Lecture 6

- **Reminder**
  - Pick up confidential info for Stakes of Engagement from the Black Folder Box
- **Debrief Jessie Jumpshot**
- **Fair Division**
- **Rothman Art Collection**

Themes

- Creating value by exploiting differences in
  - Probabilities
  - Values
- How to construct an efficient frontier
- What is Fair?
- How do divide up indivisible goods gracefully

Jessie Jumpshot

Creating Value with Contingent Contracts

Raiffa's Full Open Truthful Exchange or How to Calculate Contracts that are Un-dominated

From *Lectures on Negotiation*

By Howard Raiffa

(1996)

The Problem

- Janet and Marty must divide 20 items
- Contexts:
  - Dividing an estate
  - Dividing a partnership
  - Getting a divorce

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
J	J	M	M	M	M	J	M	M	J	M	M	M	M	J	M	J	M	J	M

Number of Contracts = 2^{20} = 1,048,576

Full Open Truthful Exchange

- Parties trust each other and are willing to exchange truthful information about their preferences for a list of items.

- They must decide how to divide them.

First Steps

- List all items
- Each party allocates points measuring the desirability of each item
- For ease of interpretation:
  - Each possesses 100 points to allocate
  - Points are non-zero and sum to 100

Janet & Marty
Must divide
20 Items

Items	Janet	Marty
1	1	2
2	1.5	1
3	8	5
4	1.5	3
5	9	7
6	2	3
7	3	8
8	14	30
9	0..5	1
10	0	1
11	7	4
12	0.5	0.5
13	25	18
14	0.5	1
15	10	5
16	4.5	3
17	3	0.5
18	8	4
19	0.5	1
20	0.5	2
	100	100

Each allocates
100 points
among 20 items

Objective

- Find all ***un-dominated*** allocations or contracts
  - An allocation is un-dominated if there does not exist an allocation preferred by ***both*** parties

- Un-dominated allocations are called "efficient" or "Pareto Optimal"
  - after the economist Vilfredo Pareto.

WHY?

- Pareto efficient allocations or contracts separate the wheat from the chaff:
  - Separates contracts for which both can do better from those for which it is not possible to improve both parties' payoffs.

- Shows where value can be created!

- Enables parties to focus on claiming value once we identify all agreements that are not dominated.

Steps

- Compute the ratio of scores for “Janet” and “Marty”: if Janet assigns 8 points to item 3 and Marty assigns 5 points, the ratio is 8/5 = 1.60.

Figure removed due to copyright restrictions. See Figure 26 from Raiffa, Howard. *Lectures on Negotiation Analysis*, Program on Negotiation at the Harvard Law School, 1998.

- Sort the list with the largest ratio for Janet/ Marty at the top and the smallest at the bottom:
  - Ratios in column 4

- Begin by allocating all items to Marty, so Janet has a score of 0 and Marty a score of 100.

