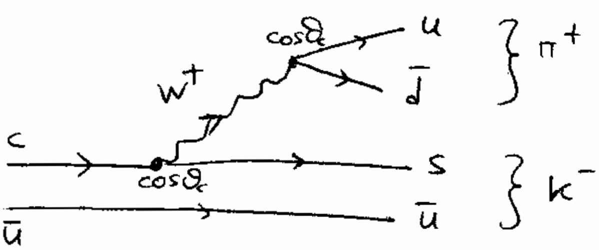
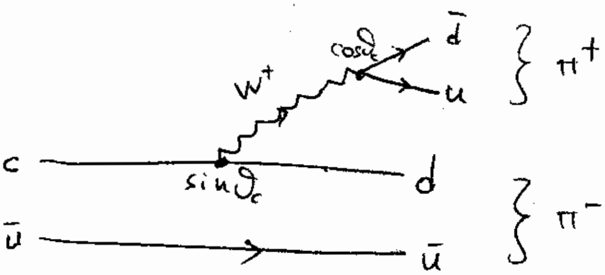
HW # 3
Solutions

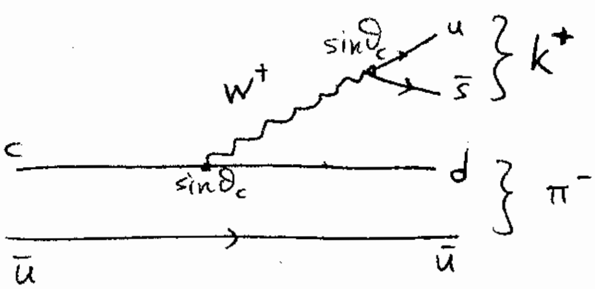
Problem 1 Q&L. 12.21



$$\mathcal{M} \sim \cos^2\vartheta_c$$



$$\mathcal{M} \sim \sin\vartheta_c \cdot \cos\vartheta_c$$



$$\mathcal{M} \sim \sin^2\vartheta_c$$

$$\frac{\Gamma(D^0\to\pi^+K^-)}{\Gamma(D^0\to\pi^+\pi^-)} = \frac{(\cos^2\vartheta_c)^2}{(\sin\vartheta_c\cdot\cos\vartheta_c)^2} \simeq 18.76$$

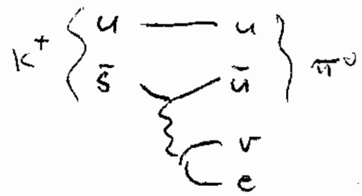
$$\frac{\Gamma(D^0\to\pi^+K^-)}{\Gamma(D^0\to\pi^-K^+)} = \left(\frac{\cos^2\vartheta_c}{\sin^2\vartheta_c}\right)^2 \simeq 352$$

$\Gamma(D^0\to\pi^+K^-) : \Gamma(D^0\to\pi^+\pi^-) : \Gamma(D^0\to\pi^-K^+) =$
$1 : 1/18.76 : 1/352$

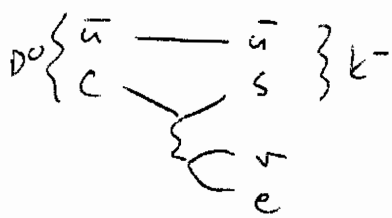
#2

given $\Gamma(K \to \pi^0 e^+ \nu) = 4 \cdot 10^6\ s^{-1}$
what is $\Gamma(D^0 \to K^- e^+ \nu)$?

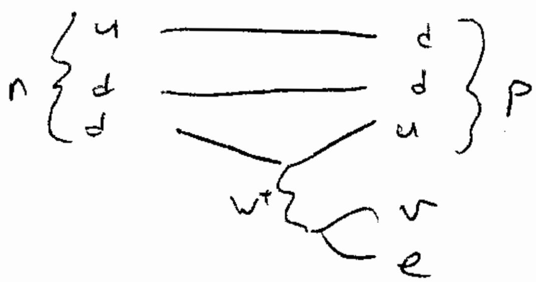
$K \to \pi^0 e^+ \nu$



$D^0 \to K^- e^+ \nu$



$n \to p e \nu$



under spectator quark model, all processes are the same to a factor of $\sin\theta_c$. Beta decay Γ goes as $(E_W)^5$

