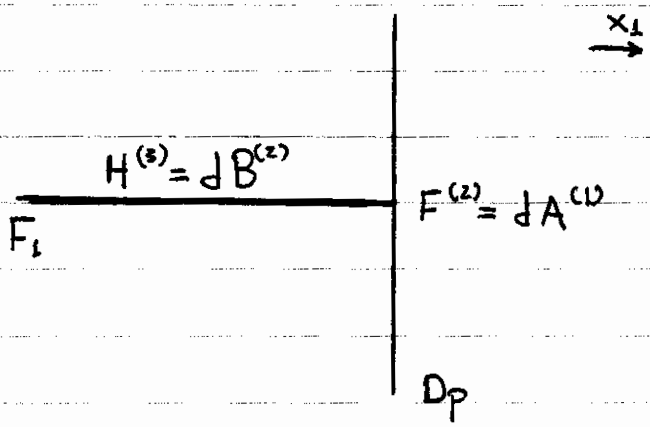
8.871

Solutions to problem set # 1

1.

(a)



e.o.m : $$d *_{(10)} H^{(3)} = Q\,\delta^{(8)}\,\theta(x_1) - \delta^{(9-p)} *_{(p+1)} F^{(2)}, \quad 1 \leq p \leq 9$$

Taking the derivative: $$d *_{(p+1)} F^{(2)} = Q\,\delta^{(p)}$$

so the endpoint of the string couples electrically to the one-form $A^{(1)}$ in the worldvolume of the Dp brane.

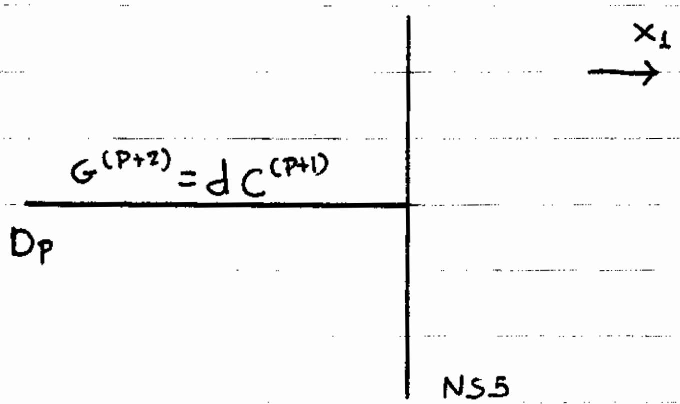
The corresponding term in the action is:

$$\int_{W^{(p+1)}} B^{(2)} \wedge *_{(p+1)} F^{(2)}$$

where $W^{(p+1)}$ is the worldvolume of the Dp-brane.

Note that for $p = 0, -1$ these equations don't make sense. A string cannot end on a D0 or D(-1) brane (it has to pass through)

(b)



Type IIA

$$p=2: \quad d *_{(10)} G^{(4)} = Q\,\delta^{(7)}\,\theta(x_1) - \delta^{(4)} *_{(6)} F^{(3)} \qquad (F^{(3)} = dA^{(2)})$$

action term: $\int_{W6} C^{(3)} \wedge *_6 F^{(3)}$

The 'endpoint' of the D2 is a string that couples electrically to $A^{(2)}$

$$p=4: \quad d *_{(10)} G^{(6)} = Q\,\delta^{(5)}\,\theta(x_1) - \delta^{(4)} F^{(1)}$$

Applying "d" we get $dF^{(1)} = Q\,\delta^{(2)}$, so the endpoint of the D4 is a vortex that couples magnetically to a zero-form in the worldvolume of the NS5 brane. The relevant action term is: $\int_{W6} C^{(5)} \wedge F^{(1)}$

p=6: The D6 brane has to pass through the NS5. Charge conservation does not allow it to end.

Type IIB

$$p=1 : \quad d *_{(10)} G^{(3)} = Q\,\delta^{(8)}\,\theta(x_1) - \delta^{(4)} *_{(6)} F^{(2)} \qquad (F^{(2)} = dA^{(1)})$$

action term: $\displaystyle\int_{W(6)} C^{(2)} \wedge *_{(6)} F^{(2)}$

$$p=3 : \quad d *_{(10)} G^{(5)} = Q\,\delta^{(6)}\,\theta(x_1) - \delta^{(4)} F^{(2)}$$

$$\int_{W6} C^{(4)} \wedge F^{(2)}$$

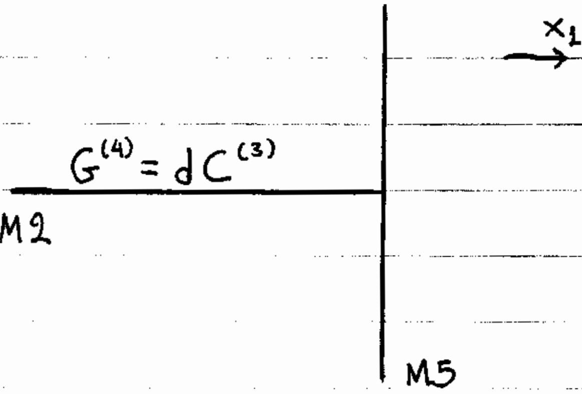
In the worldvolume of the NS5 brane this is the 'magnetic dual' of the previous case, the endpoint of the D3 coupling magnetically to the same 1-form in W^6