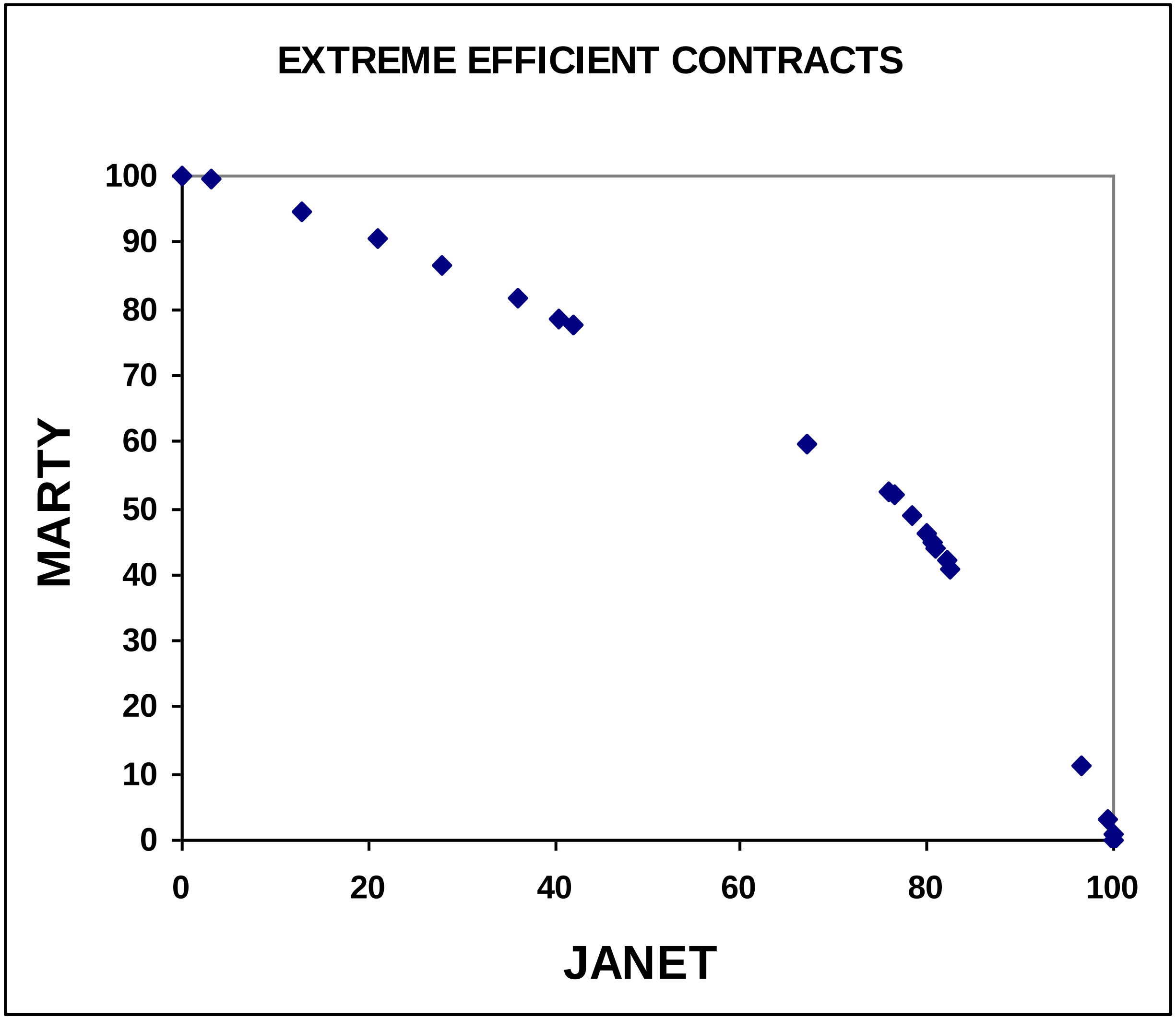
- The largest ratio, say, 6 means that for each point increment we add to Janet for the item at the top of the list, we only decrease Marty's score by 1/6:
  - Allocating item 17 to Janet gets her 3 points and reduces Marty's points by only .5

Extreme Efficient Contracts

- If we continue down the list in this fashion, we generate a set of ***extreme efficient contracts!***
- ***Plot*** the extreme efficient contracts.
- You are now on your way to deciding what constitutes a fair division of items. No one of these allocations (contracts) are dominated.

Figure removed due to copyright restrictions. See Figure 27 from Raiffa, Howard. *Lectures on Negotiation Analysis*, Program on Negotiation at the Harvard Law School, 1998.

Add'l Items to Janet	JANET	MARTY
	0	100
17	3	99.5
15	13	94.5
18	21	90.5
11	28	86.5
3	36	81.5
16	40.5	78.5
2	42	77.5
13	67	59.5
5	76	52.5
12	76.5	52
6	78.5	49
4	80	46
9	80.5	45
14	81	44
1	82	42
19	82.5	41
8	96.5	11
7	99.5	3
20	100	1
10	100	0



Extreme Efficient Contracts that Maximize the Minimum that a Party Gets

- If party 1 gets S_1 and party 2 gets S_2 finding

$$\text{Max Min } \{S_1 , S_2\}$$

  is clearly not a linear problem!

