

15.301/310, Managerial Psychology
Prof. Dan Ariely
Recitation 3: Probability

(a dash “-“ indicates student responses)

Probability: An introduction

Today we talk about probability theory

It is the first step in understanding statistics.

Probability theory – a systematic way to think about outcomes in a stochastic world

What do we know about probability?

-probabilities are not certain

Why is the world not certain?

If you go to someone in the morning, you cannot be 100% sure what they will say back.

Why not? Why do people react in different ways to different things?

-because they have the choice

If we have a pile of bricks and we kick them, they always do the same thing

If we have people and we kick them.... They react differently

1. The world is really probabilistic
2. We might not be able to take everything into account.

If could write an equation with everything included, the world might be deterministic

But as we see it, it's probabilistic.

We use intuition all the time

When you study for exams, you succeed or not

It is not always clear what will happen

What is the probability of a newborn being a male? ~50%

HH in coin toss .25

Get a 6 on a die? 1/6

Choosing the one blue ball out of a bag of 5 balls 1/5, one of the others, 4/5

Probability and Frequency – Binomial distributions

Systems with 2 possible outcomes

Like:

Die or stay alive

Sick or healthy

Yes or no

You don't always have to ask about one event.
What is the probability that out of 100 kids, 17 will be girls, and the rest boys...
Or first x girls, then y boys, then z girls, etc....

Possible outcomes of tossing 2 coins: HH HT TT
What are the probabilities of getting each?
(.25 .5 .25)

Think of 3 coins
What are the possible outcomes?
(HHH HTT HHT TTT)

Please write down the probabilities for each of these events.
($1/8$ $3/8$ $3/8$ $1/8$)

For 6 coins? (if we don't care about order?)
How many outcomes? (7)
Please write down the probability for each of these 7.

HHHHHH THHHHH TTHHHH TTTHHH TTTTHH TTTTTH TTTTTT
— — — — — — — —

How do you do it?

-the easiest way is the binomial solution

Suppose we want to enumerate it, how do we do it?
There are 2 components if we calculate by hand: how many possible outcomes? How many times would they occur? Multiply the outcomes by how many times they occur. We just have to figure out all the combinations.

It gets harder and harder for larger amounts. We have to come up with something better.

Look at these 3 distributions (see slide). What can you learn from the differences of these 3?

- more trials you get, the more it looks like a bell shaped curve
- symmetrical
- more possibilities gives a lower probability overall (for individual events)

If I threw an infinite number of coins, what is the probability that I will have half and half, heads and tails?

-Exactly 0. If there are infinite coins, you get a slimming bar corresponding to 50/50, slims to 0 as the coins go to infinity

The probability of any particular event goes to zero because you divide by infinity

If we throw 2 times or 6 times, which is the better chance of getting a 50/50 distribution?

-2 times. There are more possibilities for outcomes if we toss 6 times. Fewer trials means more width devoted to each option.

When we toss 2 coins, the probability of getting 50/50 is .5. At 6 coins, it is .3-something
It is counterintuitive to get farther away as we toss more coins.

We would expect that the more coins we throw, the closer we would get to 50/50.

How do we solve this problem?

-going discrete to continuous

It is important to focus on the range, not a specific value.

Probability of .5 boys is less the bigger the hospital is.

We need to take the probability .5 or higher to get higher precision

Or if we define the distribution from .5, define a range, a width, an interval.

Other binomial distributions

Questions about Class

We had some questions from students

The basic question – what exactly is this class? And there were many related questions.

Let me give general answer:

This class is technically known as a mish mash.

Over the years it became communication intensive – the writing, presentations.

And there is also the lab part – we cover statistics, etc.

Then the rest is managerial psychology

So how do you cover managerial psychology in 8, 9, 10 lectures?

I could have chosen a topic, lets say labor, and concentrated on that.

But your path is not clear to me, probably not to you either.

So we are doing an overview and I will give you an idea of how to apply what you are learning to management

Lets talk about the lecture on perception – how does that relate to management?

For example, Adaptation – how does adaptation link to salaries? If you have a pay level, you get used to it, and you are happy if there is a change and you get a raise.

Think about the basic principles and apply them to different scenarios

I don't want to tell you how to apply them to what is interesting to you.

Start with perception and memory, go from there

We will get more and more applied as we go through the topics.

