Real Options

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Finance Theory II
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Real options

- Managers have many options to adapt and revise decisions in response to unexpected developments.

- Such flexibility is clearly valuable and should be accounted for in the valuation of a project or firm.

Real options, cont.

Imbedded options

- Follow-up investments
- Option to abandon the project
- Option to wait before investing
- Option to expand / change production methods

Key elements

- Information will arrive in the future
- Decisions can be made after receiving this information

Our plan

Last class

- Real options: basic intuition
- Simple DCF analysis of real options (decision trees)

Today

- Review of option pricing
  - Why doesn't simple DCF work quite well?
- Identifying real options
- Valuing real options using Black Scholes

1. Review of option pricing

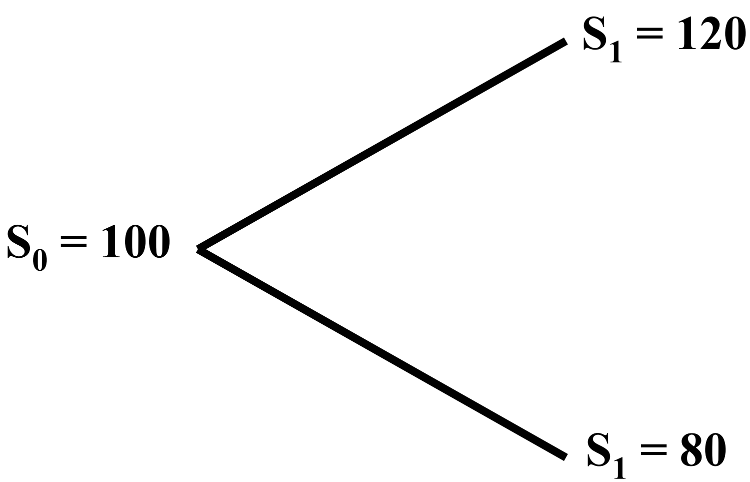
Real options and financial options

<u>Option Definition</u> The *right* (but not the obligation), to buy/sell an underlying asset at a price (the exercise price) that <u>may be</u> different than the market price.

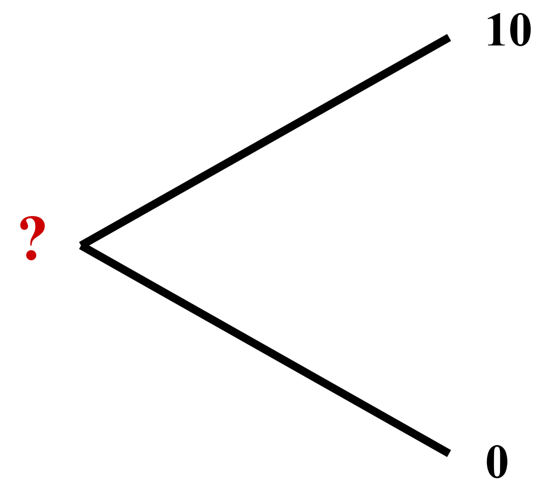
Financial Options	**Vs.**	**Real Options:**
Options on stocks, stock indices, foreign exchange, gold, silver, wheat, etc.		Not traded on exchange. Underlying asset is something other than a security

Pricing of a call option on stock

Stock



Option (X = 110)

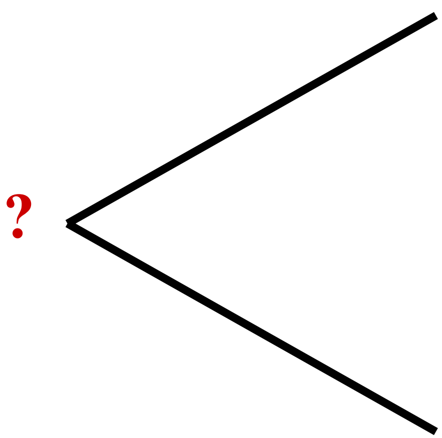


The challenge is to find the value of the call option today

Pricing of a call option on stock

- Consider the following strategy:
  - Borrow money (or sell a bond with face value of B)
  - Buy a N shares of stock

