$$\vec{x}(t,\sigma) = \frac{1}{2}(\vec{F}\overbrace{(ct+\sigma)}^{u} + \vec{G}\overbrace{(ct-\sigma)}^{v})$$
$$= \frac{1}{2}(\vec{F}(u) + \vec{G}(v))$$

$$\left(\frac{1}{c}\frac{\partial \vec{x}}{\partial t} \pm \frac{\partial \vec{x}}{\partial \sigma}\right)^2 = 1$$

$$\frac{1}{c}\dot{\vec{x}} + \vec{x}' = \vec{F}'(ct+\sigma)$$
$$\frac{1}{c}\dot{\vec{x}} - \vec{x}' = \vec{G}'(ct-\sigma)$$

$$\frac{1}{c}\dot{\vec{x}} = \frac{1}{2}(\vec{F}' + \vec{G}')$$
$$\vec{x}' = \frac{1}{2}(\vec{F}' - \vec{G}')$$

$$|\vec{F}'(u)| = |\vec{G}'(v)| = 1$$
$$\vec{x}(t, \sigma + \sigma_1) = \vec{x}(t,\sigma)$$

$$\sigma_1 = E/T_0$$

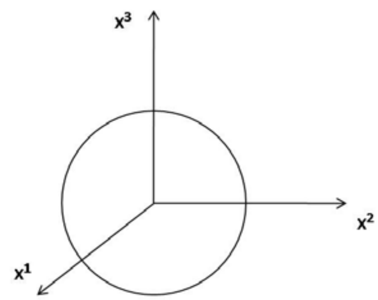
$$\vec{F}(u+\sigma_1) + \vec{G}(v-\sigma_1) = \vec{F}(u) + \vec{G}(v)$$
$$\vec{F}(u+\sigma_1) - \vec{F}(u) = \vec{G}(v) - \vec{G}(v-\sigma_1)$$

2 Periodicity Conditions:

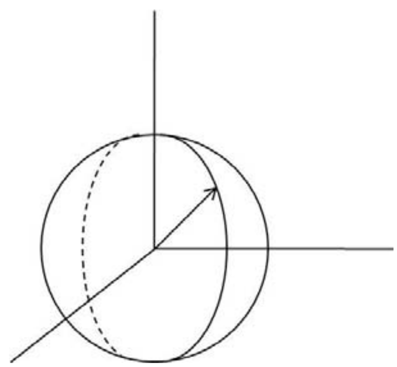
$$\vec{F}'(u+\sigma_1) = \vec{F}'(u)$$
$$\vec{G}'(v+\sigma_1) = \vec{G}'(v)$$

$$\vec{F}(u) = \underbrace{\hat{F}(u)}_{\text{strictly periodic fcn of } u} + \vec{\alpha}u$$

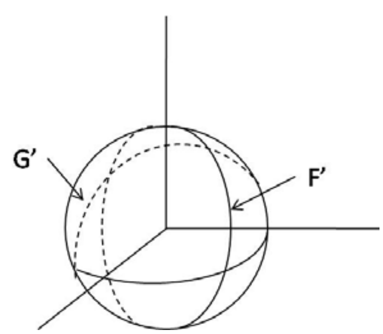
Unit Sphere in 3D



$\vec{F}'$ is a unit vector (so goes from origin to surface). Traces a closed curve on surface, periodic.



$\vec{G}'$ traces another curve



Usually, these paths cross at least two times at point (u_0,v_0) subject to $\vec{F}'(u_0) = \vec{G}'(v_0)$.

(u_0, v_0) determines (t_0, σ_0).

Thus $\frac{1}{c}\dot{\vec{x}}(t_0, \sigma_0) = \frac{1}{2}(\vec{F}'(u_0) + \vec{G}'(v_0)) = \vec{F}'(u_0)$

$$|\dot{\vec{x}}(t_0, \sigma_0)| = c|\vec{F}'(u_0)| = c$$

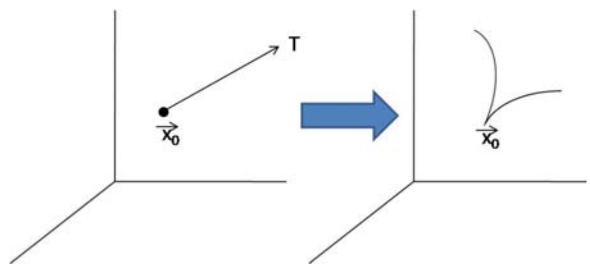
At one point at a time, string moving at speed of light!

$$\underbrace{\vec{x}'(t_0, \sigma_0)}_{0} = \frac{1}{2}(\vec{F}'(u_0) - \vec{G}'(v_0))$$

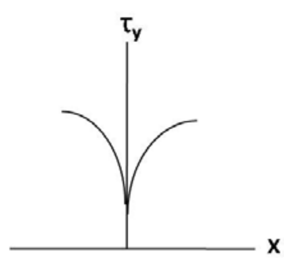
What does the string look like at this time t_0 at positions near σ_0?

$$\begin{aligned} x(t_0, \sigma \approx \sigma_0) &= x(t_0, \sigma_0) + (\sigma - \sigma_0) \cdot \overbrace{x'(t_0, \sigma_0)}^{0} + \frac{1}{2}(\sigma - \sigma_0)^2 x''(t_0, \sigma_0) + \dots \\ &= \vec{x_0} + (\sigma - \sigma_0)^2 \vec{T_0} + \dots \end{aligned}$$

where $\vec{T_0} = \frac{1}{2}x''(t_0, \sigma_0)$ etc.



