Comments by Prof. Chen after the paper was turned in:

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Tom –

As discussed earlier, you need to provide
1. the explicit formulas for $A_{FB} \equiv \frac{F-B}{F+B}$, A_LR, etc, in terms of masses, widths (with and without energy dependence due to higher order electro-weak corrections, as derived in class) of the Z's at some energies s, and show the differences of these formulas (which sort of indicate the theoretical uncertainties of the B-W form when the interaction energy is far away from the resonances!).
2. measurement and systematical errors on A_FB, etc.
in order to see if you can discover a high mass Z' at low energies.
3. use the s dependence of A_FB, A_LR, etc. to solve your problem of
that "Unfortunately, this can only measure a ratio of coupling to mass until the Z' mass is reached."

- Min

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Hello, Prof. Chen,
I have attached a response to your comments.
Thanks
Tom

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The three relevant Feynman diagrams for $e^+e^- \to \mu^+\mu^-$ are annihilation through the photon, Z, and Z'. Their (simplified) matrix elements are:

$$M_\gamma = -\frac{e^2}{s}(\bar{\mu}\gamma^\nu\mu)(\bar{e}\gamma_\nu e) \tag{1.1}$$

$$M_Z = \frac{-\sqrt{2}GM_Z^2}{s - M_Z^2 + \frac{is\Gamma_Z}{M_Z}}[c_R^\mu\left(\bar{\mu}_R\gamma^\nu\mu_R\right) + c_L^\mu\left(\bar{\mu}_L\gamma^\nu\mu_L\right)][c_R^e\left(\bar{e}_R\gamma^\nu e_R\right) + c_L^e\left(\bar{e}_L\gamma^\nu e_L\right)] \tag{1.2}$$

$$M_{Z'} = \frac{-\sqrt{2}GM_{Z'}^2}{s - M_{Z'}^2 + \frac{is\Gamma_{Z'}}{M_{Z'}}}[c'^\mu_R(\bar{\mu}_R\gamma^\nu\mu_R) + c'^\mu_L(\bar{\mu}_L\gamma^\nu\mu_L)][c'^e_R(\bar{e}_R\gamma^\nu e_R) + c'^e_L(\bar{e}_L\gamma^\nu e_L)] \tag{1.3}$$

where $c_R = c_V - c_A$ and $c_L = c_V + c_A$. With this form, it is easy to evaluate cross-sections for interactions with known chiralities. Thus you get four cross-sections of the form:

$$\frac{d\sigma}{d\Omega}(e_L^- e_R^+ \to \mu_L^- \mu_R^+) = \frac{\alpha^2}{4s}(1+\cos\theta)^2\left|1 + rc_L^\mu c_L^e + r_2 c'^\mu_L c'^e_L\right|^2 \tag{1.4}$$

where

$$r = \frac{\sqrt{2} G M_Z^2}{s - M_Z^2 + \frac{is\Gamma_Z}{M_Z}} \left(\frac{s}{e^2}\right) \tag{1.5}$$

and

$$r_2 = \frac{\sqrt{2} G' M_{Z'}^2}{s - M_{Z'}^2 + \frac{is\Gamma_{Z'}}{M_{Z'}}} \left(\frac{s}{e^2}\right) \tag{1.6}$$

To measure the Forward Backward Asymmetry, you need to use unpolarized beams, and do not measure the polarizations of the resulting muons. Thus you need to average those four cross-sections to get the actual cross-section:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}[A_0(1+\cos^2\theta) + A_1 \cos\theta] \tag{1.7}$$

In general, A_0 and A_1 are complicated factors involving all of the cross-terms from 1.4. However, if $c_i^\mu = c_i^e = c'^\mu_i = c'^e_i = c_i$ these terms become

$$A_0 = 1 + 2c_V^2[\mathrm{Re}(r) + \mathrm{Re}(r_2)] + (c_V^2 + c_A^2)^2[|r|^2 + |r_2|^2 + 2\,\mathrm{Re}(r_2 r)] \tag{1.8}$$

$$A_1 = 4c_A^2[\mathrm{Re}(r) + \mathrm{Re}(r_2)] + 8c_V^2 c_A^2[|r|^2 + |r_2|^2 + 2\,\mathrm{Re}(r_2 r)] \tag{1.9}$$