- Choose N and B so that the payoffs from the portfolio = option payoffs



$$10 = -B + N*120$$

$$0 = -B + N*80$$

$$B = 20$$
$$N = 0.25$$

Pricing of a call option on stock

- Our stock / bond portfolio has exactly the ***same payoff*** as the option
  - So, the option and the portfolio must have the ***same value today***
  - Otherwise: arbitrage opportunity

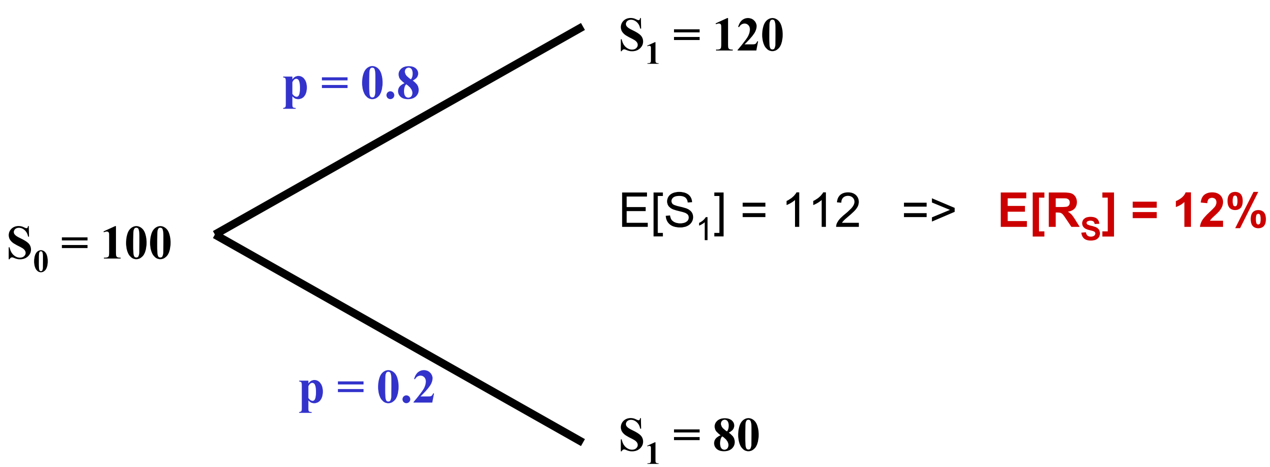
- What is the value of the portfolio today (assume risk-free rate =4%)?

$$-B / (1+r) + N * S_0 \quad = \quad -20 / 1.04 + 0.25 * 100 \quad = \quad 5.77$$

- We just priced the option. **Option value = $5.77**.

Why standard DCF doesn't work very well?

- Let's value our option using standard DCF
  - What discount rate should we use?
  - Let's try the required return on stock $E[R_S]$



Why standard DCF doesn't work very well?

- DCF gives us the following option value:

$$(0.8 * 10 + 0.2 * 0) / 1.12 = \mathbf{\$7.14} \neq \mathbf{\$5.77}$$

What's wrong?

- Discount rate of 12% is too low => the option is riskier than the underlying stock

Why?

- Option is a levered position in a stock.
  - Recall the analogy with firms' financial leverage: Higher financial leverage => higher equity betas and equity returns.

Option is a levered position in a stock

- Recall our replicating portfolio: Borrow B/(1+r) and buy N shares of stock
  - Suppose that stock beta = 1 and market premium = 8%
    - Note that this works. CAPM: 12% = 4% + 1 * 8%
- What is option beta?

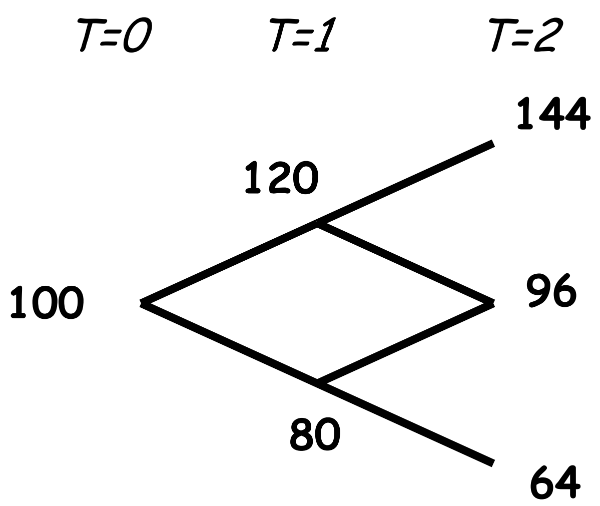
$$\beta_{option} = \underbrace{w_{bond} * \beta_{bond}}_{0} + \underbrace{w_{stock} * \beta_{stock}}_{25/5.77 * 1} = \mathbf{4.33}$$

