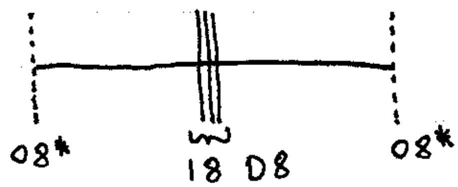
8.871

Solutions to problem set #7

1.

a,b) There are two possible enhanced simple gauge groups (rank 17)

i) SU(18)



If $\frac{1}{g_s} = 0$ at the position of an $O8^-$, we can 'extract' a **D8** brane. The $O8^-$ becomes an $O8^*$, with charge -9

The equation for $\frac{l_s}{g_s} \equiv \lambda$ is:

$$\frac{d^2\lambda}{dx^2} = -\sum_i Q_i \,\delta(x - x_i) \qquad (1)$$

Where i runs over the objects in the interval, x is the coordinate along the interval ($0 \leq x \leq R$) and Q_i, x_i are the charges and locations of the objects.

We can integrate eq.(1):

$$\frac{d\lambda}{dx} = -\frac{1}{2}\sum_i Q_i\, \varepsilon(x-x_i) \qquad (2) \qquad (\varepsilon(x) = \theta(x) - \theta(-x))$$

Here we omit the integration constant, which is putting the bacground cosmological constant to zero.

And:

$$\lambda(x) = -\frac{1}{2}\sum_i Q_i\, |x-x_i| + \lambda_0 \,, \quad (\lambda_0 \equiv \lambda(x=0)) \qquad (3)$$

To get $SU(18)$ we have to stack the D8 branes together, let's say at $x=a$

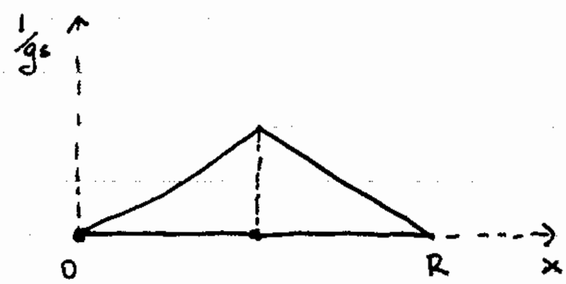
Eq. (3) becomes:

$$\lambda(x) = \frac{1}{2}\left(9|x| - 18|x-a| + 9|x-R|\right) + 9a - \frac{9}{2}R$$

We need to have $\lambda(R)=0$ in order to have an O8* there. This gives:

$$\boxed{a = \frac{R}{2}}$$

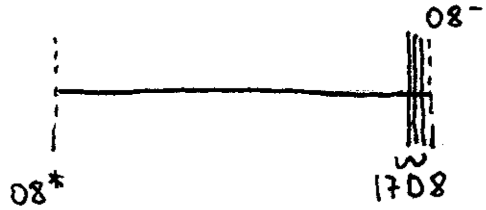
The plot of $\frac{1}{g_s}$ is:



Moving the stack of D8 branes from $R/2$ will change $1/g_s$ at the endpoints and spoil the enhancement.

The stack must be fixed, and that's why we get $SU(18)$ instead of $U(18)$.

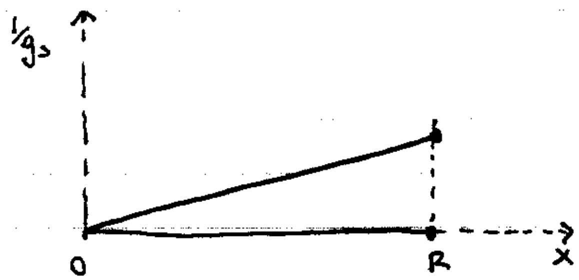
ii) $SO(34)$



Eq.(3) reads:

$$\lambda(x) = \frac{1}{2}\left(9|x| - 17|x-R| + 8|x-R|\right) + \frac{9R}{2}$$

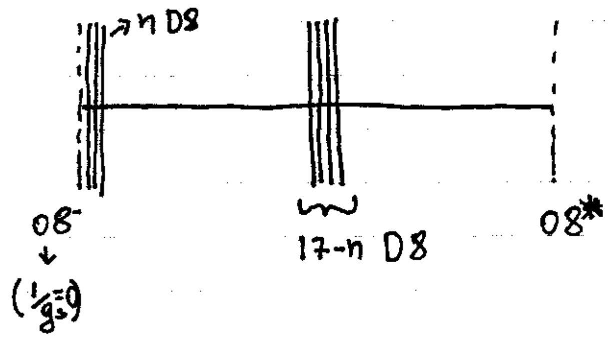
The plot of $\frac{1}{g_s}$ is:



