Problem Set 7 Solution

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Nov. 26, 2003

Bulmer Exercise 10.1
You need to first calculate the probability of throwing six on a white die. This is 3932/20000.

$$
\begin{aligned}
E(\text{throwing a 6 with the white die}) &= \frac{3932}{20000} = 0.1966 \\
Var(\text{throwing a 6 with the white die}) &= (.1966)(1 - .1966) = .1579 \\
95\% \text{ of confidence interval} &= 0.1966 \pm 1.96\sqrt{\tfrac{.1579}{20000}} \\
&= .1966 \pm .0055
\end{aligned}
$$

Bulmer Exercise 10.2
(a) Since we would like to use proportion, I will use Table 2b.

$$
\begin{aligned}
\text{Pr(stillborn in male)} &= \tfrac{.012}{.5141} = .0233 \\
95\ \% \text{ confidence interval} &= .0233 \pm 1.96\sqrt{\tfrac{.0233(1-.0233)}{368490}} \\
&= .0233 \pm .00049
\end{aligned}
$$

(b) In female case,

$$
\begin{aligned}
\text{Pr(stillborn in female)} &= \tfrac{.0109}{.4859} = .0224 \\
95\ \% \text{ confidence interval} &= .0224 \pm 1.96\sqrt{\tfrac{.0224(1-.0224)}{348250}} \\
&= .0224 \pm .00049
\end{aligned}
$$

(c) What about gender difference?

$$\begin{aligned}
\Pr(\text{difference}) &= \Pr_{\text{male}} - \Pr_{\text{female}} = .01 \\
Var(\text{difference}) &= \tfrac{.0233(1-.0233)}{368490} + \tfrac{.0224(1-.0224)}{348250} \\
&= .0000001249 \\
95\ \%\ \text{confidence interval} &= .001 \pm 1.96\sqrt{.0000001249} \\
&= .001 \pm .00069
\end{aligned}$$

Bulmer Exercise 10.5

	Hyoscyamine	Hyoscine	Difference
$\bar{x}$	.75	2.33	1.58
s	1.79	2	1.23
$t = \frac{\bar{x}}{s/\sqrt{10}}$	1.32	3.68	4.06

(a) Hyoscyamine:

$$\left|\tfrac{\bar{x}-\mu}{s/\sqrt{10}}\right| \leq 2.262, |\bar{x}-\mu| \leq 2.262 \times \tfrac{s}{\sqrt{10}}$$

$$\bar{x} - 2.262 \times \tfrac{s}{\sqrt{10}} \leq \mu \leq \bar{x} + 2.262 \times \tfrac{s}{\sqrt{10}}$$

$$.75 - 2.262 \times \tfrac{1.79}{\sqrt{10}} \leq \mu \leq .75 + 2.262 \times \tfrac{1.79}{\sqrt{10}}$$

$$.75 - 1.28 \leq \mu \leq .75 + 1.28$$

note : This is t - distribution since it is using s instead σ. You have to refer to t-table. ($d.f. = 10 - 1 = 9$)

(b) In case of hyoscine,

$$2.33 - 2.262 \times \tfrac{2}{\sqrt{10}} \leq \mu \leq 2.33 + 2.262 \times \tfrac{2}{\sqrt{10}}$$

$$2.33 - 1.43 \leq \mu \leq 2.33 + 1.43$$

(c) It is basically asking you the difference.

$$1.58 - 2.262 \times \frac{1.23}{\sqrt{10}} \leq \delta \leq 1.58 + 2.262 \times \frac{1.23}{\sqrt{10}}$$

$$1.58 - .88 \leq \delta \leq 1.58 + .88$$

$$.7 \leq \delta \leq 2.46$$

Bulmer Exercise 10.6

Now, you have to obtain confidence interval for standard deviation. In the table, we are only given s and do not know of real or true σ. We were hinted that $\frac{S^2}{\sigma^2} \sim \chi^2_{9d.f.}$, which means that σ^2 will fall into the region of $\frac{S^2}{\chi^2_{.025}}$ and $\frac{S^2}{\chi^2_{.975}}$ with 9 degrees of freedom. (S^2 is sum of square of mean deviation. that is $\sum(x_i - \bar{x})^2$.)

$$\chi^2_{.025} = 19.02 \& \ \chi^2_{.975} = 2.70$$

$$S^2 = s^2(n-1) = 4 \times 9 = 36$$

$$\frac{36}{19.02} \leq \sigma^2 \leq \frac{36}{2.7}$$

$$1.89 \leq \sigma^2 \leq 13.33$$

1. $\sigma^2 \geq 1.89, \ \sigma \geq 1.37 \text{and} \sigma \leq -1.37$

2. $\sigma^2 \leq 13.33, \ -3.65 \leq \sigma \leq 3.65$

   The range that satisfies both of (1) and (2), and also standard deviation should not be negative,

$$1.37 \leq \sigma \leq 3.65$$