

## Problem Set 1 Solution

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**Q1, Q2** I used cps85.raw. Please see attached STATA result.

**Q3** Calculating Marginal Returns to Schooling(MRS) and Marginal Returns to Experience(MRE) implies doing the first differentiation. As shown in STATA result, the regression equation with appropriate coefficients is:

$$\begin{aligned}\ln \text{ wage} &= a_0 + a_1S + a_2S^2 + a_3X + a_4X^2 + a_5SX \\ &= .7475 + .0348S + .0026S^2 + .0477X - .0006X^2 - .0007SX\end{aligned}$$

And, Marginal Return to Schooling(MRS) can be obtained through first differentiating  $\ln\text{Wage}$  with respect to  $S$ :

$$\frac{\partial \ln \text{Wage}}{\partial S} = .0348 + .0054S - .0007X$$

Since we still have experience term ( $X$ ) in the differentiated equation, marginal return is not only changes according to schooling but also to experience. Holding experience constant, we can see that the increase in schooling results in the increase in MRS. This gives you an idea that an additional education makes your wage increases more rapidly. That is, an additional education not only increases the whole amount of wage, but also speeds up the rate of increase in wage. This can be proved by checking out the second derivative.

$$\frac{\partial^2 \ln \text{Wage}}{\partial S^2} = .0054 > 0$$

The second derivative result, which is positive, shows us that the regression equation with respect to schooling has a minimum value, not maximum. Therefore, we can conclude that the more education a person receives, the higher percentage change(positively) in  $\ln\text{Wage}$ , that is, the more wage s/he can earn with increasing rate.

The same logic can be applied to experience. We first differentiate and

get Marginal Return to Experience(MRE):

$$\frac{\partial \ln Wage}{\partial X} = .0477 - .0012X - .0007S$$

Interestingly, the Marginal Return to Experience seems to decrease as a person's experience increases. (You can simply plug numbers in X) The second derivative is :

$$\frac{\partial^2 \ln Wage}{\partial X^2} = -.0012 < 0$$

Contrast to the case of schooling, the sign of the second derivative is negative. This means the experience has a maximum point at which the rate of increase becomes zero and then turn to negative. But, remember that the "wage" will be still increasing with decreasing rate until the original function hits zero.

**Q4** The question is asking you to do the first differentiation with respect to age variable, then put it equal to zero to figure out an optimal point. You can also substitute AGE for EX into the previous equation, then differentiate. Many people did the differentiation first, and replaced EX with AGE. This way of calculation, luckily, worked out in this particular case, but not necessarily all the time. It is a special case that the relationship between EX and AGE is exactly linear, which could give you a correct answer. Otherwise, you will encounter a different answers. Think about the example of  $f(x) = ax^2 + bx + c$  and  $x = ez + f$ . Then, compare when you differentiate  $f(x)$  with respect to  $x$ , then insert  $ez + f$  instead of  $x$  **vs.** differentiate  $f(z) = a(ez + f)^2 + b(ez + f) + c$  with respect to  $z$ . Unless  $e = 0$ , you will have different answers. We will probably cover this later in the course.

$$\begin{aligned} \ln \text{wage} &= a_0 + a_1S + a_2S^2 + a_3A + a_4A^2 + a_5SA \\ &= .4366 - .0151S + .0027S^2 + .055A - .0006A^2 + .0006SX \end{aligned}$$

The first differentiation will be :

$$\frac{\partial \ln Wage}{\partial A} = .055 - .0006A - .0006S$$

FOC is:

$$.055 - .0006A - .0006S = 0$$

$$A = \frac{.055 - .0006S}{.0006}$$

The age at which a person's wage reaches to maximum depends on the years of schooling. If we plug different years of education in the equation of 8, 12 and 16 years, we have 46.7, 48.4 and 50.2, respectively. Noticeably, although the education years are different by 4 years, the age of maximum does not show that much of difference. This implies that a person with higher education earns faster (as though he compensates his opportunity cost of schooling) to reach the maximum wage of his. And this result supports the previous question's answer.

Then, what about the actual value of earning at the maximum point? Simply, plug the age we just calculated into the original equation. Since the education years are given (8, 12 and 16 years), we can calculate  $\ln Wage$  at maximum for each schooling years. I obtained 1.9 for 8 years, 2.1 for 12 years and 2.5 for 16 years. This is the natural log form, so we transform these values to real wage:

$$\ln Wage = 1.9$$

$$e^{1.9} = 6.7$$

Therefore, if a person has 8 years of education, s/he will reach her/his maximum wage when s/he becomes 46.7 years old and the wage itself is \$ 67,000 a year. (I assumed that the unit might be in \$ 10,000) In the same way, I got 8 dollars per hour for 12 years of education and \$ 120,000 for 16 years of education.

**Q5** This is an integral question. You can either do the integration with respect to age or experience. Just mind the upper and lower bounds when you change the terms that you are integrating. Here, I chose experience. Writing integration for experience gives :

$$\begin{aligned} & \int_0^{51/43} a_0 + a_1S + a_2S^2 + a_3X + a_4X^2 + a_5SX dx \\ &= \int_0^{51/43} .7475 + .0348S + .0026S^2 + .0477X - .0006X^2 - .0007SX dx \\ &= \left[ (.7475 + .0348S + .0026S^2)X + \frac{1}{2}(.0477 - .0007)X^2 - \frac{1}{3}.0006X^3 \right]_0^{51/43} \end{aligned}$$

Following the integral rule, you plug upper bound and lower bound into the

equation, then subtract the lowerbound value from the upperbound value. Here, the lower bound is 0, so you only need to get the value of upperbound. (plugging 0 into the integral equation will only result in 0). The calculation gives 90.8 for 8 years of education and 107.4 for 16 years.

These numbers are the accumulation of percentage changes in wage. (Recall that the dependent variable was  $\ln Wage$ ) Thus, it does not correctly convert to the total wage a person earned. This can be obtained by:

$$\int_0^{51/43} e^{.7475+.0348S+.0026S^2+.0477X-.0006X^2-.0007SX} dx$$

This should eventually come close to a normal curve. It should be very hard to solve this equation by hand, so I used some help of Matlab. When a person has 8-year schooling, the lifetime earning of the person is \$ 2,979,533. For a 16-year schooling person, the lifetime earning is \$ 4,722,841. You can also use Excel for the calculation. Inserting each value of experience from 1 to 51 (or 43) into the equation and summing them up will give you similar result.