- Finding the set of efficient contracts that maximize the minimum any one party gets can be turned into a simple linear programming problem by a clever trick

Formulation

c_{i1} *is the point value of item i to party* 1;

c_{i2} *is the point value of item i to party* 2

$z_{ij} = 1$ *if item i goes to party j and equals* 0 *otherwise*

$$S_i = \sum_{i=1}^{n} c_{ij} \times z_{ij}\text{, total points to party } i$$

$$Max\,Min\{S_1, S_2\}$$

subject to

$$S_1 = \sum_{i=1}^{n} z_{i1} \times c_{i1} \text{ and } S_2 = \sum_{i=1}^{n} z_{i2} \times c_{i2}$$

Linear Constraints on the $z_{ij}s$

For example, $z_{i1} = 1 - z_{i2}$ *and*

sum of all $z_{ij}s$ *equals the number n of items*

TRANSLATION TO AN LP PROBLEM

Define a new variable θ *and then find*

$$Max\,\theta \text{ such that}$$

$$\theta \le S_1 = \sum_i z_{i1}c_{i1} \text{ and } \theta \le S_2 = \sum_i z_{i2}c_{i2} \text{ plus}$$

and to Linear Constraints on the $z_{ij}s$

Fair Division Schemes

Naïve, Steinhaus, Vickery

Fair Division Problem

- An Estate consisting of four indivisible items are to be shared “equally” by three children
- Each child assigns a “monetary worth” to each item

What is "FAIR"? (Brahms & Taylor 1999)

- Proportionality
- Envy Freeness
- Equitability
- Efficiency

Proportionality

- If division is among N persons, each **THINKS** he/she is getting *at least* 1/N

Envy Freeness

- No party is willing to give up the portion it receives in exchange for someone else's share
  - For two parties, this = Proportionality
  - For more than two parties, Envy Freeness is STRONGER than Proportionality
    - Someone may still be getting more than you!
  - Envy Freeness is Proportional but not conversely

Equitability

Each party THINKS—according to her/his individual preferences--that she/he received the same fraction of total value

- Coupled with envy-freeness, each of two parties would think that both exceed 50% of value, in their preference terms, by the same amount

Efficiency

- There is no other allocation that is better for one party without being worse for one or more other parties.

Impossibility Theorem (Brahms & Taylor)

- NO allocation scheme ALWAYS satisfies
  - Equitability
  - Envy Freeness
  - Efficiency

(Rijnierse and Potters in Brahms & Taylor)

Items	Ann	Ben	Carol
A	40	30	30
B	50	40	30
C	10	30	40

- 40-40-40 is both Efficient and Equitable

- However, it is not Envy Free!
  - Ann envies Ben for getting B which is worth 50 points to her

  - Allocating B to Ann and A to Ben (Carol still gets C) is Efficient but is neither Equitable nor Envy Free
    - Each now gets a different number of points
    - Ben now envies Ann

Dividing Indivisible Goods

Estate Planning

Monetary Worth To Children

	Individuals		
Items	1	2	3
A	$10,000	$4,000	$7,000
B	2,000	1,000	4,000
C	500	1,500	2,000
D	800	2,000	1,000

Side Payments?

Player	Allocated to:	Worth	Side Payments	Total
1	A	10,000		
2	D	2,000		
3	B & C	6,000		

Naive

Player	Allocated to:	Worth	Side Payments	Total
1	A	10,000	-4,000	6,000
2	D	2,000	+4,000	6,000
3	B & C	6,000		6000

Sum = 18,000/3 = 6000

NAIVE

- Accounts only for item value assigned by person who values that item the most

Steinhaus

Player	Allocated to:	Worth	Side Payments	Total
1	A	10,000		
2	D	2,000		
3	B & C	6,000		

Sum =18,000

Imagined Disagreement Point

- Each gets 1/3 of each item (at his/her evaluation)

Items	1	2	3
A	$10,000	$4,000	$7,000
B	2,000	1,000	4,000
C	500	1,500	2,000
D	800	2,000	1,000
	$4,333	$2,833	$4,667

Sum of 1/3 Values = $11,033

Allocation of Excess

Initially each gets 1/3 of each item (at his/her evaluation)

Child	Disagreement Payoff	Share of Excess	Total
1	4,433	2,022	6,455
2	2,833	2,022	4,855
3	4,667	2.022	6,689