Real Rat
$D^0 \to K^- e^+ \nu$ $3.5\% \cdot 2.4 \cdot 10^{12} = 8.5 \cdot 10^{10}$
$K \to \pi^0 e^+ \nu$ $8 \cdot 10^7 \cdot 5\% = 4 \cdot 10^6$

ratio $= 2.0 \cdot 10^4$

so ratio
$$\frac{\Gamma(D^0 \to K^- e^+ \nu)}{\Gamma(K^+ \to \pi^0 e^+ \nu)} \sim \frac{(m_{D^0} - m_{K^-})^5}{(m_{K^+} - m_{\pi^0})^5} \frac{\cos^2\theta_c}{\sin^2\theta_c} = \left(\frac{1800-500}{(500-150)}\right)^5 \frac{1}{0.04}$$
$$\sim 1.8 \cdot 10^4$$

<u>If</u> this was one of the primary decay modes (it's not), then I would estimate the total Γ to be $\Gamma(D^0 \to K^- e^+ \nu) \times (1_{e^+\nu} + 1_{\mu\nu} + 3_{u\bar{d}}) = 10^5 \cdot 4 \cdot 10^6\ s^-$

giving $\tau \sim 2.5 \cdot 10^{-12}\ s$ $\quad * \dfrac{P.S.3\,(Ke\nu)}{P.S.2\,(K^-\pi^+)} \sim 4 \times 10^{-13}\ S$

wrong, answer is $4.1 \cdot 10^{-13}$ s
$K^- e \nu$ is only 3% total Γ

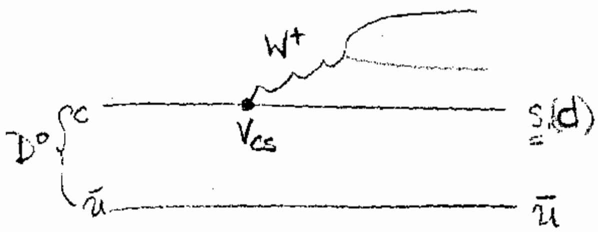
Good!

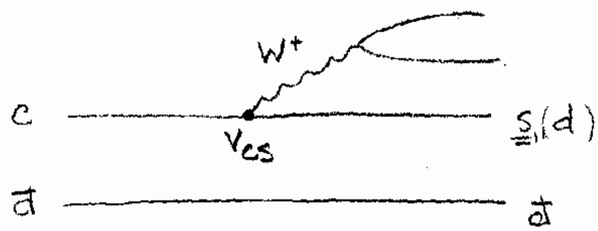
#3 12.23 Show, in the "spectator" quark model approach, that the charmed meson lifetimes satisfy:

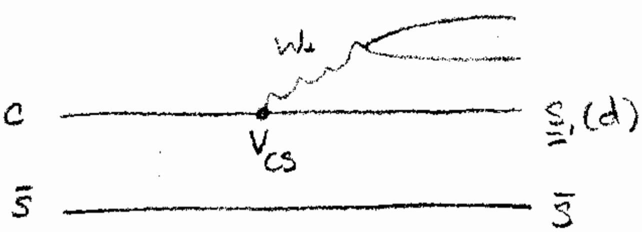
$$\tau(D^0) = \tau(D^+) = \tau(D_s^+)$$ ($= F^+$)

3

The quark compositions: $D^0 (c\bar{u})$, $D^+ (c\bar{d})$, $D_s^+ (c\bar{s})$







The dominant diagram in each of these decays is a $c \rightarrow s$ tree-level process. Thus, at this level, the decay widths are the same $\Rightarrow$ the lifetimes are equal.

#4

If $\mathcal{M}_{cp} = \mathcal{M}^+$ (hermitian), then weak interaction is CP invariant; if $\mathcal{M}_{cp} \neq \mathcal{M}^+$, then CP is violated.

$\mathcal{M}$ describes either $ab \to cd$ or $\bar{c}\bar{d} \to \bar{a}\bar{b}$

$$\mathcal{M} \sim J^{\mu}_{ca} J^{+}_{\mu bd}$$
$$\sim [\bar{u}_c \gamma^\mu (1-\gamma^5) U_{ca} u_a)(\bar{u}_b \gamma_\mu (1-\gamma^5) U_{bd} u_d)^+$$
$$\sim U_{ca} U^*_{db} (\bar{u}_c \gamma^\mu (1-\gamma^5) u_a)(\bar{u}_d \gamma_\mu (1-\gamma^5) u_b)$$

To get $\mathcal{M}_{cp}$, replace $u_i \to P(u_i)_c$, $i = a, \cdots, d$. then,

$$(J^\mu_{ca})_c = U_{ca} (\bar{u}_c)_c \gamma^\mu (1-\gamma^5)(u_a)_c$$
$$= -U_{ca} u_c^T C^{-1} \gamma^\mu (1-\gamma^5) C \bar{u}_a^T$$
$$= U_{ca} u_c^T [\gamma^\mu (1+\gamma^5)]^T \bar{u}_a^T$$
$$= (-) U_{ca} \bar{u}_a \gamma^\mu (1+\gamma^5) u_c$$

