8.821 String Theory
Fall 2008

8.821 F2008 Lecture 20: The Wider World of Gauge/Gravity Duality

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1 Introduction

Today we're going to talk more about other examples of gauge gravity duality besides the $\mathcal{N} = 4$ SYM theory. The first half of the discussion will be very survey like in which we attempt to get some feel for the possibilities beyond the $\mathcal{N} = 4$ theory. In the second half we will focus on a particularly simple extension of what we know so far that will lead to a breaking of the conformal symmetry, confinement, and a mass gap.

Once we finish up our discussion of confinement we'll be moving on to black hole mechanics: thermodynamics and hydrodynamics.

2 The Wide World In Brief

2.1 "Non-Spherical Horizons"

In the $\mathcal{N} = 4$ case the gravity dual had the form $AdS_5 \times S^5$. A natural generalization would be to consider spaces of the form $AdS_5 \times X$ where X is some compact space. The AdS isometry group was responsible for the conformal invariance in the dual field theory while the isometry group of S^5 related to the R-symmetry in the $\mathcal{N} = 4$ theory. We can consider "non-spherical horizons" by replacing the sphere S^5 with a non-spherical manifold X. This should correspond to a change in the global symmetries of the dual field theory. Such non-spherical X's arise as the locus of points equidistant from a singularity (such as orbifolds) and the dual field theory can be obtained by studying $D3$-branes probing these singularities. The coupling can even be made to run (changing the AdS part of the gravity dual) with the addition of fluxes and fractional branes (but don't ask what this means).

2.2 Dp-branes, $p \neq 3$

We've focused a lot on $D3$-branes so far, but there are other branes in string theory and they have interesting world volume theories as well. Based on our experience with the $D3$-brane, we might not be too surprised to learn that the world volume theory of a Dp-brane is the dimensional reduction of ten dimensional $\mathcal{N} = 1$ SYM on a $10-(p+1)$ torus with periodic boundary conditions. The result is a $p+1$ dimensional Yang-Mills theory with 16 supercharges. Now there is a very important difference between the $3+1$ dimensional Yang-Mills theory and Yang-Mills theory in any other dimension. To see this difference remember that the Yang-Mills action contains a term like $\int d^{p+1}x\, 1/g^2\, \mathrm{Tr}\left(F^2\right)$ (gauge kinetic term). This term allows us to figure out the mass dimension of g^2 since the dimension of A is always 1 and the dimension of F^2 is therefore 4 regardless of space-time dimension. Requiring the action to be dimensionless means that g^2 has mass dimension $3-p$ so that g^2 is dimensionful whenever $p \neq 3$.

In terms of a running effective coupling $g^2_{eff}(E) = g^2/E^{3-p}$ we find that gauge theory perturbation theory is good when $g^2_{eff} \ll 1$. The behavior of g^2_{eff} depends strongly on dimension. When $p < 3$ then g^2_{eff} is big in the IR, and when $p > 3$ we find the opposite situation where g^2_{eff} is large in the UV.

In the large N limit there exists a type II SUGRA solution with near horizon limit given by

$$ds^2/\alpha' = \sqrt{\frac{g_s N}{u^{7-p}}}du^2 + \sqrt{\frac{u^{7-p}}{g_s N}}dx^\mu dx_\mu + \sqrt{g_s N u^{p-3}}d\Omega^2_{8-p} \tag{1}$$

where $d\Omega^2_{8-p}$ is the metric on a unit S^{8-p} and μ runs from 0 to p for the $p+1$ directions on the brane world volume. When $p = 3$ this metric is just that of our old friend $AdS_5 \times S^5$, but in general the ten dimensional manifold doesn't have such a simple product structure. In addition to the metric we must also specify a flux condition

$$\int_{S^{8-p}} F_{8-p} = 2\pi N \tag{2}$$

where F_{8-p} is the field strength of some appropriate RR form. Also, the dilaton has a non-trivial profile given by

$$e^\Phi = g_s \left(\frac{g_s N}{u^{7-p}}\right)^{\frac{3-p}{4}} . \tag{3}$$

The fact that the dilaton depends on u is essentially the statement that the Yang-Mills coupling runs when $p \neq 3$.

It's important to note that both the dilaton and the curvature blow up for some values u when $p \neq 3$. When the dilaton grows large we can no longer trust perturbative string theory and when the curvature grows large corrections to supergravity become important. Conservation of evil demands that no two different descriptions of the system should be valid simultaneously, and indeed it was observed in hep-th/9802042 that the blow up occurs precisely where another description becomes valid/useful.

As an example of this important point let's consider the case of $D2$-branes.

