8.21 The Physics of Energy
Fall 2009

8.21 Lecture 9

Heat Engines

September 28, 2009

Today: heat engines

heat input → (cyclic) heat engine → work out

$\Delta Q_{\text{in}} = T_{+} \Delta S$

$\Delta W = \Delta Q_{\text{in}} - \Delta Q_{\text{out}}$

$\eta_{\max} = \frac{T_{+} - T_{-}}{T_{+}}$

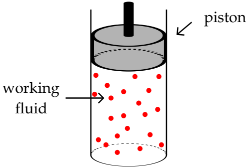
"waste" heat output

$\Delta Q_{\text{out}} \geq T_{-} \Delta S$

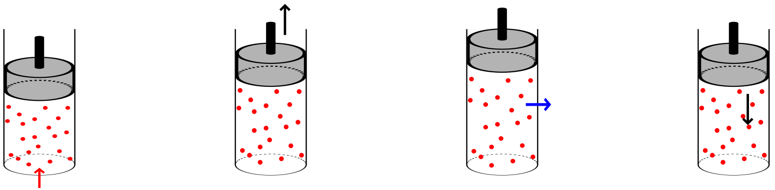
Today: closed-cycle heat engines

- working fluid in container
- no fluid leaves or enters

Friday: internal combustion engines



Components of a closed-cycle heat engine



1. Heat in

2. Expand ⇒ do work

3. Release heat

4. Contract ⇒ cycle

Processes need not be distinct

(*e.g.* isothermal expansion combines 1 + 2)

What we need from thermodynamics for heat engine analysis:

We will analyze thermodynamic processes on working fluid

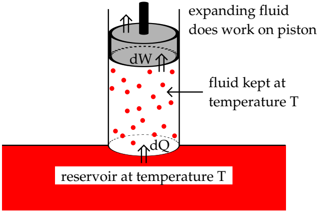
—Follow processes in terms of state variables p, V, S, T

—Assume near equilibrium at all times (quasiequilibrium)

—Combine thermodynamic processes $\Rightarrow$ heat engine cycles

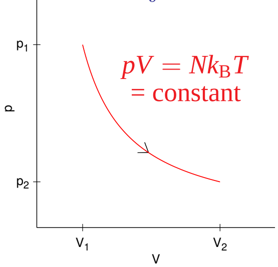
- First law$\Rightarrow$ $dQ = dU + p\,dV$ (quasieq. $\rightarrow$ no free expansion)
- Definition of temperature$\Rightarrow$ $dQ = TdS$ (quasieq.)
- Heat capacity: $dU = C_v\, dT$ (assume C_V independent of T)
- Ideal gas law: $pV = Nk_{\text{B}}T$

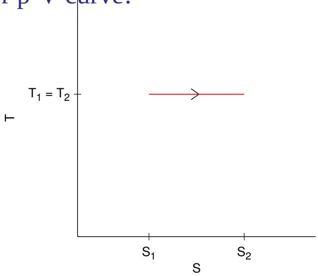
Isothermal expansion



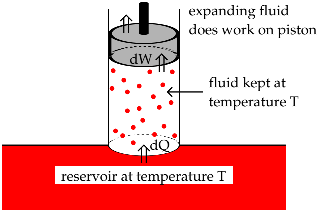
Constant temperature $\Rightarrow \; dU = 0 \; \Rightarrow \; dQ = p\, dV$

Note: $W = \int p dV$ = area under p-V curve!





Quasiequilibrium processes



In real process,
gradient ⇒ heat flow $(\mathbf{q} = -k\nabla T)$

But for slow expansion, temperature approximately constant at each time.

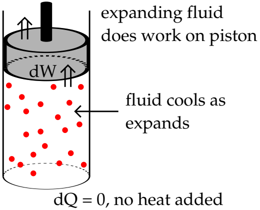
Quasiequilibrium assumption:
No gradients, system in equilibrium at each time t.

With quasiequilibrium assumption:
isothermal expansion reversible

Process reversible ⇔ $dS_{\text{total}} = 0$!

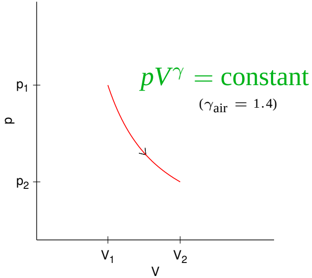
So desire reversible processes for efficient engines.