- So, the required return on the option is 38% = 4% + 4.33 * 8%
- And the option value is again: **$5.77** = 8 / 1.38

How about multiple periods?

- In principle, we can value the option the same way as before
  - Start at time T=2 and move backwards

- But several things change at each node:
  - Replicating portfolio, option beta, discount rate



- This can become quite tedious
- That's where option pricing models such as ***Black-Scholes*** come in.

Options valuation techniques

- "Dynamic" DCF (decision trees)
  - Recall our "Handheld PC" and "Copper Mine" examples
  - *Approximation* used for real-options problems
  - Not an exact answer because of ***problems with discounting***

- Binomial model
  - Similar to our one-period example from today's class
  - Requires more computations than Black-Scholes
  - Can be useful when Black-Scholes doesn't work very well

- Black-Scholes
  - We will focus on this model from now on

Black-Scholes formula

- Black-Scholes formula relies on the same valuation principles as the binomial model (replicating portfolios, no arbitrage)

$$\text{Option value} = N(d_1) * S – N(d_2) * PV(X)$$

- Note the similarities to the one-period binomial model

$$\text{Option value} = N * S – PV(B)$$

N(d): Cumulative normal probability density function
$d_1 = \ln[S/PV(X)] / (\sigma T^{1/2}) + (\sigma T^{1/2})/2$ $d_2 = d_1 - (\sigma T^{1/2})$
S = Current stock price X = Exercise price
r = Risk-free interest rate T = Time to maturity in years.
σ = Standard deviation of stock return

Recall “Handheld PC” example

Model B value in t=0 ?

Model A turns out well => good news for Model B

Model B NPV (t=3) **= 16,557**
⇨ go for it

Model A turns out badly => bad news for Model B

Model B NPV (t=3) = **-21,148**
⇨ drop it

	Model B value at t=0	
"Static" NPV	$[(16{,}557 - 21{,}148)/2]/1.12^3$	-1,634
"Dynamic" NPV	$[(16{,}557 + 0)/2]/1.12^3$	5,892
Option pricing	$[(16{,}557 + 0)/2]/???^3$	???

2. Identifying Real Options

Two Issues with Real Options

Identification

- Are there real options imbedded in this project?
- What type of options?

Valuation

- How do we value options?
- How do we value different types of options?
- Can’t we just use NPV?

Identifying Real Options

- It is important to identify the options imbedded in a project.
- There are options imbedded in all but the most trivial projects.
- All the art consists in:
  - Identifying those that are "significant", if any
  - Ignoring those that are not
- Identifying real options takes practice, and sometimes "vision".

Identifying Real Options (cont.)

- Look for clues in the project's description: "Phases", "Strategic investment", "Scenarios", …
- Examine the pattern of cash flows and expenditures over time. For instance, large expenditures are likely to be discretionary.
- Taxonomy of frequently encountered options :
  - Growth option
  - Abandonment option
  - Option to expand or contract scale
  - Timing
  - Option to switch (inputs, outputs, processes, etc.)

Is There An Option?

- **Two conditions:**

  (1) News will possibly arrive in the future;

  (2) When it arrives, the news may affect decisions.

- Search for the uncertainty that managers face:
  - What is the main thing that managers will learn over time?
  - How will they exploit that information?

Oz Toys' Expansion Program

- Oz Toys' management is considering building a new plant to exploit innovations in process technology.
- About three years out, the plant's capacity may be expanded to allow Oz Toys' entry into two new markets.

