \\ I(t) &= V(t)/R = \frac{V_0}{R} \cos \omega t = I_0 \cos \omega t \\ P(t) &= V_0 I_0 \cos^2 \omega t \\ \langle P(t) \rangle &= \frac{V_0^2}{R} \langle \cos^2 \omega t \rangle = \frac{V_0^2}{2R} \end{aligned}$$

Electric Space Heating







Electric current run through resistance

Efficiency 100% regardless of type: light bulb, radiant coil, oil ballast

So: is electricity a good way to heat your home? Compare:

(a) Burn coal in electricity generating plant at 40% efficiency (generous) → electricity → distribute (assuming no losses) → space heat

(b) Directly burn natural gas (CH_4) at 90% efficiency in your home

Carbon cost to generate one kilowatt for one day:

(a) $\Rightarrow$ 7.5 kg coal $\Rightarrow$ 27.5 kg CO_2

(b) $\Rightarrow$ 2.2 m^3 CH_4 $\Rightarrow$ 4.4 kg CO_2

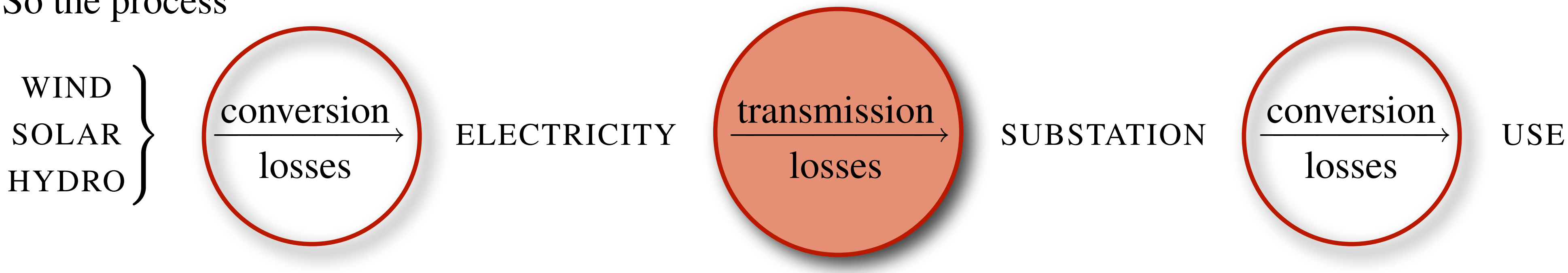
Moral: Electric space heating is not an environmentally friendly choice. In fact it is illegal in new construction in Scandanavian countries.

Power lines: losses in transmission of electromagnetic energy

Transmission of energy over long distances is problematic.

- Oil & Gas — TANKERS & PIPELINE
- Coal & Nuclear — MINES & TRAINS
- Wind, solar, hydro, tidal — ELECTRICAL

So the process

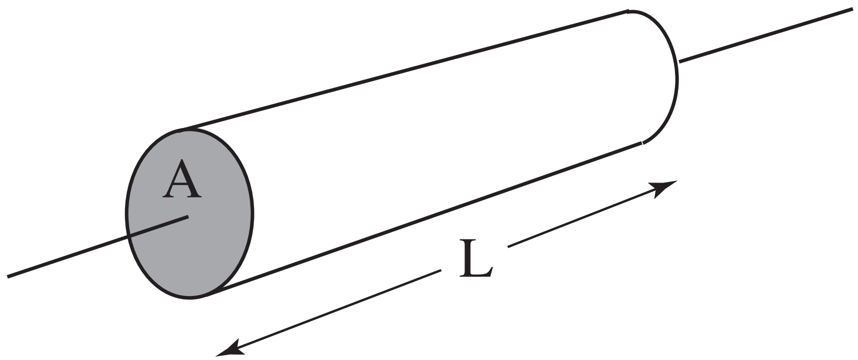


Is essential in a world where renewable energy plays a major role

Transmission losses NEXT
Conversion losses STAY TUNED

Cable

- Resistance grows linearly with length
- Resistance falls linearly with area



$$R = \frac{\rho L}{A}$$

$$\rho = \frac{RA}{L} \qquad \rho \equiv \text{resistivity}$$

$$[\rho] = [R][A/L] = \text{ohm-meter } (\Omega\text{-m})$$

Notice resemblance to heat conduction!