Again, the D8 branes are stuck at $x=R$, otherwise the enhancement at $x=0$ is broken.

c,d) There are two possible enhaneed gauge groups of the form $G_1 \times G_2$ with G_1, G_2 simple.

i) $E_{n+1} \times SU(17-n) \quad n = 0, 1 \dots 7$



If the stack of branes in the bulk is

located at $x=a$ the solution for λ is:

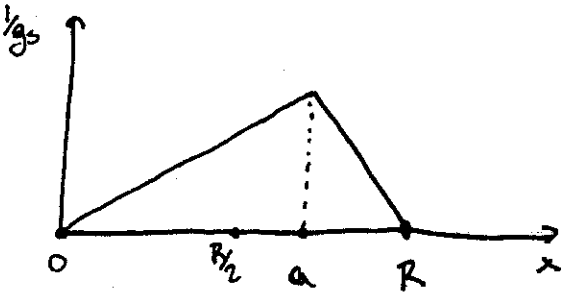
$$\lambda(x) = \frac{1}{2}\left[(8-n)|x| - (17-n)|x-a| + 9|x-R|\right] + \lambda_0$$

Demanding $\lambda(0) = \lambda(R) = 0$ gives:

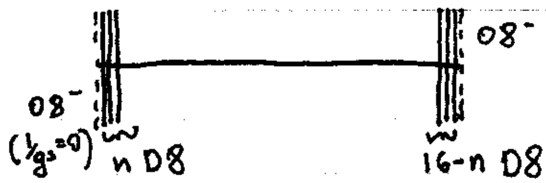
$$\boxed{\begin{gathered} \lambda_0 = 0 \\ a = \frac{9}{17-n} R \end{gathered}}$$

Note:

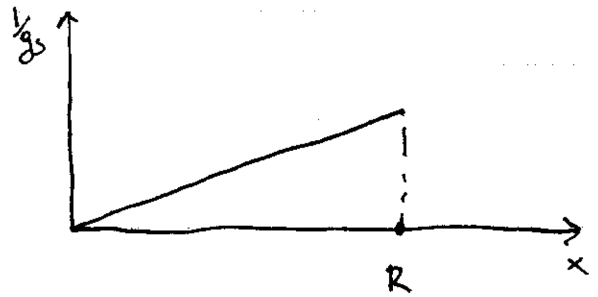
$R/2 < a < R$



ii) $E_{n+1} \times SO(2(16-n))$ $\qquad n = 0, 1, \ldots 7$



$$\lambda(x) = \frac{1}{2}\left[(8-n)|x| - (8-n)|x-R|\right] + \frac{1}{2}(8-n)R$$



2.

a) We can read off the extra W-boson state from the U(1) charge of the states under the decomposition $E_n \supset SO(2(n-1)) \times U(1)$

This U(1) quantum number is the quantised D0 brane charge and reveals the number of D0 (or $\overline{D0}$) branes in the marginal massless bound state that is the W-boson for the enhanced gauge symmetry.

So, for E_8 an E_7 we have:

$E_8 \supset SO(14) \times U(1)$

$$248 \to 91_0 + \underset{\substack{2\,D0\\ \text{b.s.}}}{14_2} + \underset{\substack{2\,\overline{D0}\\ \text{b.s.}}}{14_{-2}} + 1_0 + \underset{D0}{64_1} + \underset{\overline{D0}}{\overline{64}_{-1}}$$

$E_7 \supset SO(12) \times U(1)$

$$133 \to 66_0 + \underset{D0}{32'_1} + \underset{\overline{D0}}{32'_{-1}} + \underset{\substack{2\,D0\\ \text{b.s.}}}{1_2} + \underset{\substack{2\,\overline{D0}\\ \text{b.s.}}}{1_{-2}} + 1_0$$

For E_6 we have:

$E_6 \supset SO(10) \times U(1)$

$$78 \to 1_0 + 45_0 + 16_{-3} + \overline{16}_3$$

The same pattern persists for all lower

E_n, i.e. we get a sinlet, the adjoint of $SO(2(n-1))$, and two spinors with $U(1)$ charges $+r$ and $-r$. This seems to imply that we have bound states of r D0's and bound states of r $\bar{D0}$'s ($r=3$ for E_6). However it is possible that these $U(1)$ charges can be consistently renormalized to ± 1, and it is not immediately clear what the correct choice is.

b) The D0-D8 strings have a zero point energy $\frac{1}{2}$ in the NS sector and 0 in the R sector. The zero modes in the R sector carry an index i coming from the D8 branes. They satisfy a Clifford algebra

$$\{A^i, \Lambda^j\} = \delta^{ij}$$

and so they transform in the spinor of the $SO(2n-2)$ part of the gauge group associated with the D8 branes.

3.

a) We can think of this background as Type I compactified on $T^3 = S^1 \times S^1 \times S^1$ with a T-duality performed in every compact direction.

The result is 8 f.p. each with an $O6^-$ plane spanning the seven non-compact directions and 16 D6 branes at generic points in $T^3/(\mathbb{Z}_2)^3$.

Each $O6^-$ has charge -2, for a total charge of -16, cancelled by the 16 D6 branes.
In addition to the 16 v-plets living on the D6 branes, we have 3 v-plets coming from the dimensional reduction of the gravity multiplet.

Each of these V-plets contains 3 scalars. The Narain moduli space is:

$$SO(19,3,\mathbb{Z}) \backslash SO(19,3,\mathbb{R}) / SO(19,\mathbb{R}) \times SO(3,\mathbb{R}) \quad , \quad \dim = 19 \times 3 .$$

Replacing an $O6^-$ with an $O6^+$ (charge $+2$) increases the orientifold RR charge by four, so we have to decrease the number of D6 branes (and vector multiplets) by four to respect anomaly cancellation.

For K $O6^+$ we have:

$\#O6^-$	$\#O6^+$	$\#D6$	scalar manifold
$8-K$	K	$16-4K$	$SO(19-4K,3,\mathbb{Z}) \backslash \dfrac{SO(19-4K,3,\mathbb{R})}{SO(19-4K,\mathbb{R}) \times SO(3,\mathbb{R})}$

with $K = 1,2,3,4$.

b) The maximal rank Sp gauge group is obtained by putting all the D6 branes in the bacground on top of a single O6$^+$

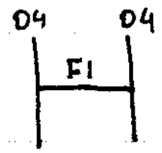
For k O6$^+$ in the backgroud the maximal Sp gauge symmetry is:

$$Sp(16-4k) \quad , \quad k = 1,2,3,4 .$$

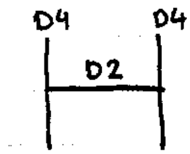
* see last page

4.

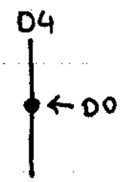
a) W-boson:



monopole:



Instanton:

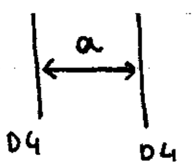


b) $m_W = \langle \phi \rangle$

$$m_m = \frac{\langle \phi \rangle}{g^2}$$

$$m_i = \frac{1}{g^2}$$

c)



$$m_w = \frac{a}{l_s^2} \qquad m_m = \frac{a}{g_s l_s^3}$$

$$m_i = \frac{1}{g_s l_s} \qquad \left(\frac{1}{g^2} = T_{D4} \cdot l_s^4 \right)$$

d) $g_s l_s = R$, $g_s l_s^3 = l_p^3$

$$m_W = \frac{aR}{l_p^3} \quad , \quad m_m = \frac{a}{l_p^3} \quad , \quad m_i = \frac{1}{R}$$

e) W-boson → wrapped M2 stretched between wrapped M5's

monopole → infinite M2 stretched between wrapped M5's

instanton → Kaluza-Klein mode.

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* A more careful study of the background in problem 3 will reveal obstructions coming from discrete torsion that limit the number of $O6^+$ orientifolds to 0, 2 or 4. Moreover, there are two inequivalent configurations for 4 $O6^+$, depending on whether they are coplanar or not.