Sum of 1/3 of each item = 11,033

→ **EXCESS = 6,067**

Pareto Optimal Sum = 18,000

Steinhaus

Player	Allocated to:	Worth	Side Payments	Total
1	A	10,000	-3,544	6,455
2	D	2,000	+2,855	4,855
3	B & C	6,000	+689	6,689

18,000 **0** **18,000**

Vickery Auction

High Bidder Wins at 2nd Highest Price

Player	Allocated to:	Worth	Side Payments	Total
1	A	10,000		
2	D	2,000		
3	B & C	6,000		

Vickery Auction Side Payments

Child	Auction Payment	Share of Receipts	Side Payment
1	7,000	3,833	-3,167
2	1,000	3,833	+2,833
3	3,500	3,833	+333

Sum =11,500

1/3 of 11,500 to Each

Vickery Auction

- Engenders HONESTY!

- Does not pay to distort values assigned to individual items

	NAIVE		STEINHAUS		VICKERY	
	Side Payment	Total	Side Payment	Total	Side Payment	Total
1	-4,000	6,000	-3,545	6,455	-3,147	6,833
2	+4000	6,000	+2,855	4,855	+2,833	4,833
3	0	6,000	+689	6,689	+333	6,333

PROBLEM!

- None of these schemes DIRECTLY take into account individual (artistic) preferences of participants—only monetary values

Nash-Raiffa Arbitration Scheme

Informal Summary

Nash Theorem

$\mathbf{x} = \text{ } A\ contract\ \ or\ negotiation\ alternative$

$\mathbf{X} = \{\mathbf{x}_1, \ldots, \mathbf{x}_N\},\ Set\ of\ all\ possible\ alternatives$

$U_Y(\mathbf{x}) = \ Utility\ to\ you\ of\ alternative\ \mathbf{x}$

$U_M(\mathbf{x}) = Utility\ to\ me\ of\ alternative\ \mathbf{x}$

Assumptions

- **Utility Invariance**:
  – If two versions of the same bargaining problem differ only in units (scale) and origins of participants' utility functions then arbitrated solutions are related by the same utility transformations

- **Pareto Optimality**:
  - Given an arbitrated solution, there exists no other arbitrated solution for which both parties are better off

- **Symmetry**
  - If an abstract version of the game places participants in completely symmetric roles, the arbitrated value will give them equal utility payoffs

- **Independence of Irrelevant Alternatives**
  - Suppose two games have the same status quo (BATNA) points and that the trading possibilities of one are included in the other.
  - If the arbitrated solution of the game with the larger set of alternatives is a feasible trade in the game with the smaller set of alternatives then it is also the arbitrated solution of the latter.

Nash-Raiffa Theorem

- The “allocation” scheme that satisfies the four assumptions stated is unique.

- The unique solution is found as follows:

Nash Arbitration Scheme

$\overline{U}_Y = Utility\ of\ "No\ Deal" -> The\ Status\ Quo\ for\ you.$

$\overline{U}_M = Utility\ of\ "No\ Deal" -> The\ Status\ Quo\ for\ me.$

$\mathbf{X} = The\ set\ of\ all\ feasible\ agreements$

$$Solution\ is\ \mathbf{x}^* \in \mathbf{X}\ satisfying$$

$$Max_{\mathbf{x} \in \mathbf{X}}\ [U_Y(\mathbf{x}) - \overline{U}_Y] \times [U_M(\mathbf{x}) - \overline{U}_M]$$

Fair Division of An Art Collection

The Rothman Art Collection

Agreed Upon Objectives

- Equal Fair Market Value
- Honest revelation of preferences
- Allow for emotional meaning attached to items
- Avoid strategic thinking
- Avoid post-decisional regret
- Take into account complementarity and substitutability

Protocol

1) **Explain** the process

2) **Compose** a list

3) **Split** list into manageable size categories

4) **Present** a few categories at once. Ask parties to state their preferences in any way that is comfortable:

   Encourage them to "star" important items, rank items, give opinions about trade-offs

5) **Follow up** statements about preferences.

Ask questions in a way that provides information without encouraging misrepresentation.

