8.871

Solutions to problem set #4

1. a) Using the results of pset #3 we have:

	d	11	IIA	IIB	9	8	7	6	5	4	3
	#1-forms	–	1	0	3	6	10	16	27	28	–
$(n=11-d)$	E_n rep.	–	1	0	2+1	(3,2)	10	16	27	56	248
			$\tilde{E}_1$	E_1							

We observe a mismatch in 4d. That's because in 4d a 1-form couples to both electric and magnetic charges, so the number of charges is 56.

b)

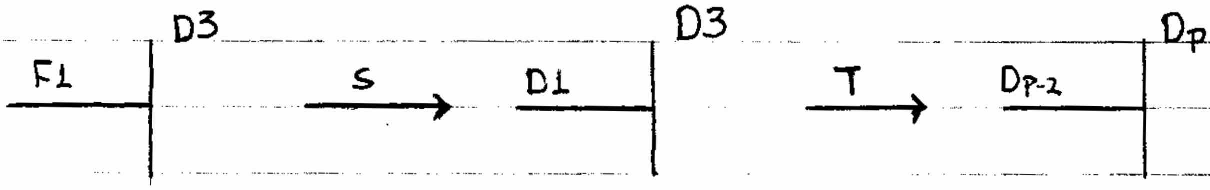
d	11	IIA	IIB	9	8	7	6
#2-forms	–	1	2	2	3	5	5
E_n rep	–	1	2	2	(3,1)	5	10
		$\tilde{E}_1$	E_1				

Again there is a mismatch (in 6d this time) for the same reason as in (a)
Also, the integer part of $SL(2,\mathbb{R})$ in Type IIB gives the electric/magnetic duality action on the 2-forms.

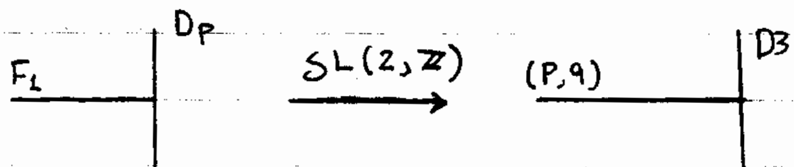
Note: $E_5 = SO(10)$, $E_4 = SU(5)$, $E_3 = SU(3)\times SU(2)$
$E_2 = SU(2)\times U(1)$, $E_1 = SU(2)$, $\tilde{E}_1 = U(1)$.

2.

a)

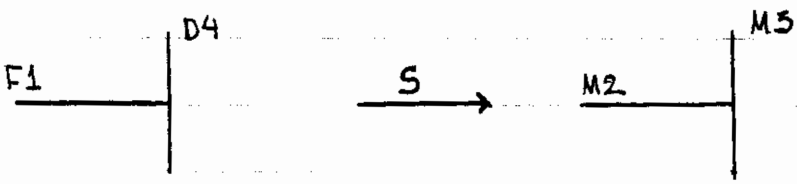


and of course:



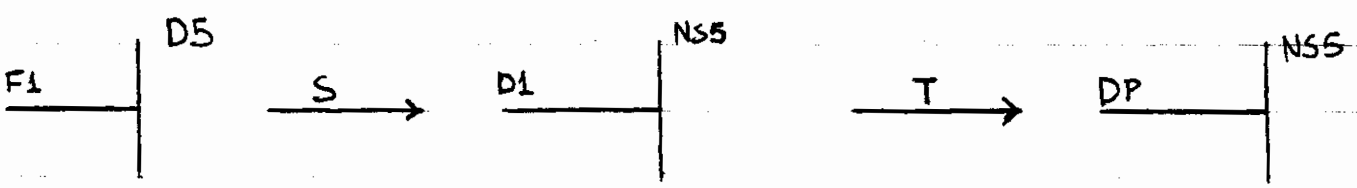
(bound state of p F1 & q D1 with p, q, coprime)

b)



c) Nothing can end on an M2

d)



(p=0,1,..6)

T-duality along a worldvolume direction of an NS5 brane turns a Type IIA NS5 brane into a Type IIB NS5 brane and vice-versa.

3.