Relativity of perception is one of the central themes.

Addressing concerns about exams:

We will give you practice questions before the exam.

Basically, you have to do the reading.

Either think about measurement or managerial psychology

Take the alien abduction article last week – what mechanisms in the paper caused people to think they were abducted? What is the main message? We are not going to ask when the article was published, etc.

Are there other questions about the structure of the class? No?

Fabulous.

Now we will talk about **binomial distribution**

Lets look at a 60/40 distribution.

(Slide)

There is a probability of .6 that people will die in a situation, what is the likelihood 10/10 survive?

-They are independent events, multiply them. It doesn't make sense to add, subtract, divide. Multiplication is the only operation I could think of that made sense.

Multiply probabilities of each because they are independent -> $.4^{10}$

What is the likelihood 9/10 will survive?

- $.4^9$ times .6

Take the probability of 9 people surviving, and multiply by .6, is that right?

We could also read it as 9/10 or 10/10 survive – at least 9 people survive.

What do we do between the two?

-add them

Why?

-It makes sense☺

Probability – the first 9 live and last one dies, could be other combinations? How many?

-10

We need to take that into account.

Is it about a number or an interval?

Example: If we form committees in this class, how many different committees of 5 people can you have from a group of 6?

-6.

Yes, kick each person out once.

You can also think of how many committees can be formed with 1 person – it is the same thing

How many committees of 8 can come from a group of 10?

-45

That is a bit more painful.

As the numbers get larger, we need more systematic way to calculate these things.

(slide)

$\binom{n}{r}$

$n!/[r!(n-r)!]$

Please calculate the number of combinations of 8/10 people

-45

Basically we do what?

$10 \times 9 \times \dots \times 8 \times 7 \dots \times 2$

Enumerate all the combinations.

A slightly more complex problem:

At least 7/10 people survive. What do we do?

Treat the problem as 4 separate events.

Multiply .4 for each survivor x .6 for those that die.

1. Find the possible combinations
2. Multiply, then add across.

To summarize binomial distribution:

Counting is all we need, we have a shortcut for counting, we can do lots of things with binomial distributions

Historical excellence: Fischer decided that if the probability is $< .5$, it is significant

Let us say the probability that you have 17 boys born one after other is .000001, we have to decide, is it by chance that this happened? Or is it not by chance?

We have a roulette wheel. The results are 600 black and 400 red. We can calculate the probability of this happening. Then we have to ask the question – is the wheel faulty or not? Do we force the casino to change wheel or not?

Fischer did agricultural experiments

More water, less light, etc.

Probability that a condition yields more growth is .2

Is this our current understanding of the truth or not? How do we act on it?

Fischer picked a number. If the probability of getting a result is less than .05, the event is not chance.

-Didn't you say the probability of one event was almost 0?

What was I not precise about?

-Distribution

Yes, the probability of getting 200 reds vs. the probability of getting the first 200 reds.

Now we will calculate probability in a more sensible way rather than calculating all of the possibilities.

This will give us a range very easily

6 coins are tossed. Find P(H) and P(T)

If half the time we get heads, .5 times q.

For the first measurement we take, the mean is $n \cdot p$

Next, calculate other things: variance, standard deviation, z score

Variance – how wide is the distribution?

Standard deviation – square root of the variance

Z score = $(r - \mu) / \sigma$

Standard deviation = σ

Variance = $\sigma^2 = n \cdot p \cdot q$

Z score gives the probability of event r.

What have you experienced that uses these measures?

-Tests at MIT

Before MIT?

-SAT

Not exactly the z score, but it has a known mean, a known standard deviation.

A score of 800 is 3 standard deviations above the mean.

Please calculate the z score of getting 6 heads.

If you have it, please raise your hand.

I'll give you a few more minutes.

-

(slide)

R is our observation.

How many heads would you expect? How many do you get? How different are those things?

N - # of times throwing a coin

P and Q are outside of what you observe – your belief about what it is.

For a coin $\rightarrow P = .5$ for each side (Heads, Tails)

With this z score, is this a fair coin?

If this was SAT, what would be the score of this?

How likely is it that the coin is a good coin with a real probability of .5? How do we transform z score to probability?

Hint – we have a paper that tells you. This table was the rave before computers. And it is still useful.

