

8.871 Lecture 2

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1 Electrically and Magnetically Charged Branes

We would like to build a general understanding of branes, the gauge fields to which they are coupled, and the gauge fields living on their world volume. First, we review electromagnetism in four dimensions. Define $F^{(2)} = dA^{(1)}$ as the field strength of the gauge field $A^{(1)}$. Then under a gauge transformation of the gauge field $A^{(1)}$ by a total derivative $d\Lambda^{(0)}$, with $\Lambda^{(0)}$ an arbitrary 0-form (or an arbitrary function):

$$A^{(1)} \rightarrow A^{(1)} + d\Lambda^{(0)}, \quad (1)$$

$F^{(2)}$ is unchanged because $dd = 0$. For a particle that carries an electric charge Q_E we have the source equation $d * F^{(2)} = Q_E \delta^{(3)}$. For a particle that carries a magnetic charge Q_M , the source equation is $dF^{(2)} = Q_M \delta^{(3)}$. We say that the electrically charged particle and the magnetically charged particle are dual to each other or they are Poincaré duals.

Now we follow the same reasoning with the gauge field $B^{(2)}$, which was introduced in the last lecture. The field strength is $H^{(3)} = dB^{(2)}$. Under a gauge transformation for the 2-form $B \rightarrow B^{(2)} + d\Lambda^{(1)}$, with $\Lambda^{(1)}$ an arbitrary 1-form, $H^{(3)}$ is invariant. An F1-string, which propagates in 9+1 dimensions, is electrically charged under $B^{(2)}$, with a charge Q . We therefore write a source equation $d * H^{(3)} = Q \delta^{(8)}$. The string is localized in 8 of the 9 space coordinates and hence we have a delta function in 8 directions. On the other hand, the NS5-brane is magnetically charged under $B^{(2)}$ with a magnetic charge Q' and the corresponding source equation is $dH^{(3)} = Q' \delta^{(4)}$. Again the 5 brane is localized in 5 out of 9 coordinates and hence we write a delta function in 4 directions. Let's generalize this idea to any $(p+1)$ -form in an arbitrary dimension d . Start with a gauge field, a $C^{(p+1)}$ form with field strength $G^{(p+2)} = dC^{(p+1)}$. Then the source equation

$$d * G^{(p+2)} = Q \delta^{(d-p-1)} \quad (2)$$

represents a p -brane which is electrically coupled with a charge Q to the $(p+1)$ -form gauge field, and the source equation

$$dG^{(p+2)} = Q' \delta^{(p+3)} \quad (3)$$

represents a $(d-p-4)$ -brane which is magnetically coupled with a charge Q' to the $(p+1)$ -form gauge field.

In Lecture 1 we listed the massless field content of 11d SUGRA. We saw that it contains a gauge field $C^{(3)}$. 11d SUGRA is the low energy effective theory of M-theory. Applying the ideas outlined above we find that in M-theory, we should have an M2-brane which is electrically charged under $C^{(3)}$ and an M5-brane which is magnetically charged under $C^{(3)}$. Similarly, we can find the branes which are charged under massless gauge fields in Type IIA string theory. Let us first find the massless fields of this theory. Recall that Type IIA theory has (1,1) supersymmetry in 10 dimensions and is nonchiral. The massless content of this theory is the 10-dimensional reduction of 11d SUGRA. As in any dimension we can classify the massless states according to irreducible representations of the little group which is $SO(8)$ for 9+1 dimensions. $C^{(3)}$ in 11 dimensions has 84 independent components which are counted as the 3-rd rank antisymmetric representation of $SO(9)$, the little group for massless states in 10+1 dimensions. Under the inclusion of $SO(8)$ into $SO(9)$, the 3-rd rank anti-symmetric representation of $SO(9)$ decomposes into a 3-rd rank antisymmetric tensor and a 2-nd rank antisymmetric tensor in 8 dimensions. We can understand this in the tensor notation as

follows: we start with $C_{\mu\nu\lambda}$ in 10+1 dimensions and pick (for example) $\lambda = 10$. Then μ and ν can be anything except 10. That leaves us with a 2-form in 9+1 dimensions which we call $B^{(2)}$. We can also have $\lambda, \mu, \nu \neq 10$ (but constrained to be antisymmetric). Then we end up with a 3-form in 9+1 dimensions. The 3-form has $\binom{8}{3} = 56$ components and the 2-form has $\binom{8}{2} = 28$ components. We can check this decomposition by computing $28 + 56 = 84$. Similarly, the graviton decomposes into g_{ab} (traceless and symmetric), $g_{a(10)}$ and $g_{(10)(10)}$, where $a, b \neq 10$. These objects have 35, 8 and 1 degrees of freedom respectively. A similar computation can be done for the gravitino, ψ_μ . The results are tabulated here:

$$SO(9) \supset SO(8) \tag{4}$$

$$C : 84 \rightarrow 56_v \oplus 28 = [3] + [2] \tag{5}$$

$$g : 44 \rightarrow 35 \oplus 8_v \oplus 1 = (2) + [1] + [0] \tag{6}$$

$$\psi_\mu : 128 \rightarrow 56_s \oplus 56_c \oplus 8_s \oplus 8_c \tag{7}$$

The notation here is that square brackets [] denote antisymmetric tensors whereas round brackets () denote symmetric traceless tensors. The number inside the brackets is the rank of the tensor. The subscript v, s, c means the vector representation, the spinor representation and the complex conjugate spinor representation, respectively. These subscripts are used since for $SO(8)$ there is a triality that acts on the 8 dimensional and the 56 dimensional representations and we therefore have three different representations of the same dimension which we distinguish by these subscripts.