Design a cable

Resistivities:

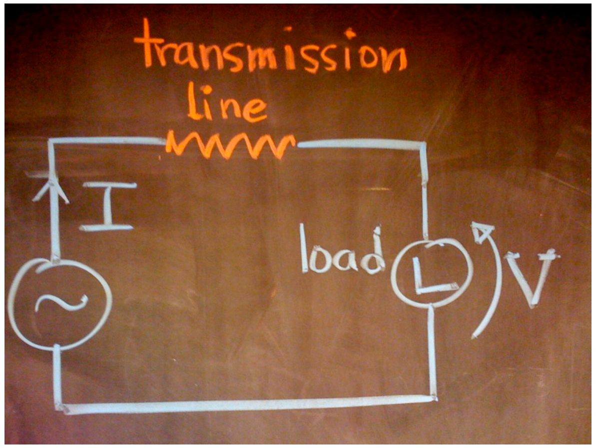
- $\rho[\text{Cu}] = 1.8 \times 10^{-8}\ \Omega\text{-m}$
- $\rho[\text{Al}] = 2.82 \times 10^{-8}\ \Omega\text{-m}$
- $\rho[\text{Fe}] = 1.0 \times 10^{-7}\ \Omega\text{-m}$

Material properties influencing choice:

- Light: Al > Cu > Fe
- Strong: Fe > Al, Cu
- Good conductor: Cu > Al > Fe
- Not subject to significant corrosion: Al > Cu > Fe
- Cost: Fe > Al > Cu

Choice: Aluminum, often with a steel support

Power loss in a transmission line



- An exercise in Ohm's Law
- Assume AC power with "power factor" $= 1/2 = \langle \cos^2 \omega t \rangle$

- I is current in line
- V is voltage across the load
- R is resistance in the transmission line $R = \rho L/A$.

$$P = I\,V \Rightarrow I = P/V$$

$$P_{\text{lost}} = \frac{1}{2} I^2 R = \frac{1}{2}\frac{P^2}{V^2} R$$

$$\frac{P_{\text{lost}}}{P} = \frac{1}{2}\frac{PR}{V^2} = \frac{1}{2}\frac{P\rho L}{AV^2}$$

Note that the "V" that appears in the boxed form at the left is the *peak* voltage. Often AC voltages are quoted as root mean square. For a simple sinusoidal voltage

$$V_{\text{RMS}} = \frac{1}{\sqrt{2}} V_{\text{peak}}$$

So the boxed formula can be rewritten

$$\frac{P_{\text{lost}}}{P} = \frac{P\rho L}{AV_{\text{RMS}}^2}$$

For fixed material (ρ), cable (A), length, and power, *maximize the voltage*

But maximum voltage is limited by other issues like corona discharge to $V \lesssim 250$ kilovolts

Specific example...

$$\frac{P_{\text{lost}}}{P} = \frac{1}{2}\frac{PR}{V^2} = \frac{1}{2}\frac{P\rho L}{AV^2}$$

How much power can be carried in an aluminum cable if a 7% power loss is maximum acceptable, with the following design conditions...

$L = 200$ km.

$A = 1\ \text{cm}^2$

$V = 200$ KV

First, calculate resistance

$$R = \frac{\rho L}{A} = 2.82 \times 10^{-8}\Omega\text{m} \times \frac{200 \times 10^3\ \text{m}}{10^{-4}\ \text{m}^2} = 56.4\,\Omega$$

Then, given acceptable loss, voltage, and resistance, compute deliverable power

$$\frac{P_{\text{lost}}}{P} = \frac{PR}{2V^2} \quad \Rightarrow \quad P = 0.14\,\frac{V^2}{R}$$

$$P = 0.14 \times \frac{4 \times 10^{10}\ \text{V}^2}{56\Omega} \approx 100\ \text{MW}$$

An individual cable can carry about 100 Megawatts, so a 1 Gigawatt power plant should need about 10 such cables.

