Recitation Note

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1 Some Integration Theorem

1. A constant can be moved through an integral sign.

$$\int cf(x)dx = c\int f(x)dx$$

2. An integration of a sum is the sum of the integrations.

$$\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$$

3. An integration of a difference is the difference of the integrations.

$$\int [f(x) - g(x)]dx = \int f(x)dx - \int g(x)dx$$

2 Integration Rules

1. Power Rule

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c$$

2. Exponential Rule I

$$\int e^x dx = e^x + c$$

3. Exponential Rule II

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

4. Logarithm Rule I

$$\int \frac{1}{x}dx = lnx + c$$

5. Logarithm Rule II

$$\int \frac{f'(x)}{f(x)}dx = lnf(x) + c$$

Examples:

1) $\int 2e^{2x} + \frac{14x}{7x^2+5}dx$
2) $\int \frac{4x}{2x^2+1}dx$
3) $\int \frac{x}{3x^2+5}dx$

3 Substitution Rule

When you have integration with high power or with complicated form, you can use substitution rule. First, you put the complicated part as u and find $\frac{du}{dx}$. Then, substitute the original equation with u and $\frac{du}{dx}$ to solve it in u term.

Example:

1) $\int 2x(x^2+1)^{50}dx$
2) $\int 6x^2(x^3+2)^{99}dx$

4 Integration by Parts

The integral of v with respect to u is equal to uv minus the integration of u with respect to v.

$$\int vdu = uv - \int udv$$

Here, you use the less complicated part (usually one term) as v and the rest of the integration including integral sign and dx as u. For example, if you have $\int x(x+1)^{\frac{1}{2}}dx$, v should be x and u should be $\int (x+1)^{\frac{1}{2}}dx$. Then, find dv and du to plug them in the formula.

Example:

1) $\int xe^x dx$
2) $\int ln\ x\ dx$

5 Properties of Sigma Notation

The following properties of sigma notation will help to manipulate sums:

1.
$$\sum_{k=1}^{n} ca_k = c\sum_{k=1}^{n} a_k$$

2.
$$\sum_{k=1}^{n}(a_k + b_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$$

3.
$$\sum_{k=1}^{n}(a_k - b_k) = \sum_{k=1}^{n} a_k - \sum_{k=1}^{n} b_k$$

Question Are these true?

1) $\int \left[\sum_{i=1}^{n} f_i(x)\right] dx = \sum_{i=1}^{n} \left[\int f_i(x)dx\right]$

2) $\frac{d}{dx}\left[\sum_{i=1}^{n} f_i(x)\right] = \sum_{i=1}^{n} \left[\frac{d}{dx}[f_i(x)]\right]$

3) $\sum_{i=1}^{n} a_i b_i = \sum_{i=1}^{n} a_i \sum_{i=1}^{n} b_i$

4) $\sum_{i=1}^{n} \frac{a_i}{b_i} = \frac{\sum_{i=1}^{n} a_i}{\sum_{i=1}^{n} b_i}$

5) $\sum_{i=1}^{n} a_i^2 = \left(\sum_{i=1}^{n} a_i\right)^2$

Quick Question
Let $\bar{x}$ denote the arithmetic average of the n numbers $x_1, x_2, \cdots, x_n$. Prove

that

$$\sum_{i=1}^{n}(x_i - \bar{x}) = 0$$