Parity operator is γ^0, so

$$P^{-1} \gamma^\mu (1+\gamma^5) P = \gamma^{\mu +} (1-\gamma^5)$$

$$\therefore (J^\mu_{ca})_{cp} = (-) U_{ca} \bar{u}_a \gamma^{\mu +} (1-\gamma^5) u_c$$

In the same way we get $J^+_{\mu bd}$, so we get:

$$\mathcal{M}_{cp} \sim U_{ca} U^*_{db} [\bar{u}_a \gamma^\mu (1-\gamma^5) u_c][\bar{u}_b \gamma_\mu (1-\gamma^5) u_d]$$

$\mathcal{M}'$ describes antiparticle process $\bar{a}\bar{b} \to \bar{c}\bar{d}$ (or $cd \to ab$),

$$\mathcal{M}' \sim (J^\mu_{ca})^+ J_{\mu bd}$$
$$\sim U^*_{ca} U_{db} [\bar{u}_a \gamma^\mu (1-\gamma^5) u_c][\bar{u}_b \gamma_\mu (1-\gamma^5) u_d]$$

$$\mathcal{M}' = \mathcal{M}^+$$

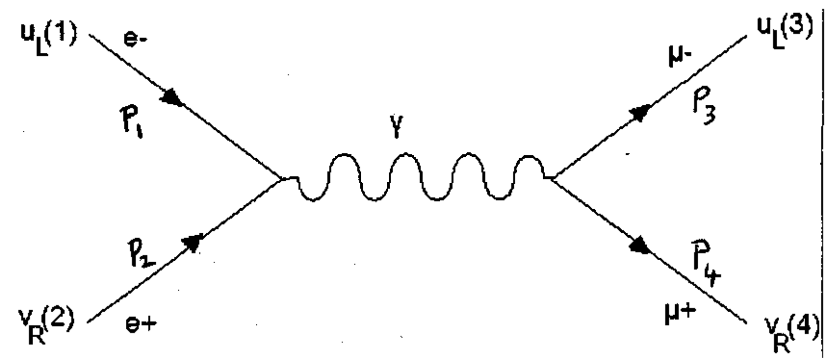
Compare $\mathcal{M}_{cp}$ and $\mathcal{M}^+$. if the mixing matris is all reall,

$$U_{ca} U^*_{db} = U^*_{ca} U_{db} \Rightarrow \mathcal{M}_{cp} = \mathcal{M}^+$$

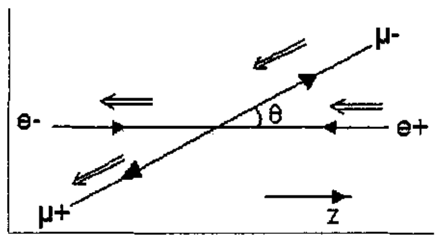
then is CP invariant, Or else it is ~~as~~ CP violated.

#5

Problem: e+ e- annihilation into muons
Georgios Choudalakis
(10/23/04)



I will calculate the matrix element $\mathfrak{M}$ for the following combination of helicities and angles.



For this $e^- e^+ \to \mu^+ \mu^-$ I find this matrix element:

$$-i\mathfrak{M} = \left[\overline{v}_{R(2)} ie\gamma^{\mu} u_{L(1)}\right]\left[-i\frac{g_{\mu\nu}}{q^2}\right]\left[\overline{u}^{\theta}_{L(3)} ie\gamma_{\nu} v^{\theta}_{R(4)}\right]$$

Where, for $m \to 0$

$$\frac{u_{L(1)}}{\sqrt{E}} = \frac{u^2_{(p_1)}}{\sqrt{E}} = \begin{pmatrix} \begin{pmatrix}0\\1\end{pmatrix} \\ \frac{\vec{\sigma}\cdot\vec{p}_1}{E_e + m_e}\begin{pmatrix}0\\1\end{pmatrix} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix}0\\1\end{pmatrix} \\ \vec{\sigma}\cdot\hat{p}_1\begin{pmatrix}0\\1\end{pmatrix} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix}0\\1\end{pmatrix} \\ \vec{\sigma}\cdot\hat{z}\begin{pmatrix}0\\1\end{pmatrix} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix}0\\1\end{pmatrix} \\ \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} \end{pmatrix} = \begin{pmatrix}0\\1\\0\\-1\end{pmatrix}$$

$$\frac{v_{R(2)}}{\sqrt{E}} = \frac{u^4_{(-2)}}{\sqrt{|E|}} = \begin{pmatrix} \vec{\sigma}\cdot\hat{p}_2\begin{pmatrix}0\\1\end{pmatrix} \\ \begin{pmatrix}0\\1\end{pmatrix} \end{pmatrix} = \begin{pmatrix} +\vec{\sigma}\cdot\hat{z}\begin{pmatrix}0\\1\end{pmatrix} \\ \begin{pmatrix}0\\1\end{pmatrix} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix}+1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} \\ \begin{pmatrix}0\\1\end{pmatrix} \end{pmatrix} = \begin{pmatrix}0\\-1\\0\\1\end{pmatrix}$$