Conclusion: 7% transmission loss is probably acceptable given other inefficiencies, so electrical transmission is a relatively efficient process.

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Transforming (to and from) Electromagnetic Energy

- Dynamos/motors
- Phases and Power
- Transformers

Basic idea: Induction

Time changing magnetic field induces an electric field.

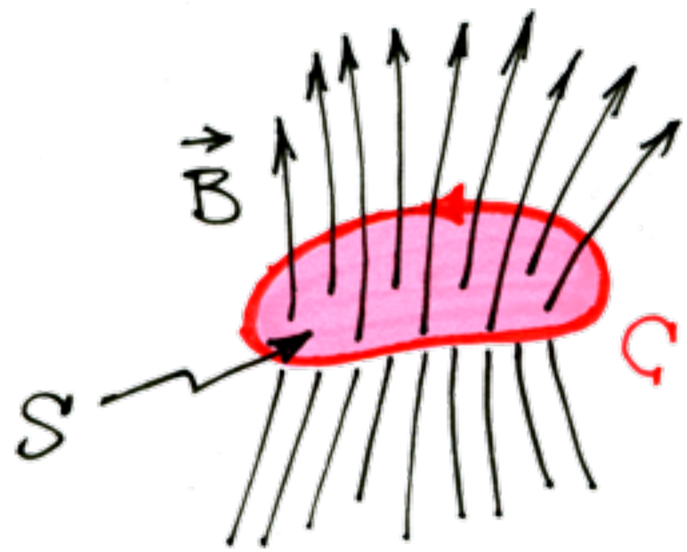
- Dynamos/motors

Long range transmission ⇒ electromagnetic forms. Hence

MECHANICAL ENERGY GENERATION → EM TRANSMISSION → MECHANICAL ENERGY USE

All based on Faraday's Law of Induction

$$\oint_C \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \iint_S \vec{B} \cdot d\vec{A}$$



$$\oint_C \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\iint_S \vec{B}\cdot d\vec{A} = -\frac{d}{dt}\Phi$$

$\mathcal{EMF}$ plays the same role in electrodynamics as voltage in electrostatics

$$\mathcal{EMF} = -N\frac{d\Phi}{dt}$$

Number of turns of wire loop

Rate of change of magnetic flux through the loop

$$\begin{aligned}
\mathcal{EMF} &= -N\frac{d\Phi}{dt} \\
\Phi &= |\vec{B}|A\cos\omega t \\
\mathcal{EMF} &= NBA\omega\,\sin\omega t \equiv V_0\sin\omega t
\end{aligned}$$

$$V_0 = NBA\omega$$

Units and magnitudes

Magnetic flux	Φ	1 Weber = 1 Volt-second
Magnetic field	$\lvert\vec{B}\rvert$	1 Tesla = 1 Weber/meter^2

1 Tesla = 10,000 gauss

$|\vec{B}_{\text{earth}}| \approx 0.3$ gauss

Example

Generate peak 120 Volts at 60 cycles/second

$B = 1000\,\text{gauss} = 0.1\,\text{Weber/m}^2$

$\omega = 2\pi \times 60\,\text{sec}^{-1}$

$V_0 = BAN\omega \Rightarrow AN = \frac{10}{\pi}\,\text{m}^2$

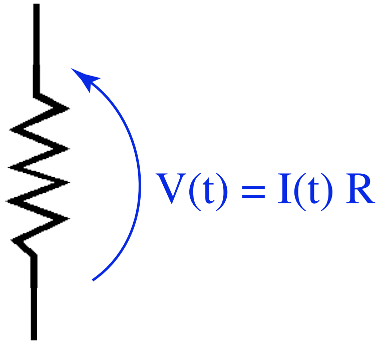
But *remember power requirements*. It you want to get 746 Watts from this generator you will need a *horse* to power it!