	2000	2001	2002	2003	2004	2005	2006
EBIT * (1-t)		2.2	4.0	-10.0	11.5	13.7	17.4
Depreciation		19.0	21.0	21.0	46.3	48.1	50.0
CAPX	120.0	8.1	9.5	307.0	16.0	16.3	17.0
ΔNWC	25.0	4.1	5.5	75.0	7.1	8.0	9.7
FCF	-145.0	9.0	10.0	-371.0	34.7	37.5	40.7
TV							610.5
NPV (WACC=12%)	**-19.8**						

Oz Toys: Is There An Option?

(1) Oz Toys might learn (or not) about:

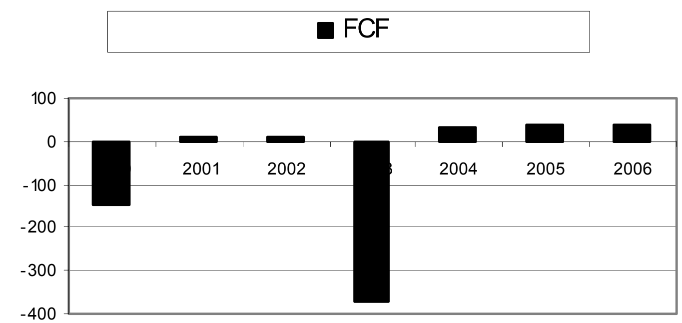
- The demand for the current and/or new products
- The possibility of rivals entering the market
- Etc.

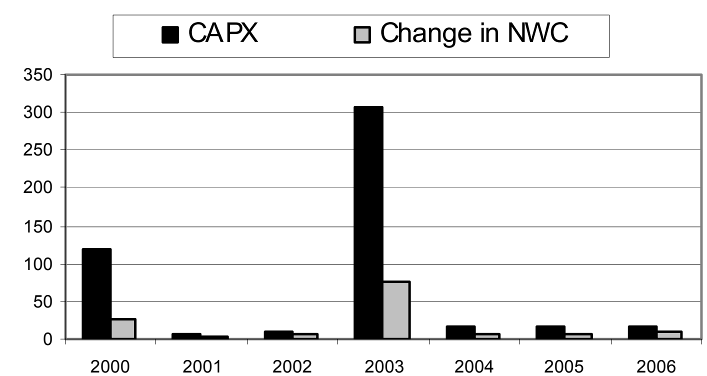
(2) The information might affect (or not) Oz Toys' decision:

- Whether or not to undertake expansion phase 1 at all
- Whether to undertake phase 2 (or even phase 3,…)
- Whether to push one new product or the other
- Etc.

Oz Toys: Identifying the Option

- Project's description refers to two distinct phases
  - Phase 1: New plant
  - Phase 2: Expansion
- Spike in spending: Probably discretionary
- Possibly, an imbedded growth option





Practical Issue #1: Simplifications

- Real projects, especially long-horizon ones, are complex:
  - They often combine assets-in-place and options.
  - Options are nested.

- Simplifying assumptions are needed:
  - To allow the technical valuation analysis
  - To keep the model flexible
  - To keep the model understandable to you and others (especially others involved in the decision process)

Practical Issue #1: Simplifications (cont.)

- Cut the project into pieces corresponding to simple options.
- Search for the **primary** uncertainty that managers face
- A simplified model that dominates (is dominated) by the project gives an upper (a lower) bound for the project's value, e.g.,:
  - Using European rather than American options
  - Ignoring some of the options
  - Ignoring some adverse effects of waiting (e.g. possible entry)

Oz Toys: Simplifications

- Value phase 1 and phase 2 separately.
- Focus on the option to undertake expansion phase 2 or not.
  - Assume all other options are “negligible”
- Assume that phase 2 is to be undertaken in 2003 or never.

⇨ European Call option

3. Valuing Real Options

Valuation of Real Options

- Tools developed to value financial options can be useful to estimate the value of real options embedded in some projects.
- Real options are much more complex than financial options.
- The aim here is to develop numerical techniques to “keep score” and assist in the decision-making process, not provide a recipe to replace sound business sense.