There is a table for negative and positive values. It is photocopied from your book.

If you find 2.4, to over to .05, it gives the probability at 2.45.

Is the coin fare or not?

You have to decide right now. Would you gamble against it? Would you gamble on tails?

Often people will say something to you. But if you get people to act, it reveals their true beliefs.

If 44/50 infants born at MGH yesterday were female, what could you argue?

What probability would you give this event?

Would you argue that the probability of a female being born at MGH is above .5?

-Do we assume $P = .5$ for females born normally? Isn't that circular? We are concluding that the original assumption is wrong.

Is something odd at MGH? Maybe the people, the water, the atmosphere is different for some reason.

Imagine they were above average intelligence instead of female, should we investigate?

The event doesn't matter, take a particular event – is this an oddity, should we investigate, or did it happen by chance?

Have you solved it, anyone?

-5.38

Everything above what number will be significant in Fischer's terms?

-1.6

1.6 what?

1.65 for the example is very close to 5 percent (see table).

Every time you see something around that, you can take notice

-We can't quantize how likely that is fluke vs. what is not

Basically we say: is this from a distribution that is close to 50 percent?

It is a very tricky question. We will come back to it.

Example:

10/12 times, a roulette wheel landed on red. What could you argue, what probability would you give to this event? Would you argue that the probability of the roulette wheel landing on red is above 50 percent?

Solve this at home.

Last time on the test, without practice, people didn't get this. This time we will test with practice and see.

-can you work backwards to find out which is better?

There is no way to take a z and say "this is the right probability." You can say, given this observation, what is the range in which I expect this z to fall? You can't find out the specific interval

-the range is based on .05, right?

Yes.

-Can p and q be less than 1?

Together? No. The sum has to be 1.

(slide)

Linda is vegetarian, wears sandals even in winter, and is active in a women's literary society. Her hair usually needs a trim.

What are the probabilities that:

Linda bank teller

Linda is a feminist bank teller

Write these probabilities.

Many people write a higher probability for feminist bank teller.
Linda has to be more likely a bank teller than a feminist bank teller.
It is a conjunctive fallacy.
Both have to occur for the second option to be true

Conjunctive - all overlapping sections happen at the same time
Disjunctive - one of the overlapping sections happens at a time

(Slides) disjunctive questions
Can we answer this? No
What if I say the clean students are always on time?
Sorry, if all those on time are also clean.
Basically we need to think about the dependency.

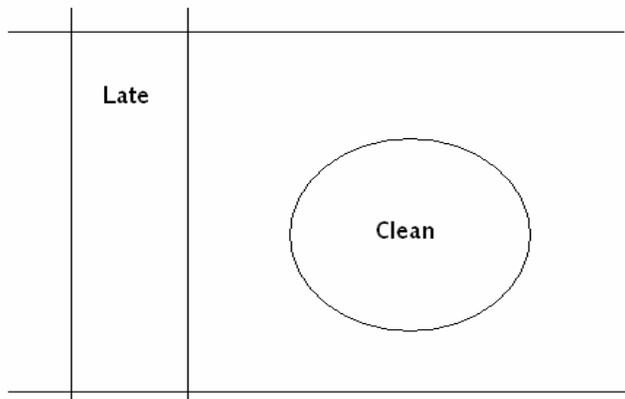
Conjunctive: AND

Example: Our famous bag (slide)
Likelihood of 2 red balls

-with replacement?

Yes.
Basically multiply probabilities

Take those same late and clean students –



probability(tardy and clean student)?
0 in current picture

If 10% are late and 25% shower at least once a week,
How many are tardy or clean? In this situation, 35%

Say all of the late people also showered at least once a week.
 $P(\text{late or unclean}) = 75\%$

Disjunctive: OR
Important to know overlap

If we don't know, multiply probabilities
AND – joint probability, the overlap.
It is important to understand the dependency. It is easy to calculate if they are independent, hard if they are not.

Central Limit Theorem

It is about the behavior of real data
The more samples we take, the closer it gets to the distribution.

This is surprising. No matter what distribution we start from, take average

Example – uniform dist 1-8 (slides)

Even though started from a uniform distribution, distribution of samples will resemble the uniform distribution at the end of the day.

Probability is the basis for stats
Binomial is the easiest way to look at the probability of events
Central limit theorem