Phases

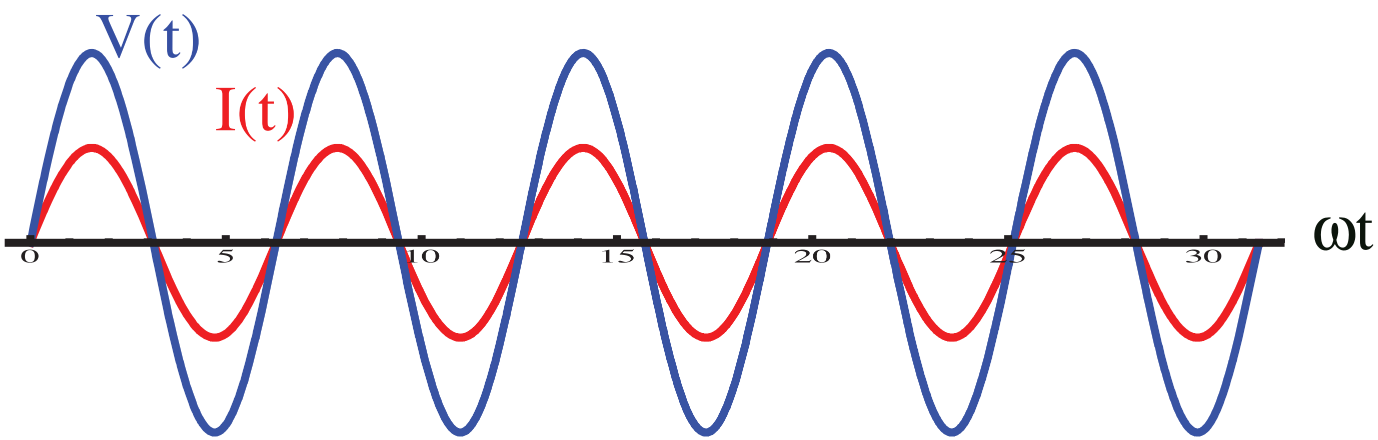
In AC electric circuits the current and voltage need not be in phase, and this has enormous effects on the power.

$$V(t) = V_0 \sin \omega t$$

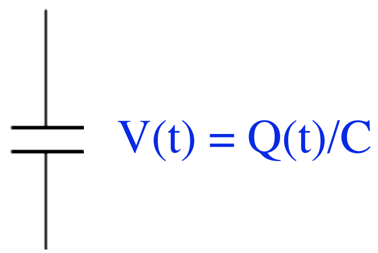
Resistive Load: $I(t) = V(t)/R = \frac{V_0}{R} \sin \omega t$



$$\begin{aligned} \langle P(t) \rangle &= \langle I(t) V(t) \rangle \\ &= \left\langle (\frac{V_0}{R} \sin \omega t)(V_0 \sin \omega t) \right\rangle \\ &= = \frac{V_0^2}{R} \langle \sin^2 \omega t \rangle = \frac{V_0^2}{2R} \end{aligned}$$

