Lecture 21 - Topics

- Wrap-up of Closed Strings

Wrapup of Closed Strings

String Coupling Constant

Dimensionless number that sets the strength of the interaction.

Example of coupling constants:

1. Fine structure constant $\alpha = \frac{e^2}{4\pi\hbar c} \approx \frac{1}{137}$. Derives from interactions between charged particles, magnetic fields. Could imagine particles with $e = 0$.

Action $S = \int d^p x(-\frac{1}{4}F^2) - mc\int dS + e\int_{\mathcal{P}} A_{\mu(x)dx^\mu}$. Doesn't talk about interaction between charged particle and field (both are free).

2. Binding energy of an electron in a H atom. $E_{binding} \propto e^4$ since $V \propto \frac{e^2}{r}$ $r \approx \frac{1}{e^2}$ in Bohr atom.

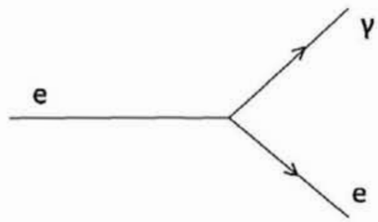
$E_{binding} = \frac{1}{2}\alpha^2(M_e C^2) \approx \frac{1}{2}\frac{1}{137}1137(500,000eV) \approx 13eV$.

Similar effect in string theory unless gravitons interacting with matter, no gravity!

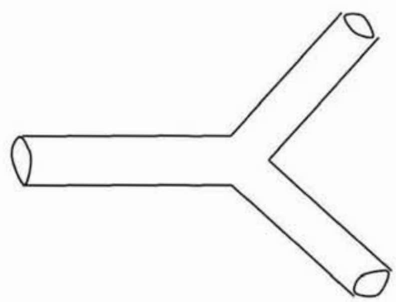
D-dim Newton's Constant: $[G^(D)] = L^{D-2}$ (natural units). $D = 10$: $[G^{(10)}] = L^8 \Rightarrow G^{(10)} = g^2(\alpha')^4$ where g is the string coupling constant.

Planck length l_p related to string length l_s: $l_p^8 = g^2 l_s^8 \Rightarrow l_p \approx g^{\frac{1}{4}} l_s$

Particle View:



String View:



Close string splits into 2 strings. Same coupling constant $g\forall$ string interactions.

g for closed strings. $\sqrt{g}$ for open strings (too complicated for this class)

How do you fix g?

$\phi(x)$ =dilation $\leftrightarrow$ $a_1^{+I}\overline{a}_1^{I+}|\Omega\rangle$ massless state. $g \approx e^{\phi(x)}$. Field sets value of coupling constant. So g changes with $\phi(x)$. Not a constant, but a dynamical value. But usually work with constant field.

4D:

$$G^{(4)} = \frac{G^{(10)}}{V^{(6)}} = \frac{g^2\alpha'^4}{V^{(6)}} = \frac{g^2\alpha'}{(V^{(6)}/\alpha'^3)}$$

Superstrings

Everyone uses superstrings more than superstrings. Takes a long time to develop all background, so will present intuitively-reasonable results from QFT.

Pauli exclusion principle: multiple fermions cannot occupy the same state.

$X^\mu(\tau,\sigma)$: Classical variables. Commute $X^I(\tau,\sigma)X^J(\tau',\sigma') = X^J(\tau',\sigma')X^I(\tau,\sigma)$. The X's behave as boson fields in (τ,σ) space.

In quantum theory, things don't quite commute. For operators A, B commutator $[A,B] = A\cdot B - B\cdot A$. A and B commute if and only if $[A,B]=0$.

Define: $\psi_1^\mu(\tau,\sigma)$, $\psi_2^\mu(\tau,\sigma)$. Classical, anticommuting variables $\psi_\alpha^\mu(\tau,\sigma)$.

Let B_1, B_2 be classical anticommuting variables. Then $B_1B_2 = -B_2B_1$, $B_1B_1 = -B_1B_1 \Rightarrow B_1B_1 = 0$. Same for all indices.

Set of anticomm. variables B_i:

$$B_iB_j = -B_jB_i$$

$$B_iB_i(\text{not summed}) = 0$$

Example with matrices:

$$\gamma_1 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\gamma_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\gamma_1\gamma_2 = -\gamma_2\gamma_1$$
$$\gamma_1\gamma_1 \neq 0 \neq \gamma_2^2$$

Quantum operators f_1,f_2

$$\{f_1, f_2\} = f_1 f_2 + f_2 f_1$$

Operators anticommute if and only if $\{f_1, f_2\} = 0$.