We now look at the bosonic massless fields to determine the kinds of branes that can exist in Type IIA theory. Note that there are three kinds of forms. The 3-form couples electrically to a D2-brane and magnetically to a D4-brane. The 2-form, $B^{(2)}$, couples electrically to the F1 string, and magnetically to the NS5-brane. (One might ask why $B^{(2)}$ couples to the fundamental string and NS5-brane, not the D1-brane and D5-brane. Indeed, there is no such distinction if we are just looking at the supergravity multiplet reduced to 10 dimensions. But in perturbative string theory, Dp -branes must carry Ramond-Ramond charges, and so we can distinguish them from NS-branes and F-strings.) Finally, the 1-form in equation (6) couples electrically to a D0-brane and magnetically to a D6-brane.

Exercise: Repeat this analysis for Type IIB theory. List the massless fields and find out what types of branes are present in Type IIB.

We can relate the branes of M-theory to the branes of Type IIA string theory via S-duality. M-theory on $M_{10} \times S^1$ is S-dual to Type IIA theory on M_{10} , where M_j is an arbitrary j -dimensional manifold. When the M2-brane wraps one of its own dimensions on the S^1 , it is dual to an F1 string in Type IIA. If instead we use dimensional reduction on M-theory, the M2-brane will reduce to a D2-brane. Similarly, if the M5-brane wraps around the circle, it becomes a D4-brane in Type IIA, but it dimensionally reduces to the NS5-brane.

Just to complete the picture, we'll briefly go over the massless multiplet in string theories with 16 supercharges, and thus deduce the types of branes in the Type I and heterotic theories. First, the gravity multiplet contains the graviton, g , which has $\frac{8 \cdot 9}{2} - 1 = 35$ components. Then there is the 2-form $B^{(2)}$ which has 28 components as usual. The dilaton ϕ adds one more degree of freedom. We also have a spinor ψ_c with 8 components and a spinor with a vector index, ψ_μ^c (the gravitino), which has 56 components. The vector multiplet consists of the vector A_μ , which has 8 components, and λ , a chiral spinor we denote 8_s . In the Type I theory, we will therefore find a D1-brane (electrically coupled to $B^{(2)}$) and the D5-brane (magnetically coupled to $B^{(2)}$). Similarly, in heterotic theories, we have an F1-string and NS5-brane. In addition to these objects we also have small instantons which are branes that are present in any theory that has non-abelian massless gauge fields. An instanton is an object which is localized in 4 space coordinates and therefore in 9+1 dimensions it will be a 5 brane. The Type I string theory has a gauge group $SO(32)$ and the two Heterotic theories have gauge groups $SO(32)$ and $E_8 \times E_8$, respectively. As a result we have two additional 5 branes together with what we listed above: the $SO(32)$ small instanton and the E_8 small instanton

2 Central Charges and Branes

The existence of branes (solitonic solutions) and the central charges in the supersymmetry algebra are intimately connected. Central charges are due to non-zero boundary terms that appear when computing the conserved supercharges for topologically nontrivial states, like branes. For example, in 11d SUGRA, we can have the following maximal extension to the supersymmetric algebra:

$$\begin{aligned}
 \{Q_r, Q_s\}_{sym} &= P_\mu + Z^{(5)} + Z^{(2)} & (8) \\
 &= \binom{11}{1} + \binom{11}{5} + \binom{11}{2} \\
 &= \binom{11}{1} + \binom{11}{5} + \binom{11}{9} \\
 &= 11 + 55 + 462 \\
 &= 528
 \end{aligned}$$

Note that $528 = \frac{32 \cdot 33}{2}$, because the LHS of (8) is a symmetric 32×32 matrix. The 5-form on the RHS corresponds to the charge of the M5-brane. The 2-form corresponds to the M2-brane. To get this result, we could also decompose $(32 \otimes 32)_{sym}$, the symmetrized tensor product of spinor representations of $SO(11)$, into the direct sum of vector representations of $SO(11)$. We find that $(32 \otimes 32)_{sym} = 11 \oplus 55 \oplus 462$. Now we can further decompose this into representations of $SO(10)$ by doing dimensional reduction, and find almost the full list of objects in Type IIA theory. We find that the 2-form $B_{\mu\nu}$ becomes $B_{\mu(10)}$, a 1-form, and B_{ab} ($a, b \neq 10$) a 2-form in 10 dimensions. These correspond to the F1-string and the D2-brane. The 1-form P_μ becomes a 1-form in 10 dimensions and a scalar P_{10} . The scalar corresponds to a D0-brane. Similarly, the 5-form decomposes into a 5-form and 4-form in 10 dimensions. These are the NS5 and D4-branes. (Note that the D6-brane and D8-brane are missing in this analysis.) Furthermore, all these objects are BPS states: they break half the supersymmetries in the theory. They also saturate the BPS bound, which roughly means that the tension (or energy density) is equal to the absolute value of the charge on the brane.