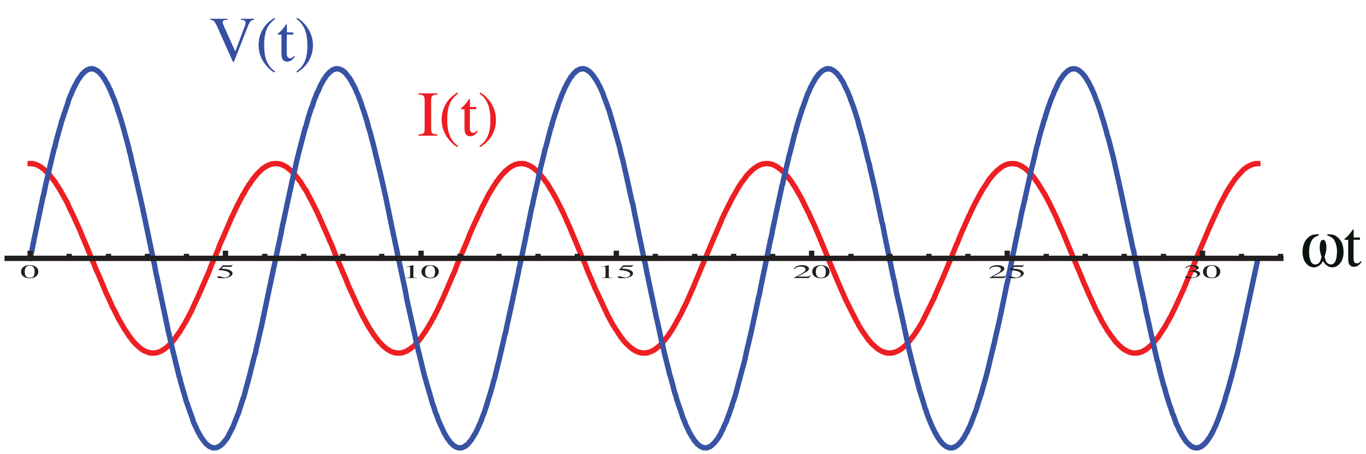


Capacitive Load:
$$
\begin{aligned}
Q(t) &= CV(t) \\
\Rightarrow I(t) &= C\frac{dV}{dt} \\
&= CV_0\omega \cos \omega t = CV_0 \omega \sin(\omega t + \pi/2)
\end{aligned}
$$



$$
\begin{aligned}
\langle P(t) \rangle &= \langle I(t) V(t) \rangle \\
&= \langle (CV_0 \cos \omega t)(V_0 \sin \omega t) \rangle \\
&= = CV_0^2 \langle \sin \omega t \cos \omega t \rangle = 0\,!
\end{aligned}
$$

Current and voltage are 90° out of phase so *no power is delivered to load*



General case

R	Resistance	$V(t) = I(t)R$
C	Capacitance	$V(t) = Q(t)/C$
L	Inductance	$V(t) = L\, dI/dt$

Theory of impedance — domain of electrical engineering!!

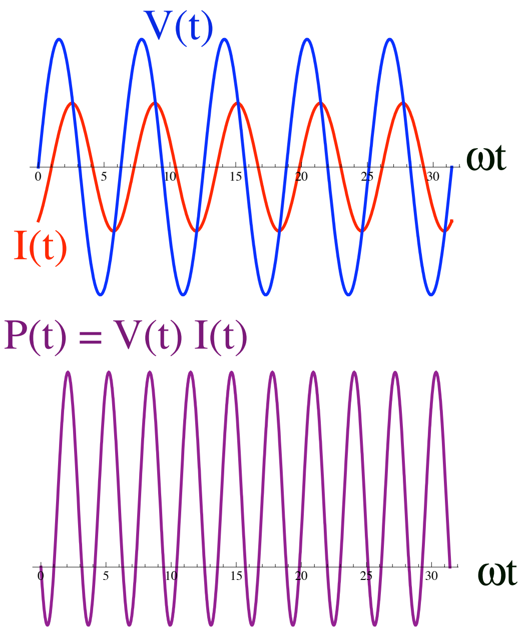
General result:

If $V(t) = V_0 \sin \omega t$

Then $I(t) = I_0 \sin(\omega t - \phi)$

Where I_0/V_0 and ϕ depend on R, L, and C

$$\begin{aligned} \langle P(t) \rangle &= \langle I(t) V(t) \rangle \\ &= \langle (I_0 \sin(\omega t + \phi))(V_0 \sin \omega t) \rangle \\ &= V_0 I_0 \langle (\sin \omega t \cos \phi + \cos \omega t \sin \phi) \cos \omega t \rangle \\ &= \frac{1}{2} I_0 V_0 \cos \phi \end{aligned}$$



The ratio of actual power delivered to “naive” power ($I_0 V_0/2$) is the power factor — here $\cos \phi$. Important issue in electrical power grids.