Quantized a scalar field. Got particle state of n_k particles

$$(a_{p_1}^+)^{n_1}(a_{p_2}^+)^{n_2)\ldots(a_{p_k})^+)^{n_k}|\Omega\rangle}$$

Electron Dirac Field

$$f_{p_1,s_1}^+ f_{p_2,s_2}^+ \ldots f_{p_k,s_k}^+ \Omega$$

p: momentum, s: spin.

Creation operators have nonzero anticomm. operators with annihilation operators. All f^+'s anticommute. This relates to the Pauli exclusion principle and Fermi statistics.

Action: $S = S_{\text{Bosonic}} + S_{\text{Fermionic}}$

$$S_B = \frac{1}{4\alpha'} \int d\tau d\sigma (\dot{X}^I \dot{X}^I - X^{I\prime} X^{I\prime})$$

Action for LC coordinates. Tells you pretty much everything about dynamics of LC variables.

$$S_F = \frac{1}{2\pi} \int d\tau d\sigma [\psi_1^I (\frac{\partial}{\partial\tau} + \frac{\partial}{\partial\sigma}) \psi_1^I + \psi_2^I (\frac{\partial}{\partial\tau} - \frac{\partial}{\partial\sigma}) \psi_2^I]$$

What Dirac would have written for a fermion in 2D on the worldsheet, but we can a fermion in spacetime.

Dirac equation usually written as $\psi i \gamma \partial \psi$

Usually, $A\partial_\tau A = \frac{1}{2}\partial_\tau A^2$ but here the $A's$ (ψ_α^Is) are anticommuting so $A^2 = 0$.

Instead, $A\partial_\tau A = \partial_\tau (AA) - (\partial_\tau A)A$. $A\partial_\tau A + (\partial_\tau A)A = \partial_\tau (AA)$.

Varying $\delta\psi_1^I$, $\delta\psi_2^I$

$$\delta S_F = \frac{1}{\pi} \int d\tau d\sigma (\delta\psi_1^I (\partial_\tau + \partial_\sigma) \psi_1^I + \delta\psi_2^I (\partial_\tau - \partial_\sigma) \psi_2) + \frac{1}{2\pi} \int d\tau (\psi_1^I \delta\psi_1^I - \psi_2 \delta\psi_2^I)_{\sigma=0}^{\sigma=\pi}$$

$$\boxed{(\partial_\tau + \partial_\sigma)\psi_1^I = 0}$$

$$\boxed{(\partial_\tau - \partial_\sigma)\psi_2^I = 0}$$

BC: $\psi_1^I(\tau, \sigma_*) \delta\psi_1^I(\tau, \sigma_*) - \psi_2^I(\tau, \sigma_*) \delta\psi_2^I(\tau, \sigma_*) = 0$

$$\boxed{\psi_1^I = \Psi_1^I(\tau - \sigma)}$$

$$\boxed{\psi_2^I = \Psi_2^I(\tau + \sigma)}$$

$$\boxed{\psi_1^I(\tau, \sigma_*) = \pm \psi_2^I(\tau, \sigma_*)}$$

Satisfies BC without too much violence.

$$\boxed{\delta\psi_1^I(\tau,\sigma_*) = \pm\delta\psi_2^I(\tau,\sigma_*)}$$

$\sigma = 0$:

$$\psi_1^I(\tau,0) = \pm\psi_2^I(\tau,0)$$

Choose sign to be positive since action doesn't care, but then can't change sign of field. We have two choices:

$$\psi_1^I(\tau,\pi) = +\psi_2^I(\tau,\pi)$$

or

$$\psi_1^I(\tau,\pi) = -\psi_2^I(\tau,\pi)$$

$$\Psi_I(\tau,\sigma) = \begin{cases} \psi_1^I(\tau,\sigma) & \sigma \in [0,\pi] \\ \psi_2^I(\tau,-\sigma) & \sigma \in [-\pi,0] \end{cases}$$

Continuous field over $[-\pi,\pi]$ since $\psi_1^I(\tau,0) = \psi_2^I(\tau,0)$

$$\Psi^I(\tau,\pi) = \psi_1^I(\tau,\pi) = \pm\psi_2^I(\tau,\pi) = \pm\Psi(\tau,-\pi)$$

Ψ fermion field is periodic if choose positive sign, antiperiodic if choose negative sign.

Suppose choose periodic (Rammond sector)

$$\Psi(\tau,\sigma) = \sum_{n\in Z} d_n^I e^{-in(\tau-\sigma)}$$

Suppose choose antiperiodic (Neveu-Schawrz sector)

$$\Psi^I(\tau,\sigma) = \sum_{r\in Z+\frac{1}{2}} b_r^I e^{-ir(\tau-\sigma)}$$