Here I use results from problem set #1

a) Electric : F1 (coupling to a worldvolume 1-form)
Magnetic : Dp-2

b) Electric / Magnetic : M2 (self-dual)

(coupling to a world-volume 2-form)

c) Nothing

d) Type IIB : Electric : D1 (coupling to a 1-form)
Magnetic : D3

The D5 brane ending on an NS5 is a domain wall and does not have a magnetic dual in the same way that a D8 brane does not in Type IIA.

Type IIA : Electric / Magnetic : D2 (self-dual) (coupling to a 2-form)

Electric : Euclidean D0 (coupling to a 0-form potential)
Magnetic : D4

4.

a) $$\mathcal{L} = -\frac{1}{4g^2} \mathrm{Tr}(F_{MN} F^{MN}) - \frac{i}{2g^2} (\bar{\lambda} \Gamma^M D_M \lambda)$$

where λ is a Majorana-Weyl spinor in 10 dimensions.

b) $A_M \to A_\mu, A_9$

$$F_{MN} F^{MN} \to F_{\mu\nu} F^{\mu\nu} + 2 D_\mu A_9 D^\mu A_9$$

$$D_M \lambda \to D_\mu \lambda + i [A_9, \lambda]$$

$$\mathcal{L}_{9d} = -\frac{1}{4g^2} \mathrm{Tr}\left(F_{\mu\nu} F^{\mu\nu} + 2 D_\mu \phi D^\mu \phi\right) - $$
$$- \frac{i}{2g^2} \mathrm{Tr} \left(\bar{\lambda} \Gamma^\mu D_\mu \lambda + i \bar{\lambda} \Gamma^9 [\phi, \lambda]\right)$$

with $\phi = A_9$. Moduli space: $\dfrac{\mathbb{R}^{\mathrm{rank}\, G}}{\mathrm{Weyl}(G)}$

c) Let $G = SU(2)$ and $\langle \phi \rangle = a \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

The $D_\mu \phi D^\mu \phi$ term in the action contains a $([A_\mu, \phi])^2$ part.

If we write $A^\mu = \begin{pmatrix} A_1^\mu & A_2^\mu - iA_3^\mu \\ A_2^\mu + iA_3^\mu & -A_1^\mu \end{pmatrix}$

the explicit calculation gives

$$\mathrm{Tr}\left([A_\mu, \phi]^2\right) \sim a^2 \, |A_2 + iA_3|^2$$

This is a mass term for the W-bosons $A_2 \pm iA_3$. Their mass is a constant times $|a|$.

For $SU(n)$, the mass matrix for the W-bosons would be $m_{ij} \propto |a_i - a_j|$ where $\langle \phi \rangle = \begin{pmatrix} a_1 & & 0 \\ & \ddots & \\ 0 & & a_n \end{pmatrix}$, $\Sigma a_i = 0$.

d) Further reduction to 8 dimensions will give an extra term, coming from the $i[A_\mu, A_\nu]$ part of $F_{\mu\nu}$.

The Lagrangian density is:

$$\mathcal{L}_{8d} = -\frac{1}{4g^2} \mathrm{Tr}\left(F_{\mu\nu} F^{\mu\nu} + 2 D_\mu \phi_i D^\mu \phi_i - 2[\phi_8, \phi_9]^2 \right)$$
$$- \frac{i}{2g^2} \mathrm{Tr}\left(\bar{\lambda} \Gamma^\mu D_\mu \lambda + i \bar{\lambda} \Gamma^i [\phi_i, \lambda] \right)$$

where $\phi_8, \phi_9 = A_8, A_9$, $\mu, \nu = 0, 1 \ldots 7$, $i = 8, 9$.

The moduli space is $\dfrac{\mathbb{R}^{2 \times \mathrm{rank}(G)}}{\mathrm{Weyl}(G)}$

e) In any dimension $d<9$ the bosonic part of the action is

$$\mathcal{L}_d = -\frac{1}{4g^2} \mathrm{Tr}\left(F_{\mu\nu} F^{\mu\nu} + 2 D_\mu \phi_i D^\mu \phi_i - \sum_{i,j} [\phi_i, \phi_j]^2 \right)$$

$$- \frac{i}{2g^2} \mathrm{Tr}\left(\bar{\lambda} \Gamma^\mu D_\mu \lambda + i \bar{\lambda} \Gamma^i [\phi_i, \lambda] \right)$$

with $\mu, \nu = 0, 1, \ldots d-1$, $i, j = d, \ldots 9$.