Phases "lite"

The fact that the electromagnetic power delivered to a load in an oscillating circuit depends on the relative phase is an example of a general phenomenon.

$$P(t) = \vec{\boldsymbol{F}}(t) \cdot \vec{\boldsymbol{v}}(t)$$

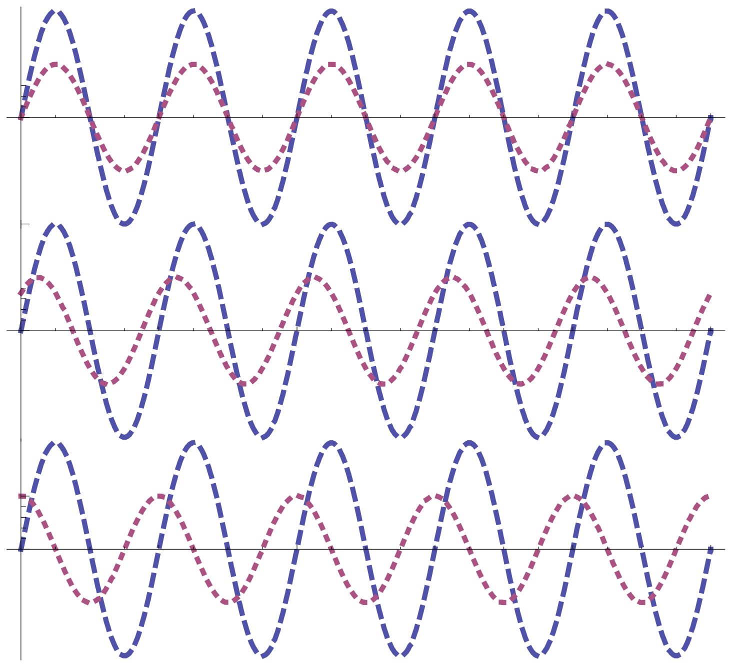
$$\text{or } \left(P(t) = V(t)I(t)\right)$$

Mechanical case is most familiar...

One child pushing another on a swing can

(a) Add energy by pushing in phase with velocity
(b) Remove energy by pushing out of phase with velocity
(c) Do nothing by pushing a 90° to velocity

Phase matching is an important ingredient in electrical power grids.



Conversion of the form of EM energy

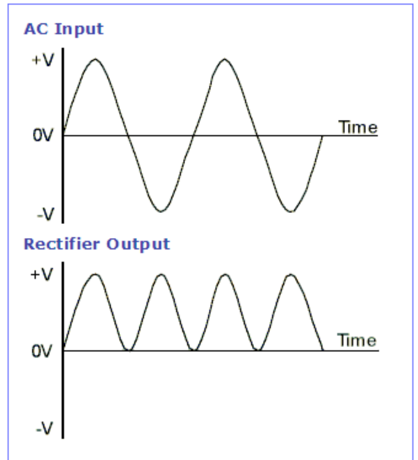


Figure 4: Full-wave rectification

A huge subject...

$AC \leftrightarrow DC$	RECTIFIERS
Low voltage $\leftrightarrow$ High Voltage	TRANSFORMERS
Phase matching	POWER FACTOR

For now, some random comments

- High voltage is desirable for transmission, but applications use a huge range of voltages, therefore...
- Transformers occur everywhere! Even though they are typically high efficiency (theoretically they can approach 100%), energy losses in transformers are significant fraction of lost electrical energy in U. S.
- Transformers only work on AC.
- But DC has transmission advantages (stay tuned)