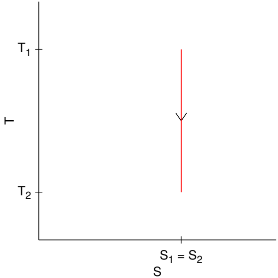
Adiabatic expansion (reversible)



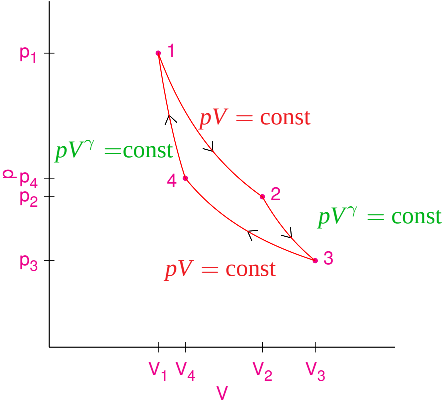
$$dU = \hat{c}_v N k_B dT$$
$$= \hat{c}_v (pdV + Vdp) = -pdV$$
$$\Rightarrow (\hat{c}_v + 1)pdV + \hat{c}_v Vdp = 0$$
$$\Rightarrow pV^\gamma = \text{constant}, \; \gamma = \hat{c}_p/\hat{c}_v$$

No heat added $\Rightarrow dQ = 0 \;\Rightarrow\; dU = -p\,dV,\; dS = 0$





Carnot Heat Engine



1→2: Isothermal expansion at T_H
Heat in, work out $\Delta Q_{\text{in}} = W_{1\to 2}$

2→3: Adiabatic expansion,
$\Delta Q = \Delta S = 0$, work $W_{2\to 3}$

3→4: Isothermal compression (T_C)
Heat out, work in $\Delta Q_{\text{out}} = W_{3\to 4}$

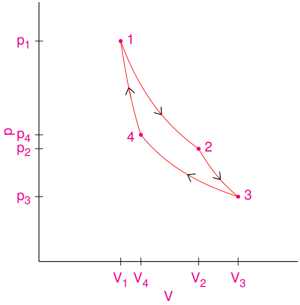
4→1: Adiabatic compression,
$\Delta Q = \Delta S = 0$, work in $W_{4\to 1}$

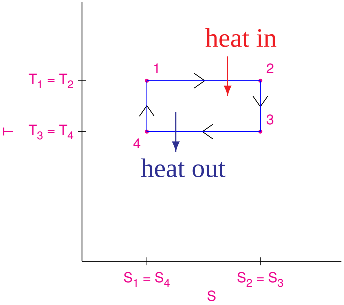
Total work: area

$$W = W_{12} + W_{23} - W_{34} - W_{41}$$
$$= Q_{\text{in}} - Q_{\text{out}} = (T_H - T_C)\Delta S$$

$$\eta = W/Q_{\text{in}} = (T_H - T_C)/T_H = \eta_C$$

Compare: pV and TS plots





$W = \int p\,dV = \Delta Q = \int T\,dS$

Net work out = net heat in = area on both graphs

Cycle analysis: $\sim$ Sudoku–4 relations $p_1 V_1 = p_2 V_2$, $p_2 V_2^\gamma = p_3 V_3^\gamma$, $\ldots$

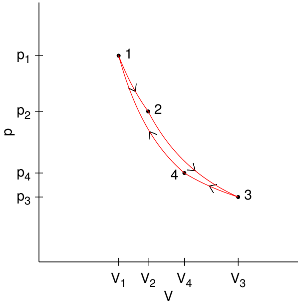
$\Rightarrow$ solve given 4 independent variables (5 including T's or N)

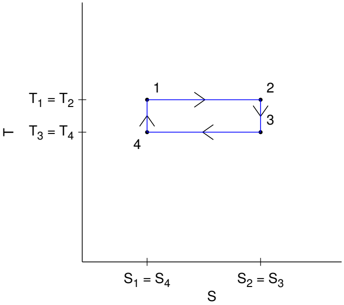
Carnot engines achieve optimal efficiency

But generally very little work/cycle

Previous example had unrealistic $\gamma = 2.5$.

With $\gamma = 1.4$

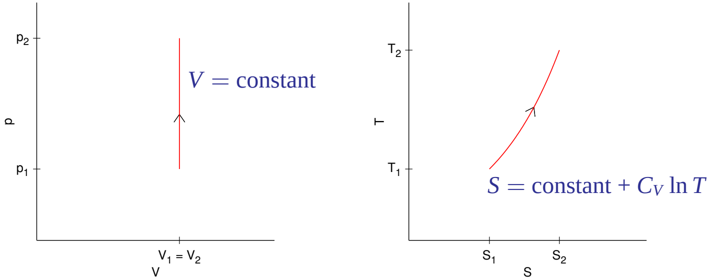




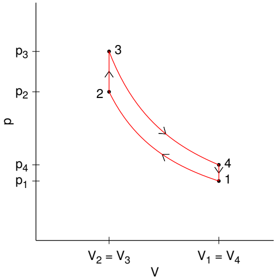
Better: Stirling engine

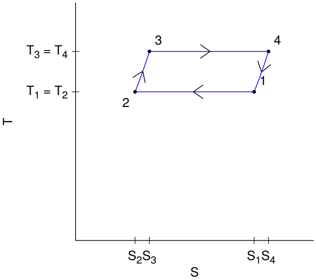
Need new process — Isometric heating
Easy to do irreversibly, more tricky reversibly

No change in volume $\Rightarrow dQ = dU = C_V dT \Rightarrow dS = C_V dT/T$



Stirling cycle (note different [engineering] labeling 1-4!)