String has cusp singularity.
Align xy axis subject to:



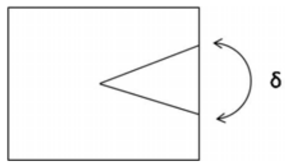
Find $y \approx |x|^{\frac{2}{3}}$

Imagine string survived early universe, very long (across universe), with lots of tension, gigantic μ_0. What would happen if crossing the room?

We wouldn't feel any gravitational attraction! Einstein's gravity says sometihng with mass will gravitate. But the μ_0 and the tension conspire to make no gravity.

Does affect geometry of space. If measure radisu of string and go in same radius circle around, don't get circumference/radius = 2π.

Creates a conical singularity!



Conical singularity of a cosmic string $\exists$ ongoing searches for these. False alarm 1.5 years ago.

Deficit angle: $\delta = 8\pi G T_0/c^4 = 8\pi G\mu_0/c^2$. $\mu_0 = T_0/c^2$.

Recall $G = \hbar c/M_p^2$. M_p: Planck Mass. Got M_p from $G, \hbar, c$. Construct now M_p from $\mu_0, \hbar, c$.

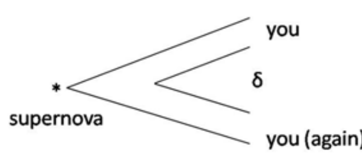
$$[\mu_0] = M/L$$
$$[t] = ML^2/T = ML[c]$$
$$[\mu_0] = M/[\hbar][c] = M^2[c]/[\hbar]$$

So $\mu_0 = M_s^2 c/\hbar$. M_s is string mass.

$$\boxed{\delta = 8\pi\left(\frac{\mu_s}{\mu_p}\right)^2}$$

$\delta \approx 5.2\text{"}(G\mu_0/10^{-6})$. ": seconds or arc, $c = 1$.
Example: If knew $\delta = 0.5''$, $\mu_0 = 1.3 \times 10^{20}$ kg/m.

Can look for δ



See 2 images with angle between.

What is $\mathcal{P}^\sigma_\mu$?

Recall Maxwell: $\nabla \cdot E = \rho$. $\nabla \times B = \frac{1}{c}\vec{J} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$.

$$\nabla \cdot (\nabla \times \vec{B}) = 0 \qquad \text{(for any } B\text{)}$$

$$\frac{1}{c}\nabla \cdot \vec{J} + \frac{1}{c}\frac{\partial \nabla \cdot E}{\partial t} = 0$$

$$\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

Charge Conservation:

$$\frac{1}{c}\frac{\partial}{\partial t}(\rho c) + \partial_i J^i = 0$$

$$\frac{\partial}{\partial x^\mu} J^\mu = 0$$

Define Charge:

$$Q = \int d^3 x \rho(t, \vec{x})$$

$$\frac{dQ}{dt} = \int d^3 x d\rho(t, \vec{x})/dt = -\int d^3 x \nabla \cdot \vec{J} = -\int d\vec{a} \cdot \vec{J}$$

ρ: charge density Q/L^3 (density over volume)
$\vec{J}$: current density Q/TL^2 (density over area)

Mechanics

Lagrangian: $L(q(t), \dot{q}(t); t)$
Symmetry: a rule to change any path $q(t)$ subject to L is unchanged. There exists other kinds of symmetries too. Rule may depend on path.

Rule:

$$q(t) \to q(t) + \delta q(t)$$

where $\delta q(t) = \epsilon \cdot h(q(t); t)$. h is a magic function. ϵ: small number. Accordingly:

$$\dot{q}(t) = q(t) + \frac{d}{dt}(\delta q(t))$$

Theorem: If under these changes the terms linear is δq vanish in δL, then we get a conserved charge:

$$\epsilon Q = \left(\frac{\partial L}{\partial \dot{q}}\right)\delta q$$

Claim: $\frac{dQ}{dt} = 0$

$$\frac{d}{dt}(\epsilon Q) = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)\delta q + \frac{\partial L}{\partial \dot{q}}\frac{d}{dt}(\delta q)$$

Euler Lagrange Equation of Motion:

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial q}\right) - \frac{\partial L}{\partial \epsilon} = 0$$

Used to write:

$$\frac{d(EQ)}{dt} = \left(\frac{\partial L}{\partial q}\right)\delta q + \left(\frac{\partial L}{\partial \dot{q}}\right)\delta \dot{q} = \delta L = 0$$

Example:
$L = L(\dot{q}(t))$. L only dependent on velocity. Then have symmetry $\delta q(t) = \epsilon$. The have $\delta \dot{q} = 0$. L invariant (so is symmetric).

$$Q = \frac{\partial L}{\partial \dot{\epsilon}} = p$$

$$S = \int d\xi^0 d\xi^1 \dots d\xi^p \mathcal{L}(\phi^a, \partial_\alpha \phi^a)$$

$$\alpha \to 0, \dots, p$$

Fields: ϕ^a.
Nambu-Gotta:

$$\xi^\alpha(\tau, \sigma) = \int d\tau \int d\sigma \mathcal{L}\left(\frac{\partial x^\mu}{\partial \tau}, \frac{\partial x^\mu}{\partial \sigma}\right.$$

Will find there are conserved currents of form $\frac{\partial}{\partial \xi^\alpha}(J_i^\alpha)$