Options vs. DCF

- The real options approach is often presented as an alternative to DCF.
- In fact, the real options' approach does not contradict DCF: It is a particular form that DCF takes for certain types of investments.
- Recall that option valuation techniques were developed ***because discounting is difficult***
  - I.e., due to the option, one should not use the same discount rate (e.g. WACC) for all cash flows.

Options vs. DCF (cont.)

- DCF method:
  - “expected scenario” of cash flows,
  - discount the expected cash flows
- This is perfectly fine as long as:
  - expected cash flows are estimated properly
  - discount rates are estimated properly
- Precisely, it is complex to account for options in estimating:
  - expected cash flow
  - discount rates

Start with the "static" DCF Analysis

- Begin by valuing the project as if there was no option involved
  - Pretend that the investment decision must be taken ***immediately***.
- This benchmark constitute a ***lower bound*** for the project's value.
  - NPV<0 does not mean that you will never want to undertake the investment.
  - NPV>0 does not mean that you should go ahead immediately with the invest (nor that you will definitely invest in the future).

Oz Toys: DCF Analysis

- Disentangling the two phases.
- Requires making judgments about:
  - Which expenses are discretionary vs. non-discretionary
  - Which cash inflows/outflows are associated with each phase
- Note: Sometimes, simply retrieve disaggregated data used to construct the summary DCF analysis.

Oz Toys: Valuing Phases 1 and 2

	2000	2001	2002	2003	2004	2005	2006
Phase 1							
Cash flow		9.0	10.0	11.0	11.6	12.1	12.7
Investment	145.0						
TV (5% growing perpetuity)							191.0
NPV (WACC=12%)	**-3.7**						
Phase 2							
Cash flow					23.2	25.4	28.0
Investment				382.0			
TV (5% growing perpetuity)							419.5
NPV (WACC=12%)	**-16.1**						
Total							
Cash flow		9.0	10.0	11.0	34.7	37.5	40.7
Investment	145.0			382.0			
TV							610.5
NPV (WACC=12%)	**-19.8**						

Oz Toys: DCF Analysis (cont.)

- Both phases have negative NPV
- Phase 2's NPV is probably largely overstated:
  - Investment ($382M) is likely to be less risky than cash flows.
  - Using the three-year risk-free rate of 5.5%

DCF Analysis of Phase 2 Discounting the Investment at 5.5%							
	2000	2001	2002	2003	2004	2005	2006
Phase 2							
Cash flow					23.2	25.4	28.0
Investment				382.0			
TV (5% growing perpetuity)							419.5
NPV (WACC=12%)	**-69.5**						

Valuing the Option

- The strategy is to map the project into a simple option and use financial valuation tools to price the option: Black-Scholes formula.

- Oftentimes, this involves making somewhat heroic assumptions about the project.

Mapping: Project → Call Option

Project		Call Option
Expenditure required to acquire the assets	X	Exercise price
Value of the operating assets to be acquired	S	Stock price (price of the underlying asset)
Length of time the decision may be deferred	T	Time to expiration
Riskiness of the operating assets	σ^2	Variance of stock return
Time value of money	r	Risk-free rate of return

Oz Toys: The 5 Variables

X	Investment needed in 2003 to obtain the phase 2 assets.	$382M
S	PV of phase 2’s cash flows.	$255.8
T	It seems that phase 2 can be deferred for 3 years (Check with managers).	3 years
r	3-year risk-free rate (Check yield curve).	5.5%
σ^2	Variance per year on phase 2 assets. Can’t get it from DCF spreadsheet.	Say 40%

Phase 2	2000	2001	2002	2003	2004	2005	2006
Cash flow					23.2	25.4	28.0
TV							419.5
PV (WACC=12%)	**255.8**						

Practical Issue #2: What Volatility?

- Volatility (σ) cannot be looked up in a table or newspaper.
  - Note: Even a rough estimate of σ can be useful, e.g., to decide whether to even bother considering the option value.

1. Take an informed guess:

- Systematic and total risks are correlated: High β projects tend to have a higher σ.

- The volatility of a diversified portfolio within that class of assets is a lower bound.

- 20-30% per year is not remarkably high for a single project.

Practical Issue #2: What Volatility? (cont.)

