8.871 Lecture 3

M. Padi

November 14, 2004

Let's use dimensional analysis to make some general remarks about the tensions of various branes. We'll start with the M2- and M5-branes. Note that the tension of an object that fills 2 space dimensions must have units of $[L^{-3}] = \frac{[E]}{[L^2]}$. The only length scale we have is l_p, the Planck length. Therefore we set the tension of the M2-brane to $T_2 = \frac{1}{l_p^3}$. Similarly, the tension of the M5-brane must be proportional to $\frac{1}{l_p^6}$. The constant of proportionality between these two tensions is determined by the ratio $\frac{T_5}{T_2^2}$, which is in turn set by Dirac-quantization. Following Schwarz [1] we find

$$T_5 = \frac{1}{2\pi} T_2^2. \tag{1}$$

In the following we will ommit factors of 2 and π.

For D-branes in Type II theories the tension is given in terms of the two parameters of the theory: g_s and l_s, where g_s is the string coupling, and the fundamental length scale of the string is denoted by l_s. Since the D-brane arises in the open string sector, its tension will come with a factor of g_s^{-1}. The string scale dependence is determined by scaling and therefore a general Dp-brane has tension

$$T_{Dp} = \frac{1}{g_s l_s^{p+1}}. \tag{2}$$

We can also determine the relationships between tensions of M-branes and D-branes using S-duality between M-theory compactified on a circle and Type IIA string theory. We use the fact that the M2-brane compactified over one of its dimensions becomes an F1-string in Type IIA. Thus we can write

$$T_{F1} = \frac{1}{l_s^2} = \frac{R}{l_p^3} \ . \tag{3}$$

An M2 which does not wrap the M theory circle becomes a D2 brane in Type IIA and we can write a relation between the tensions:

$$T_{D2} = \frac{1}{g_s l_s^3} = \frac{1}{l_p^3} \ . \tag{4}$$

From these two equations we can write a relation between the two fundamental quantities of M theory on a circle and the two fundamental quantities of Type IIA:

$$R = g_s l_s, \qquad l_p^3 = g_s l_s^3. \tag{5}$$

As a check, we can confirm formulas for the tension of the D4-brane and the NS5-brane. We note that the M5-brane becomes the D4-brane or the NS5-brane, depending on whether we compactify on a world volume direction of the M5 brane or not, respectively. Thus, using equation (5) we get

$$T_{M5} = \frac{1}{l_p^6} = \frac{1}{g_s^2 l_s^6} = T_{NS5} \ . \tag{6}$$

Indeed the NS5-brane is a solitonic object and should have a tension which is proportional to the inverse square of the string coupling. Furthermore,

$$T_{D4} = RT_{M5} = \frac{R}{l_p^6} = \frac{1}{g_s l_s^5} \ , \tag{7}$$

in agreement with equation (2).

We also know that the Type IIB theory at coupling g_s is S-dual to itself at coupling $g'_s = 1/g_s$. Under this duality, the D1-brane becomes the F1-string, the D3-brane is self-dual and the D5-brane becomes the NS5-brane. So now we have the following equalities:

$$T_{D1} = \frac{1}{g_s l_s^2} \quad = \quad T'_{F1} = \frac{1}{l_s'^2} \tag{8}$$

$$T_{F1} = \frac{1}{l_s^2} \quad = \quad T'_{D1} = \frac{1}{g'_s l_s'^2} \tag{9}$$

From these equations we can derive the following relation between l_s and l'_s:

$$l_s'^2 = g_s l_s^2 \ . \tag{10}$$

References

[1] J. H. Schwarz, "Lectures on superstring and M theory dualities," Nucl. Phys. Proc. Suppl. **55B**, 1 (1997) [arXiv:hep-th/9607201].