With F = the rate of μ^+ going in the forward direction ($0<\theta<\pi/2$)
and B = the rate of μ^+ going in the backward direction ($\pi/2<\theta<\pi$),

$$A_{FB} \equiv \frac{F - B}{F + B} \tag{1.10}$$

A simple integral shows that

$$A_{FB} = \frac{3A_1}{8A_0} \tag{1.11}$$

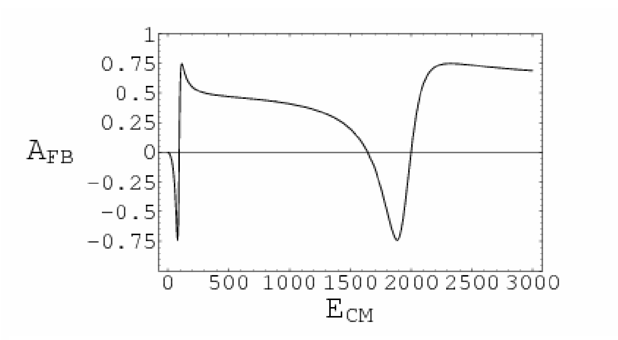

Fig. 1: A_{FB} plotted with $M_{Z'}$ = 2TeV, $\Gamma_{Z'}$ = 20 GeV, and G' = G_W / 1000

Fig. 1 shows this function plotted with some possible Z' parameters. The main features of the plot are the two interference patterns at the Z and Z' masses. In the region where $M_Z \ll E_{CM} \ll M_{Z'}$ you find that $r_2 \approx -\frac{\sqrt{2} G' s}{e^2}$ and A_{FB} can be Taylor expanded as

$$\frac{1}{2} + 2c_A^2 \operatorname{Re}(r_2) = \frac{1}{2} - 2\sqrt{2}c_A^2 \left(\frac{G'}{e^2}\right)s \tag{1.12}$$

Thus in this region, it is possible to find the strength of the interaction without knowing the mass of the Z'.

Fig.2 shows a sample data set in this region with some estimated error bars. The errors in the y-direction come from uncertainties in measuring the flux in the forward and backward directions. The main sources of this error would be uncertainties in acceptance, uncertainties in reconstructing the positive versus negative muons, and uncertainties in momentum reconstruction near $\theta = \pi/2$. Uncertainties in acceptance can be minimized by having a rotating detector, allowing that error to be calibrated to 1-2%. The other errors can be minimized by having good position resolution in the tracker. These errors should

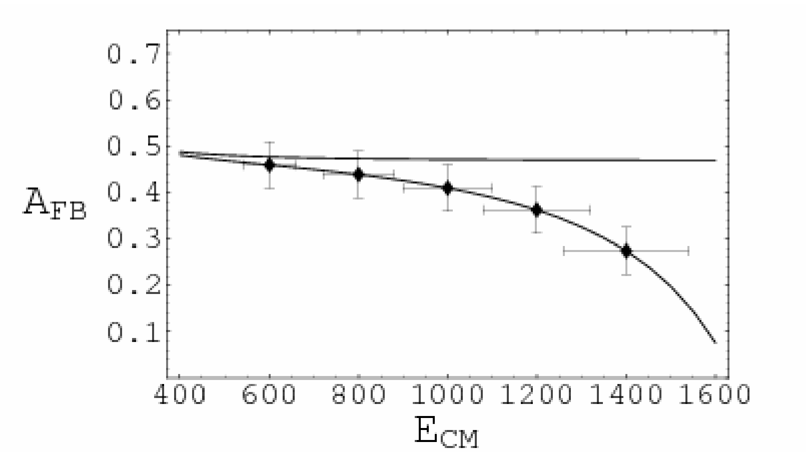

Fig. 2: A sample data set for discovery of the Z'.
The flat line is the Standard Model prediction, while the curved line is the prediction from fig.1. The error bars are a worst-case scenario, and should be twice as small in a well-run experiment.

also be at 1-2%. Fig. 2 shows 5% error bars in the y-direction, and the signal is still clearly visible.

The errors in the x-direction are due to uncertainties in the beam energies. Beam-beam interactions give a spread in the beam energy of between 1% and 10%. Reconstructing the center-of-mass energy of each collision could allow that spread to be taken into account in a maximum-likelihood fit. However, there are errors associated with the momentum reconstruction. At high energies, these errors should be on the order of 1-2%. In fig. 2, I plotted the worst-case scenario of a 10% uncertainty in momentum. Even with that worst-case scenario, the signal is still visible.

Some sources of error, such as Poisson statistics and cosmic-ray muons have not been included here because they should be small. The $e^+e^- \rightarrow \mu^+\mu^-$ reaction has a very large cross-section even without a Z', so gathering enough statistics should not be a problem. For that same reason, the flux of cosmic-ray muons should not be large enough to cause problems. Even if there were many cosmic-ray muons, their only effect would be to shrink A_{FB} by a constant factor, so the decrease as a function of energy would still be evident.

Thus even below the Z' mass, it is possible to show the existence of a Z'. In a worst-case scenario, the signal looks good, and is probably at least a 2σ deviation from the standard model. With more work to reduce the errors and a better analysis method, this experiment should be able to show the existence of a Z' at the 5σ level required for discovery.