For $\vec{p} = (p\sin\theta, 0, p\cos\theta)$ we have already calculated the helicity eigenspinors:

$$u^{s,\theta}_{(p)} = \begin{pmatrix} \cos\frac{\theta}{2} - i\sigma_2\sin\frac{\theta}{2} & 0 \\ 0 & \cos\frac{\theta}{2} - i\sigma_2\sin\frac{\theta}{2} \end{pmatrix} u^{s,0(:along\ z-axis)}_{(p)}$$

Which gives:

$$u^{\theta}_{\lambda=+}=\begin{pmatrix}\cos\frac{\theta}{2}\\ \sin\frac{\theta}{2}\\ \cos\frac{\theta}{2}\\ \sin\frac{\theta}{2}\end{pmatrix}\sqrt{E}\qquad u^{\theta}_{\lambda=-}=\begin{pmatrix}-\sin\frac{\theta}{2}\\ \cos\frac{\theta}{2}\\ \sin\frac{\theta}{2}\\ -\cos\frac{\theta}{2}\end{pmatrix}\sqrt{E}$$

So,

For the assumed left-helicity μ^- we need to rotate by θ a spinor which is identical to $u_{L(1)}$, with the difference that now it is $u^{along\ z}_{L(3)}$, but after ignoring the mass of the muon as I did for the electron, the momentum amplitude cancels out. So,

$$\frac{u^{\theta}_{L(3)}}{\sqrt{E}}=\begin{pmatrix}\cos\frac{\theta}{2} & -\sin\frac{\theta}{2} & 0 & 0\\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} & 0 & 0\\ 0 & 0 & \cos\frac{\theta}{2} & -\sin\frac{\theta}{2}\\ 0 & 0 & \sin\frac{\theta}{2} & \cos\frac{\theta}{2}\end{pmatrix}\begin{pmatrix}0\\1\\0\\-1\end{pmatrix}=\begin{pmatrix}-\sin\frac{\theta}{2}\\ \cos\frac{\theta}{2}\\ \sin\frac{\theta}{2}\\ -\cos\frac{\theta}{2}\end{pmatrix}$$ for μ^-_L at θ

Similarly, for the assumed right-helicity μ^+, we need to rotate by θ a spinor $v^{along\ negative\ z-axis}_{R(4)}$. As before, this spinor that we must rotate by θ is identical to $v_{R(2)}$, because of the approximation of zero masses. i.e. momentum amplitudes cancel and only their directions count. So,

$$\frac{v^{\theta}_{R(4)}}{\sqrt{E}}=\frac{u^{4,(\theta)}_{(-4)}}{\sqrt{|E|}}=\begin{pmatrix}\cos\frac{\theta}{2} & -\sin\frac{\theta}{2} & 0 & 0\\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} & 0 & 0\\ 0 & 0 & \cos\frac{\theta}{2} & -\sin\frac{\theta}{2}\\ 0 & 0 & \sin\frac{\theta}{2} & \cos\frac{\theta}{2}\end{pmatrix}\begin{pmatrix}\frac{\vec{\sigma}.\left(|\vec{p}_4|\hat{z}\right)}{E_4+m_4}\begin{pmatrix}0\\1\end{pmatrix}\\ \begin{pmatrix}0\\1\end{pmatrix}\end{pmatrix}$$

$$\simeq\begin{pmatrix}\cos\frac{\theta}{2} & -\sin\frac{\theta}{2} & 0 & 0\\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} & 0 & 0\\ 0 & 0 & \cos\frac{\theta}{2} & -\sin\frac{\theta}{2}\\ 0 & 0 & \sin\frac{\theta}{2} & \cos\frac{\theta}{2}\end{pmatrix}\begin{pmatrix}\begin{pmatrix}+1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}\\ \begin{pmatrix}0\\1\end{pmatrix}\end{pmatrix}$$

$$=\begin{pmatrix}\cos\frac{\theta}{2} & -\sin\frac{\theta}{2} & 0 & 0\\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} & 0 & 0\\ 0 & 0 & \cos\frac{\theta}{2} & -\sin\frac{\theta}{2}\\ 0 & 0 & \sin\frac{\theta}{2} & \cos\frac{\theta}{2}\end{pmatrix}\begin{pmatrix}0\\-1\\0\\1\end{pmatrix}=\begin{pmatrix}+\sin\frac{\theta}{2}\\ -\cos\frac{\theta}{2}\\ -\sin\frac{\theta}{2}\\ \cos\frac{\theta}{2}\end{pmatrix}$$ for μ^+_R at $\pi-\theta$