Energy Storage and Propagation in EM Fields

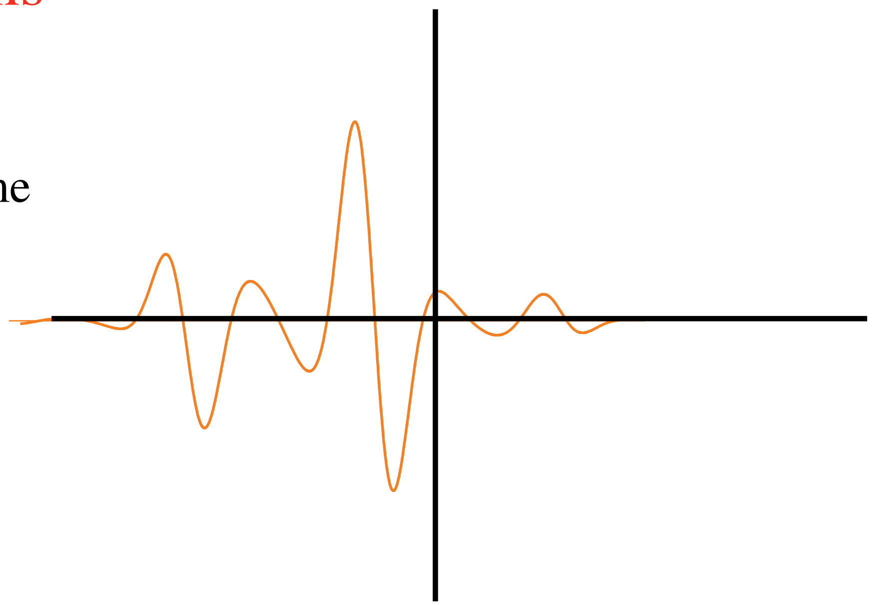
- Electromagnetic fields propagate as waves in empty space: light!
- Electromagnetic fields store and transport energy and momentum and exert pressure

- Sun!
- Absorption and emission in the atmosphere & the greenhouse effect.
- Radiative energy transport and conservation.

- Maxwell's equations have wave solutions

What's a wave?

- A fixed shape that propagates through space and time $f(x - vt)$.
- $t \to t + \Delta t$: $f(x - vt - v\Delta t)$ has same value at $x \to x + v\Delta t$.
- v is speed of the wave.



Wave Solution (moving up the $\hat{\boldsymbol{x}}$ axis):

$$\vec{\boldsymbol{E}}(\vec{\boldsymbol{x}},t) = E_0 \cos(kx - \omega t)\hat{\boldsymbol{y}}$$

$$\vec{\boldsymbol{B}}(\vec{\boldsymbol{x}},t) = B_0 \cos(kx - \omega t)\hat{\boldsymbol{z}}$$

$v = \omega/k$

Satisfy Maxwell's equation with prediction $v_{\text{wave}} = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ which equals c, the speed of light

Energy and Momentum in EM Fields

Energy per unit volume: $$u(\vec{\boldsymbol{x}},t) = \frac{dU}{dV} = \frac{1}{2}\epsilon_0 \vec{\boldsymbol{E}}^2(\vec{\boldsymbol{x}},t) + \frac{1}{2\mu_0}\vec{\boldsymbol{B}}^2(\vec{\boldsymbol{x}},t)$$

Energy also flows as electromagnetic fields propagate $\Rightarrow$ Energy flux $\vec{\boldsymbol{S}}(\vec{\boldsymbol{x}},t)$

Poynting's Vector: $\vec{\boldsymbol{S}}(\vec{\boldsymbol{x}}) \implies$ Energy per unit time, per unit area crossing a plane in the direction $\hat{\boldsymbol{S}}$ at the point $\vec{\boldsymbol{x}}$.

$$\vec{\boldsymbol{S}} = \frac{1}{\mu_0}\vec{\boldsymbol{E}} \times \vec{\boldsymbol{B}}$$

Conceptually similar to heat flux from last lecture (and to charge flux $\equiv$ current density, and mass flux in fluid dynamics)

Maxwell's equations have wave solutions

Gauss's Law	$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$
No magnetic charge	$\vec{\nabla} \cdot \vec{B} = 0$
Faraday's Law	$\vec{\nabla} \times \vec{E} + \frac{d\vec{B}}{dt} = 0$
Ampere/Maxwell Law	$\vec{\nabla} \times \vec{B} - \mu_0\epsilon_0 \frac{d\vec{E}}{dt} = \mu_0 \vec{J}$

Empty space: $\rho = \vec{j} = 0$

$$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$

$$\vec{\nabla} \times \vec{B} - \mu_0\epsilon_0 \frac{\partial \vec{E}}{\partial t} = 0$$

Wave motion: $\omega = k v_{\text{wave}}$ (or $\lambda\nu = v_{\text{wave}}$)