2. Data:

- For some industries, historical data on investment returns.

- Implied volatilities can be computed from quoted option prices for many traded stocks
  - Note: These data need adjustment because equity returns being levered, they are more volatile than the underlying assets.

Practical Issue #2: What Volatility? (cont.)

3. Simulation:

- Step 1: Build a spread-sheet based (simplified) model of the project's future cash flows
  - Model how CFs depend on specific items (e.g. commodity prices, interest and exchange rates, etc.)
- Step 2: Use Monte Carlo simulation to simulate a probability distribution for the project's returns and infer σ.

Black-Scholes Formula

- Two numbers suffice:

$$A = \frac{S \times (1+r)^T}{X} \quad \text{and} \quad B = \sigma \times \sqrt{T}$$

- A table that gives the Black-Scholes' call option value as a fraction of the stock price S (see handout)

Black-Scholes Formula: Columns: A Lines: B						
	0.60	0.65	0.70	0.75	0.80	0.86
0.50	5.1	6.6	8.2	10.0	11.8	14.2
0.55	6.6	8.3	10.0	11.9	13.8	16.1
0.60	8.3	10.1	11.9	13.8	15.8	18.1
0.65	10.0	11.9	13.8	15.8	17.8	20.1
0.70	11.9	13.8	15.8	17.8	19.8	22.1
0.75	13.7	15.8	17.8	19.8	21.8	24.1

Black-Scholes Formula (cont.)

- The number A captures phase 2's value if the decision could not be delayed (but investment and cash flows still began in 2003).

- Indeed, in that case, A would be phase 2's ***Profitability Index***:

$$PI = \frac{PV(cf)}{PV(inv.)} = \frac{S}{\left(\frac{X}{(1+r)^T}\right)} = A$$

and $A > 1 \Leftrightarrow NPV > 0$

- The option's value increases with A (as shown in the table).

Black-Scholes Formula (cont.)

- The number B, ***Cumulative Volatility***, is a measure of “how much S can change” between now and the decision time T.

- Intuitively, S can change more:
  - when S has more variance per year, i.e., σ is large
  - when there is more time for S to change, i.e., T is large

- B captures the value of being able to delay the decision.

Note: When B=0, only the project’s NPV matters (whether A>1) because either the decision has to be taken now (T=0) or it might just as well be taken now as no news will arrive (σ =0).

Oz Toys: Valuation

$$A = \frac{S \cdot (1+r)^T}{X} = \frac{255.8 \cdot (1.055)^3}{382} = 0.786 \qquad \text{and} \qquad B = \sigma \cdot \sqrt{T} = 0.4 \cdot \sqrt{3} = 0.693$$

Black-Scholes Formula: Columns: A Lines: B						
	0.60	0.65	0.70	0.75	0.80	0.86
0.50	5.1	6.6	8.2	10.0	11.8	14.2
0.55	6.6	8.3	10.0	11.9	13.8	16.1
0.60	8.3	10.1	11.9	13.8	15.8	18.1
0.65	10.0	11.9	13.8	15.8	17.8	20.1
0.70	11.9	13.8	15.8	**17.8**	**19.8**	22.1
0.75	13.7	15.8	17.8	19.8	21.8	24.1

- The value of phase 2 is (roughly): V_2 = 19% * S = .19 * 255.8 = **$48.6M**
- The value of the expansion program is: $V_1 + V_2$ = -3.7 + 48.6 = **$44.9M**

Practical Issue #3: Checking the Model

- Formal option pricing models make distributional assumptions.
- Approach 1: Try and find a model that is close to your idea of the real distribution (More and more are available).
- Approach 2: Determine the direction in which the model biases the analysis, and use the result as an upper or lower bound.
- Approach 3: Simulate the project as a complex decision tree and solve by brute force with a computer (i.e., not analytically).

Practical Issue #4: Interpretation

- Since we use simplified models, the results need to be taken with a grain of salt and interpreted.

- Put complexity back into the model with:
  - Sensitivity analysis
  - Conditioning and qualifying of inferences

- Iterative process.

- Helps you identify the main levers of the project, and where you need to gather more data or fine tune the analysis.