1 $\longrightarrow$ 2: Isothermal (T_C) compression, heat out Q_{out}, work in W_{in}.

2 $\longrightarrow$ 3: Isometric heating $T_C \rightarrow T_H$, heat in Q_h, no work.

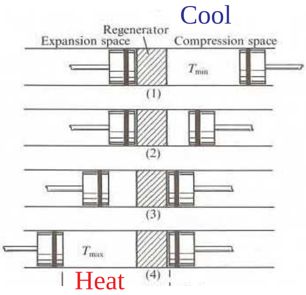
3 $\longrightarrow$ 4: Isothermal (T_H) expansion, heat in Q_{in}, work W_{out}.

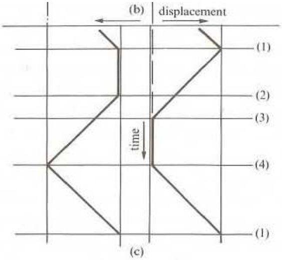
4 $\longrightarrow$ 1: Isometric cooling $T_H \rightarrow T_C$, heat out Q_c, no work.

Key: heat output Q_c stored in *regenerator*, returned as Q_h

Achieves Carnot efficiency– but greater work output than Carnot!

Stirling engine implementation (example)

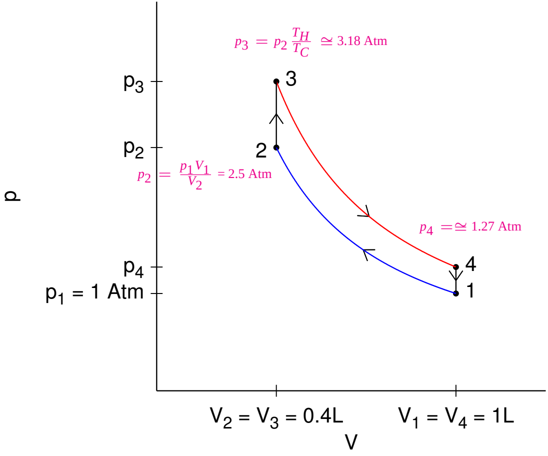




Stirling cycle: example

Use$T_H = 100°C$, $T_C = 20°C$, $V_1 = 1$ L, $V_2 = 0.4$ L, $p_1 = 1$ Atm

1. Use $pV = Nk_B T \Rightarrow p$'s



2. Work = $\Delta Q = \int p\, dV$

$$\int_3^4 p\, dV = p_3 V_3 \ln \frac{V_4}{V_3}$$

$Q_{in} \cong 129$ J (ln 2.5) $\cong 118$ J

$Q_{out} \cong 101$ J (ln 2.5) $\cong 92.5$ J

W = $Q_{out} - Q_{in} \cong 25.5$ J

Compare Carnot: W $\cong 8.6$ J

Stirling engines

- Maximum (Carnot) efficiency
- Higher work/cycle than Carnot for given p, V extremes
- Theoretically very promising
- Variant: Ericsson cycle–constant pressure regenerative process
- External combustion, arbitrary fuel source (solar, nuclear, rice. . .)
- Currently used in niche apps (space, mini cryocoolers. . .)
- Technically challenging!
  —Hard to get true isothermal expansion
  —Seal problems at high pressure (Beale: free piston SE)
  —Materials exposed to high T for long time
- Maybe widely used in future? (vehicles, solar thermal?)

SUMMARY

- Thermodynamic processes/cycles: use state variables p, V, T, S, U
  quasiequilibrium assumption, reversible processes.

- Isothermal expansion/compression: add heat, do work
  $dQ = dW, dU = 0$. $pV =$ constant, $T =$ constant.

- Adiabatic expansion/compression: no heat added
  $dU = -pdV, dQ = 0$. $pV^\gamma =$ constant, $S =$ constant.

- Isometric heating/cooling: reversible with *regenerator*, no work
  $dQ = dU$. $V =$ constant, $S = C_V \ln T +$ constant.

- Carnot cycle: IT exp., Ad exp., IT compr., Ad compr.
  Maximum (Carnot) efficiency, but small work done per cycle.

- Stirling cycle: IT compr., IM heat, IT exp., IM cool.
  Maximum efficiency, more work/cycle than Carnot