Maxwell's equations:

$$\vec{\nabla} \cdot \vec{E} = \vec{\nabla} \cdot \vec{B} = 0$$

Wave Solution (moving up the $\hat{x}$ axis):

$$\vec{E}(\vec{x}, t) = E_0 \cos(kx - \omega t)\hat{y}$$

$$\vec{B}(\vec{x}, t) = B_0 \cos(kx - \omega t)\hat{z}$$

Substitute in Maxwell's equations

$$\vec{\nabla} \cdot \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$$

$$\vec{\nabla} \times \vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_0 \cos(kx - \omega t) & 0 \end{vmatrix} = -E_0 k \sin(kx - \omega t)\hat{z}$$

$$\frac{\partial \vec{B}}{\partial t} = +\omega B_0 \sin(kx - \omega t)\hat{z}$$

$$\vec{\nabla} \times \vec{E} + \partial \vec{B}/\partial t = (\omega B_0 - k E_0)\sin(kx - \omega t)\hat{z} = 0$$

And likewise for second two Maxwell's equations

Conservation of energy: In a fixed volume, energy change inside must come from energy flowing in or out

In a time dt, an amount of energy $\vec{\boldsymbol{dA}} \cdot \vec{\boldsymbol{S}}\, dt\, dA$ crosses the surface element $\vec{\boldsymbol{d}A}$.

$$\frac{d}{dt}\iiint_V u(\vec{\boldsymbol{x}},t)d^3x = -\iint_{\substack{\text{boundary}\\ \text{of } V}} \vec{\boldsymbol{S}}(\vec{\boldsymbol{x}},t)\cdot \vec{\boldsymbol{d}A}$$

Classic "conservation law"

In a fixed volume V, use Gauss's Theorem

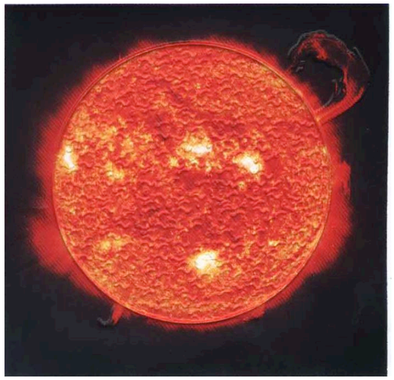
$$\iiint_V \frac{\partial}{\partial t}u(\vec{\boldsymbol{x}},t)d^3x = -\iiint_V \vec{\boldsymbol{\nabla}}\cdot \vec{\boldsymbol{S}}(\vec{\boldsymbol{x}},t)d^3x$$

$$\frac{\partial}{\partial t}u(\vec{\boldsymbol{x}},t) + \vec{\boldsymbol{\nabla}}\cdot \vec{\boldsymbol{S}}(\vec{\boldsymbol{x}},t) = 0$$

Poynting's vector

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

Energy flowing in our light wave...



$$\begin{aligned}\vec{S} &= \frac{1}{\mu_0}E_0\cos(kx-\omega t)\hat{\boldsymbol{y}} \times \frac{E_0}{c}\cos(kx-\omega t)\hat{\boldsymbol{z}} \\ &= \sqrt{\frac{\epsilon_0}{\mu_0}}E_0^2\cos^2(kx-\omega t)\hat{\boldsymbol{x}}\end{aligned}$$

flows in the direction perpendicular to $\vec{E}$ and $\vec{B}$, which is direction of wave propagation.

Summary

Capacitance:	?	Energy storage	$E = \frac{1}{2}CV^2$
Resistance:		Energy loss	$P = \left(\frac{1}{2}\right) I^2 R$
	X	Space heating	
	⋆	Transmission	$\frac{P_{\text{lost}}}{P} = \frac{PR}{2V_{\text{max}}^2}$
Inductance:	⋆	Conversion	Generators Transformers Motors
Radiation:	⋆	Transmission	$\boldsymbol{S} = \frac{1}{\mu_0}\boldsymbol{E} \times \boldsymbol{B}$