6) **Keep all information presented to you *strictly confidential.***

7) **Construct** a preliminary allocation in which items are distributed such that:

- ***All parties do about equally well on their own subjective scales and***

- ***Fair market values of the participant's allocations are roughly equal***

Key to Success: Differences in Relative Preferences

- Give each brother **more than he expected while treating each equally in $ allocated.**
- A random division by flipping a coin doesn’t necessarily yield equal value:
  - One brother receives 1st, 3rd, 5th, 7th…
  - The other receives 2nd, 4th, 6th, 8th,….
- If Lorin goes first he gets $39K and Paul gets $31K.

Artist Selda Gund

This allocation is described in the paragraphs below Table 1 in the Rothman Art Collection case

		($1,000)							($1,000)
		Fair Market	**Rank**		**Allocation**		**Market Values**		**Fair Market**
Item	**Description**	**Value**	**Lorin**	**Paul**	**Lorin**	**Paul**	**Lorin**	**Paul**	**Value**
1	Brown Bear	$10	2	2	0	1	$0	$10	$10
2	Lion	$9	1	6	1	0	$9	$0	$9
3	Pig	$8	12	8	0	1	$0	$8	$8
4	Monkey	$6	13	7	0	1	$0	$6	$6
5	Polar Bear	$6	5	9	1	0	$6	$0	$6
6	Rabbit	$6	9	12	1	0	$6	$0	$6
7	Turtle	$6	10	11	0	1	$0	$6	$6
8	Robin	$2	8	13	0	1	$0	$2	$2
9	Small Bear	$2	7	3	0	1	$0	$2	$2
10	Swallow	$9	3	10	1	0	$9	$0	$9
11	Turkey	$3	6	5	1	0	$3	$0	$3
12	Dog	$2	4	4	1	0	$2	$0	$2
13	Cat	$1	11	1	0	1	$0	$1	$1
					Market Value to Each:		**$35**	**$35**	**$70**

I **Lorin ranks Lion 1, Paul 6: -→ Lion to Lorin**

II **Paul ranks Cat 1, Lorin 11: Assign Cat to Paul**

III **Paul states that he likes Cat and Brown Bear much more than the other 11 paintings .Give Paul Brown Bear**

IV **Assign Pig, Monkey, Turtle, Robin & Small Bear to Paul**

V **Assign Polar Bear, Rabbit, Swallow, Turkey, & Dog to Lorin**

Lorin gets his 1st,3rd, 4th,5th,6th and 9th ranked painting **5 of top 6**

Paul gets his 1st,2nd,3rd,7th,8th,11th and 13th ranked painting **5 of top 8**

Typical Problems

- Items are really discrete and opinions may be "lumpy".
- Both may have similar rankings of preferences.
- Confusing signals: Paul ranked all items in a group but starred some of them. Some starred items were ranked *below* unstarred items.

- A star next to an item low in ranking signals that it was worth, to the evaluator, far more than its associated fair market value.

- **Complementarity**: One brother may insist that a “block” of painting not be split up while the other won’t accept that the entire block go to one party.

Diffusing Attention from a Single Painting

- Ask for comparisons of four or five groups of paintings with the disputed painting among some of these groups.

Strategic Misrepresentation

- This system is predicated on each brother having complete information about his own preferences, but only probabilistic information about the other brother.
- With only an impressionistic understanding of the other brother's preferences, distorting your own to gain advantage MAY BACKFIRE!

The Potential for Strategic Misrepresentation Limits the Usefulness of Joint Fact Finding

- Mediators often ask parties to discuss issues face to face to attain convergence of beliefs.
- A skilled analyst can exploit this to her advantage, because she will learn about the preferences of her counterpart.
- This would destroy differences in preferences which we use to generate joint gains.

Sense of Loss

- Even when a brother received more than he expected, he wasn't overly enthusiastic.
- Emotional attachment engendered a sense of what was lost.
- Reduce expectations with "…this was an extremely difficult category to divide…but we did the best we could…"

Gaining Closure

- Get the brothers to sign off on the first half of the estate before moving on.
- This will smooth discussion of the second half if the brothers are pleased with allocations on the first half.

Concerns about “Finality”

- Suggest at the outset that we might meet in a year to discuss trades, a ‘Post-Settlement Settlement’.

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