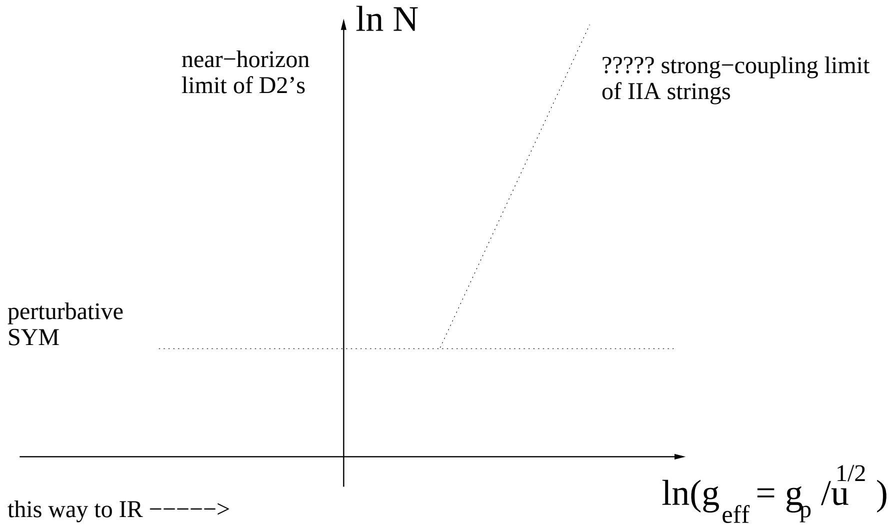

Figure 1: Regimes of validity of various descriptions of N D2-branes, from [1].

2.3 M-theory

M-theory also provides further examples of gauge gravity duality. Let's recall the basics of M-theory very briefly. The theory has a vacuum with eleven non-compact dimensions forming $\mathbb{R}^{10,1}$ and no coupling. The low energy limit of the theory is eleven dimensional supergravity with 32 supercharges which is the mother of all supergravity theories. The theory also has a massless 3-form potential with field strength $G_4 = dC_3$. This field is coupled minimally to branes in the theory called $M2$-branes. The dual of the 4-form field strength is a 7-form field strength $G_7 = dC_6$ which strongly suggests that the theory also has $M5$-branes that source G_7 electrically and hence G_4 magnetically.

What does M-theory have to do with string theory? Compactifying the theory on a circle of radius R_{10} gives IIA string theory with string coupling $g_s \sim (R_{10}M_P^{11})^{3/2}$. This claim can be fleshed out by identifying the IIA fields in terms of M-theory objects. The Kaluza-Klein gauge boson $G_{11\,\mu}$ becomes the RR 1-form. The $D0$-branes that source the RR 1-form arise as Kaluza-Klein excitations. They have a mass given by $m \sim n/R_{10} \sim n/(g_s\sqrt{\alpha'}$ which agrees with the mass of KK excitations and the result from IIA which is protected by supersymmetry. $M2$-branes perpendicular to the circle become $D2$-branes in IIA, and thus the M-theory 3-form C_3 becomes the IIA RR 3-form when it is perpendicular to the circle. An $M2$-brane wrapped on the circle becomes a fundamental string, and so the M-theory 3-form when pointing along the circle becomes the NS-NS 2-form B_2 as $C_3 = B_2 \wedge dx^{10}$.

M-theory is useful here in part because it provides the answer to the question, what happens to the $D2$-branes at strong coupling? The answer according to M-theory is that we should think about $M2$-branes on a large circle instead of $D2$-branes when the gauge theory effective coupling is large. The world volume theory on the $M2$-branes should flow in the IR to some superconformal field theory that fills in the blank in our diagram above.

There is one limit in particular in which we know how to make much more sense of this statement. In the limit of a large number N of $M2$-branes we expect a supergravity solution (in eleven dimensions) that takes back reaction of the branes into account. Taking the near horizon limit of this solution we find the space $AdS_4 \times S^7$ with $\int_{S^7} G_7 = 2\pi N$ which describes the IR limit of many $M2$-branes in the gravity dual language. Without the large N limit the IR limit of the $M2$-brane theory is a superconformal field theory with $\mathcal{N} = 8$ and an $SO(8)_R$ R symmetry. This theory is just the IR limit of three dimensional super Yang-Mills (which we remember can be obtained by dimensionally reducing $\mathcal{N} = 4$ SYM on a circle). This theory has 7 scalars X^i representing transverse fluctuations of the $M2$-branes leading to a visible $SO(7)$ symmetry. To make the $SO(8)$ symmetry manifest we must somehow find another coordinate. The belief is that this coordinate is related to the three dimensional dual of a $U(1)$ gauge field $da = \star_3 d\phi$ with ϕ essentially the eighth coordinate on the circle.

Recently, there has been progress in formulating a Lagrangian description of the theory of $M2$-branes. The theory has a discrete parameter related to a Chern-Simons coupling, and indeed the theory looks like a Chern-Simons theory with special matter content.