$$p=5 : \quad d *_{(10)} G^{(7)} = Q\,\delta^{(4)}\,\theta(x_1) - \delta^{(4)} F^{(0)}$$

$$\int_{W6} C^{(6)} \wedge F^{(0)}$$

The endpoint of the D5 is a domain wall in the worldvolume of the NS5 brane.

(c)



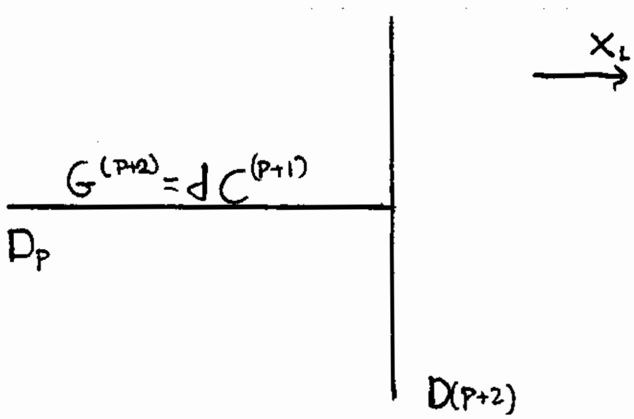
Note: M2, M5 live in 11d

$$d *_{(11)} G^{(4)} = Q\, \delta^{(8)}\, \theta(x_1) - \delta^{(5)} *_{(6)} F^{(3)}$$

action term: $\displaystyle\int_{W^6} C^{(3)} \wedge *_{(6)} F^{(3)}$

The field strength $F^{(3)}$ is middle-dimensional in the world-volume of the M5 and can be taken to be self-dual (or anti-self-dual).

(d)



$$d *_{(10)} G^{(p+2)} = Q\,\delta^{(9-p)}\, \theta(x_1) - \delta^{(7-p)} F^{(2)}$$

action term: $\displaystyle\int_{W(p+3)} C^{(p+1)} \wedge F^{(2)}$

The endpoint of the Dp brane couples magnetically to the one-form living in the worldvolume of the D(p+2) brane. This is the same one-form that the fundamental string ending on the D(p+2) couples to electrically.

2.

11d supergravity — little group: $SO(9)$

Bosonic

Metric $g_{\mu\nu}$ (symmetric, traceless) 44

Antisymmetric three-form $C^{(3)}$ 84

Fermionic

Vector-spinor gravitino Ψ_μ 128

$44+84=128$

Type II A — little group: $SO(8)$

Bosonic

Scalar ϕ 1

Metric $g_{\mu\nu}$ 35

One-form $C^{(1)}$ 8_v

Two-form $B^{(2)}$ 28

Three-form $C^{(3)}$ 56_t

Fermionic

Two gravitini of opposite chirality : $\Psi_{\mu\pm}$ $56_c + 56_s$ (non-chiral)

Two spinors " " " : $\lambda_{\pm}$ $8_c + 8_s$

$$1+8+35+28+56 = 128 = 2\times 56 + 2\times 8$$

Type IIB

little group : SO(8)

Bosonic

Two scalars : $\phi, C^{(0)}$ 2×1

Two two-forms : $B^{(2)}, C^{(2)}$ 2×28

Metric : $g_{\mu\nu}$ 35

Self-dual four-form : $C^{(4)}_{+}$ 35_{+}

Fermionic

Two gravitini of the same chirality : $56_s + 56_s$ (chiral)

Two conjugate spinors " " " " : $8_c + 8_c$

$$2\times 1 + 2\times 28 + 2\times 35 = 128 = 2\times 56 + 2\times 8$$

Type I

little group: SO(8)

gauge group: SO(32)

Bosonic

Scalar : ϕ 1

Metric : $g_{\mu\nu}$ 35

Two-form : $C^{(2)}$ 28

Vector : A_μ $(8_v, \text{adj})$ (adjoint of SO(32))

Fermionic

Gravitino : 56_s

Spinor : 8_c

Gaugino : $(8_s, \text{adj})$

$$1 + 28 + 35 + 8 \times 496 = 8 + 56 + 8 \times 496$$

Heterotic

little group : $SO(8)$

gauge group : $SO(32)$ or $E_8 \times E_8$

$SO(32)$ Heterotic string theory has the same massless spectrum as Type I. For $E_8 \times E_8$ the only thing different is the adjoint representation which is $(248, 1) \oplus (1, 248)$. The $SO(8)$ quantum numbers are the same. Note that Type I and Heterotic $SO(32)$ have different massive spectra.