

8.251 Test

B. Zwiebach

Thursday March 18, 2004

Only personal 2 page notes allowed.

Test duration: 90 minutes.

Problem 1. (20 points) 4 short questions (5 points each) – Correct answers suffice !

- (a) What are the units (M, L, T) of the Planck constant \hbar . Why does the Planck constant have the same units in all numbers of spacetime dimensions ?
- (b) Consider the variation $\delta\phi(x)$ of the field in the action

$$S = \int d^D x \left(-\frac{1}{2} \eta^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right).$$

Compute the variation of the action (dropping boundary terms) and write it as

$$\delta S = \int d^D x \delta\phi(x) (\dots).$$

Find the terms inside the parenthesis.

- (c) Consider two static D1-branes in four dimensional spacetime (ct, x^1, x^2, x^3) . The first one is at $x^1 = x^2 = 0$ – it stretches along the x^3 axis. The second one is parallel to the first and goes through $x^1 = 0, x^2 = a > 0$. Sketch the branes. Consider open strings with $\sigma \in [0, \sigma_1]$ that stretch from the first brane ($\sigma = 0$) to the second brane ($\sigma = \sigma_1$). State the boundary conditions (Free or Dirichlet, with value) for the string coordinates $X^\mu(t, \sigma_*)$ (list the eight conditions – $\mu = 0, 1, 2, 3$ and $\sigma_* = 0, \sigma_1$).
- (d) Use the explicit (but general) expression for \mathcal{P}_μ^σ (recall that $\mathcal{P}_\mu^\sigma = \frac{\partial \mathcal{L}}{\partial \dot{X}^{\mu\sigma}} = \dots$) to calculate the value of $\mathcal{P}_\mu^\sigma \dot{X}^\mu$. (Just in case: $\mathcal{L} = -\frac{T_0}{c} \sqrt{(X' \cdot \dot{X})^2 - \dot{X}^2 X'^2}$.)

Problem 2. (20 points) Rotating open string.

Consider an open string of energy E which rotates rigidly on a plane with angular velocity ω . Recall that the length ℓ of such string is $\ell = \frac{2}{\pi} \frac{E}{T_0}$.

- (a) What is ω in terms of E, T_0 , and c .
- (b) Calculate the magnitude p of the relativistic momentum carried by any one-half of the string stretching from the middle point to the endpoint. Give your answer in terms of E and c .
- (c) Let q denote the momentum of a *massless* particle that carries half of the energy of the string. What is the value of p/q ?

Problem 3. (20 points) Constructing the T^2/\mathbb{Z}_3 orbifold (almost!)

Consider the complex plane $z = x + iy$ subject to the following two identifications

$$z \sim T_1(z) = z + 1, \quad \text{and} \quad z \sim T_2(z) = z + e^{i\pi/3}.$$

($e^{i\pi/3}$ is a unit vector at an angle of $\pi/3$ with respect to the horizontal axis.) A fundamental domain (with its boundary) is the parallelogram with corners at $z = 0, 1$, and $e^{i\pi/3}$.

- Where is the fourth corner of the parallelogram? Make a sketch (large please!) and indicate the identifications on the boundary. Describe the space obtained by the identification.
- Consider now the *additional* \mathbb{Z}_3 identification

$$z \sim R(z) = e^{2\pi i/3} z.$$

We would like to determine the fixed points of the orbifold action on the parallelogram. One fixed point is the origin (which is identified with the other corners). The other two nontrivial fixed points z_1 and z_2 (take $|z_1| < |z_2|$) lie on the long diagonal of the parallelogram. Find them. [Hint: make a nice large sketch and examine the action of R on the diagonal.]

- Does the composition $T_1 \circ R$ of the R and T_1 action leave invariant z_1 or z_2 ?

Problem 4. (40 points) Closed strings and time development.

The time development of closed strings in the static gauge is governed by usual wave equations and parameterization conditions:

$$\frac{\partial^2 \vec{X}}{\partial \sigma^2} - \frac{1}{c^2} \frac{\partial^2 \vec{X}}{\partial t^2} = 0, \quad \left(\frac{\partial \vec{X}}{\partial \sigma} \pm \frac{1}{c} \frac{\partial \vec{X}}{\partial t} \right)^2 = 1.$$

- Assume that $\frac{\partial \vec{X}}{\partial t}(0, \sigma) = 0$, and write the general solution for $\vec{X}(t, \sigma)$ in terms of a vector function $\vec{F}(u)$ of a single variable.
- What do the parameterization conditions require on \vec{F} ?
- On a closed string the parameter σ lives on the circle $\sigma \sim \sigma + \sigma_1$. What condition would you impose on $\vec{X}(t, \sigma)$ to implement this feature? What does it imply for \vec{F} ?
- Consider the closed string motion represented by:

$$\vec{F}'(u) \equiv \frac{d\vec{F}}{du} = \left(-\sin \frac{2\pi u}{\sigma_1}, \cos \frac{2\pi u}{\sigma_1} \right).$$

Describe the string at $t = 0$.

Consider henceforth the closed string motion represented by:

$$\vec{F}'(u) = \left(-\sin \left[\gamma \sin \frac{2\pi u}{\sigma_1} \right], \cos \left[\gamma \sin \frac{2\pi u}{\sigma_1} \right] \right), \quad \gamma > \frac{\pi}{2}.$$

- Find a simplified expression for the value of $\frac{1}{c} \left| \frac{\partial \vec{X}}{\partial t} \right|$ for arbitrary values of t and σ .
- Find an equation for the smallest time $t_* > 0$ at which cusps suddenly appear.
- Assume $\frac{3\pi}{2} < \gamma < 2\pi$. What is the maximum number of cusps simultaneously present on the closed string?