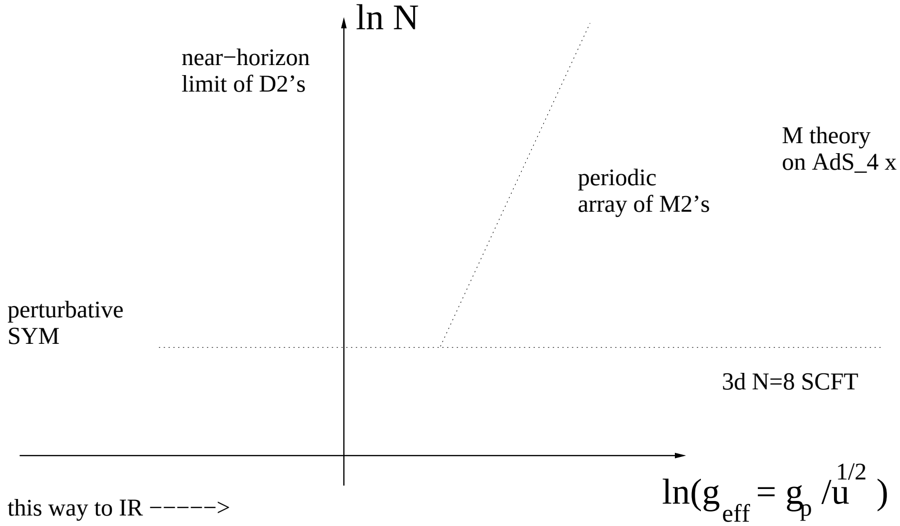

Figure 2: Regimes of validity of various descriptions of N D2-branes, after incorporating the fact that the strong-coupling limit of type IIA is M-theory.

A similar story applies for $M5$-branes, where at low energy the theory becomes some superconformal field theory in six dimensions. Very little is known about this theory. For a large number of $M5$-branes some information about the theory can be obtained from black holes. The free energy goes as $F_{M5} \sim T^6 V N_{M5}^3$ suggesting that the theory is not best formulated in terms of the matrices of SYM! It is tempting to mumble words about fields which are $N \times N \times N$ objects, but not much progress has been made with that idea.

3 Confinement

To motivate our picture of confinement let's remember that the radial coordinate behaves like a spectrograph separating the theory into energy scales. In the case of AdS we know that $z = 0$ is like the UV while $z \to \infty$ is like the IR. The fact that AdS goes forever in the z direction seems to be related to the fact that the dual theory is a conformal field theory with degrees of freedom at all energies. We can make this more precise using the "warp factor" $W(z)$ which is defined to be W in

$$ds^2 = \frac{dz^2}{z^2} + W^2(z) dx_\mu dx^\mu.$$

The warp factor for AdS is simply $1/z$, and we can interpret the existence of modes at arbitrarily low energy as the statement that the warp factor has a zero at $z = \infty$. More precisely, there are $\mathcal{O}(N^2)$ degrees of freedom at every energy scale in the CFT.

What would we expect for a theory exhibiting confinement? We might expect the theory to have a mass gap as in pure Yang-Mills theory. On the other hand, there may be Goldstone bosons from symmetry breaking caused by the strong dynamics. Such "pions" would represent low energy degrees of freedom, but the important point is that we wouldn't expect the number of pion modes to scale with N (they are color neutral). Thus a confining large-N gauge theory might reasonably have a mass gap except for a few low energy modes. The absence of modes at low energy should therefore be reflected in the bulk geometry and the warp factor. In particular, we would like to interpret a minimum of the warp factor as a signal of a mass gap in the dual field theory.

The simplest realization of this idea [2] is the $\mathcal{N} = 4$ theory in four dimensions with one direction compactified into a circle. Call the coordinate along the circle y and identify $y \sim y + 2\pi R_y$. We give the fermions anti-periodic boundary conditions around the circle so that $\Psi(y + 2\pi R_y) = -\Psi(y)$. To the bosons and gauge fields we give periodic boundary conditions. Such boundary conditions are called "Scherk-Schwarz" or "thermal" boundary conditions. Indeed, if the direction we compactify is Euclidean time then we are considering precisely the $\mathcal{N} = 4$ theory at finite temperature. Alternatively, if we compactify some other spatial direction then we get Yang-Mills theory in one lower dimension (for energies lower than $1/R_y$)

The boundary conditions treat bosons and fermions differently so supersymmetry is broken. The fermionic modes begin at energies of order $1/R_y$ because they are required to have non-zero angular momentum around the circle. The bosons also get a mass from fermions running in loops because the protection from supersymmetry is lost at low energy. All that remains therefore are the gauge fields at energies small compared to $1/R$ and we might expect that we have essentially Yang-Mills theory in three dimensions at low energy. However, we should be a little careful since there are factors of λ floating around and because the energy scale of the bosonic and fermionic modes isn't separated from the scale at which the theory looks four dimensional again.

References

[1] N. Itzhaki, J. M. Maldacena, J. Sonnenschein and S. Yankielowicz, "Supergravity and the large N limit of theories with sixteen supercharges," Phys. Rev. D **58**, 046004 (1998) [arXiv:hep-

th/9802042].

[2] E. Witten, "Anti-de Sitter space, thermal phase transition, and confinement in gauge theories," Adv. Theor. Math. Phys. **2**, 505 (1998) [arXiv:hep-th/9803131].