So, substituting those spinors to the first equation we get:

$$\mathfrak{M} = -\frac{e^2}{q^2}\left\{\overbrace{(0,1,0,-1)}^{\gamma^0}{}_{\wedge}\gamma^\mu \begin{pmatrix}0\\1\\0\\-1\end{pmatrix}\right\}\left\{\overbrace{\left(-\sin\frac{\theta}{2}, \cos\frac{\theta}{2}, \overset{-\sin\frac{\theta}{2}}{\downarrow}\sin\frac{\theta}{2}, +\cos\frac{\theta}{2}\right)}^{\gamma^0}{}_{\wedge}\gamma_\mu \begin{pmatrix}-\sin\frac{\theta}{2}\\ \cos\frac{\theta}{2}\\ -\sin\frac{\theta}{2}\\ \cos\frac{\theta}{2}\end{pmatrix}\right\} \quad E^2$$

Using the convention

$$\gamma^0 = \begin{pmatrix}\mathrm{I} & 0\\ 0 & -\mathrm{I}\end{pmatrix}, \vec{\gamma} = \begin{pmatrix}0 & \vec{\sigma}\\ -\vec{\sigma} & 0\end{pmatrix}$$

we get the result:

$$\mathfrak{M} = -\frac{e^2}{q^2}4(1+\cos\theta) \ E^2 \qquad\qquad q^2 = s = 4E^2$$

$$\Rightarrow |\mathfrak{M}|^2 \propto \frac{e^2}{4}(1+\cos\theta)^2$$

where, I've made use of the fact that $q^\mu = p^\mu_{e^+} + p^\mu_{e^-} - = \left(2E_e, \vec{0}\right) \Rightarrow q^2 = (2E_e)^2 = s$

The angular dependence of this result is in agreement with the angular momentum conservation.

#6.

✓ 6

Show that the same angular dependence of #5 can be obtained using the rotational operator defined in problem 2.6 of Q&L.

The rotational operator for spin 1 is given as the matrix R in eq (*) below. If we think about the physical situation ...

incoming: e^+ (RH) + e^- (LH) $\Rightarrow |\tfrac{1}{2},-\tfrac{1}{2}\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle = |1,-1\rangle$

choose the spin-1 direction as our 1-axis, thus the γ^* can be written as the vector:

$$\gamma^* : \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

outgoing: μ^+ (RH) + μ^- (LH) $\Rightarrow |\tfrac{1}{2},-\tfrac{1}{2}\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle = |1,-1\rangle$ (a)

LH RH $\Rightarrow |\tfrac{1}{2},\tfrac{1}{2}\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle = |1,1\rangle$ (b)

but these guys are at an angle to the incoming e^+e^- axis... thus their vectors:

$$\text{(a)} \quad \overbrace{\begin{pmatrix} \frac{1}{2}(1+\cos\theta) & -\frac{1}{\sqrt{2}}\sin\theta & \frac{1}{2}(1-\cos\theta) \\ \frac{1}{\sqrt{2}}\sin\theta & \cos\theta & -\frac{1}{\sqrt{2}}\sin\theta \\ \frac{1}{2}(1-\cos\theta) & \frac{1}{\sqrt{2}}\sin\theta & \frac{1}{2}(1+\cos\theta) \end{pmatrix}}^{R} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(1+\cos\theta) \\ \frac{1}{\sqrt{2}}\sin\theta \\ \frac{1}{2}(1-\cos\theta) \end{pmatrix} \quad (*)$$

for (b), this is just $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

multiply (inner product) w/ γ^*:

$$M_a \propto (1\ 0\ 0)(a) = \tfrac{1}{2}(1+\cos\theta)$$

$$M_b \propto (1\ 0\ 0)(b) = \tfrac{1}{2}(1-\cos\theta)$$

there's no interference, so add cross-sections

yay!! :)

$$|M|^2 = |M_a|^2 + |M_b|^2 \propto \tfrac{1}{4}\left[(1+\cos\theta)^2 + (1-\cos\theta)^2\right] = \tfrac{1}{4}(1+\cos^2\theta)$$