Risk and return (1)

Class 9
Financial Management, 15.414

Today

Risk and return

- Statistics review
- Introduction to stock price behavior

Reading

- Brealey and Myers, Chapter 7, p. 153 – 165

Road map

Part 1. Valuation

Part 2. Risk and return

Part 3. Financing and payout decisions

Cost of capital

DCF analysis

$$NPV = CF_0 + \frac{CF_1}{(1+r)} + \frac{CF_2}{(1+r)^2} + \frac{CF_3}{(1+r)^3} + \frac{CF_4}{(1+r)^4} + \frac{CF_5}{(1+r)^5} + \ldots$$

r = opportunity cost of capital
The discount rate equals the rate of return that investors demand on investments with comparable risk.

Questions

- How can we measure risk?
- How can we estimate the rate of return investors require for projects with this risk level?

Examples

- In November 1990, AT&T was considering an offer for NCR, the 5th largest U.S. computer maker. How can AT&T measure the risks of investing in NCR? What discount rate should AT&T use to evaluate the investment?

- From 1946 – 2000, small firms returned 17.8% and large firms returned 12.8% annually. From 1963 – 2000, stocks with high M/B ratios returned 13.8% and those with low M/B ratios returned 19.6%. What explains the differences? Are small firms with low M/B ratios riskier, or do the patterns indicate exploitable mispricing opportunities? How should the evidence affect firms' investment and financing choices?

Background

The stock market

- **Primary market**
  New securities sold directly to investors (via underwriters)
  Initial public offerings (IPOs)
  Seasoned equity offerings (SEOs)

- **Secondary market**
  Existing shares traded among investors
  Market makers ready to buy and sell (bid vs. ask price)
  Market vs. limit orders

  NYSE and Amex: Floor trading w/ specialists
  NASDAQ: Electronic market
  Combined: 7,022 firms, $11.6 trillion market cap (Dec 2002)

Background

Terminology

- **Realized return**

$$r_t = \frac{D_t + P_t - P_{t-1}}{P_{t-1}} = \frac{D_t}{P_{t-1}} + \frac{P_t - P_{t-1}}{P_{t-1}} \qquad \text{(DY + cap gain)}$$

- **Expected return** = best forecast at beginning of period

$$E[r_t] = \frac{E[D_t] + E[P_t - P_{t-1}]}{P_{t-1}}$$

- **Risk premium**, or expected excess return

Risk premium = $E[r_t] - r_f$

Statistics review

Random variable (x)

- **Population parameters**

mean = μ = E[x]

variance = σ^2 = E[(x – μ)^2], standard deviation = σ

skewness = E[(x – μ)^3] / σ^3

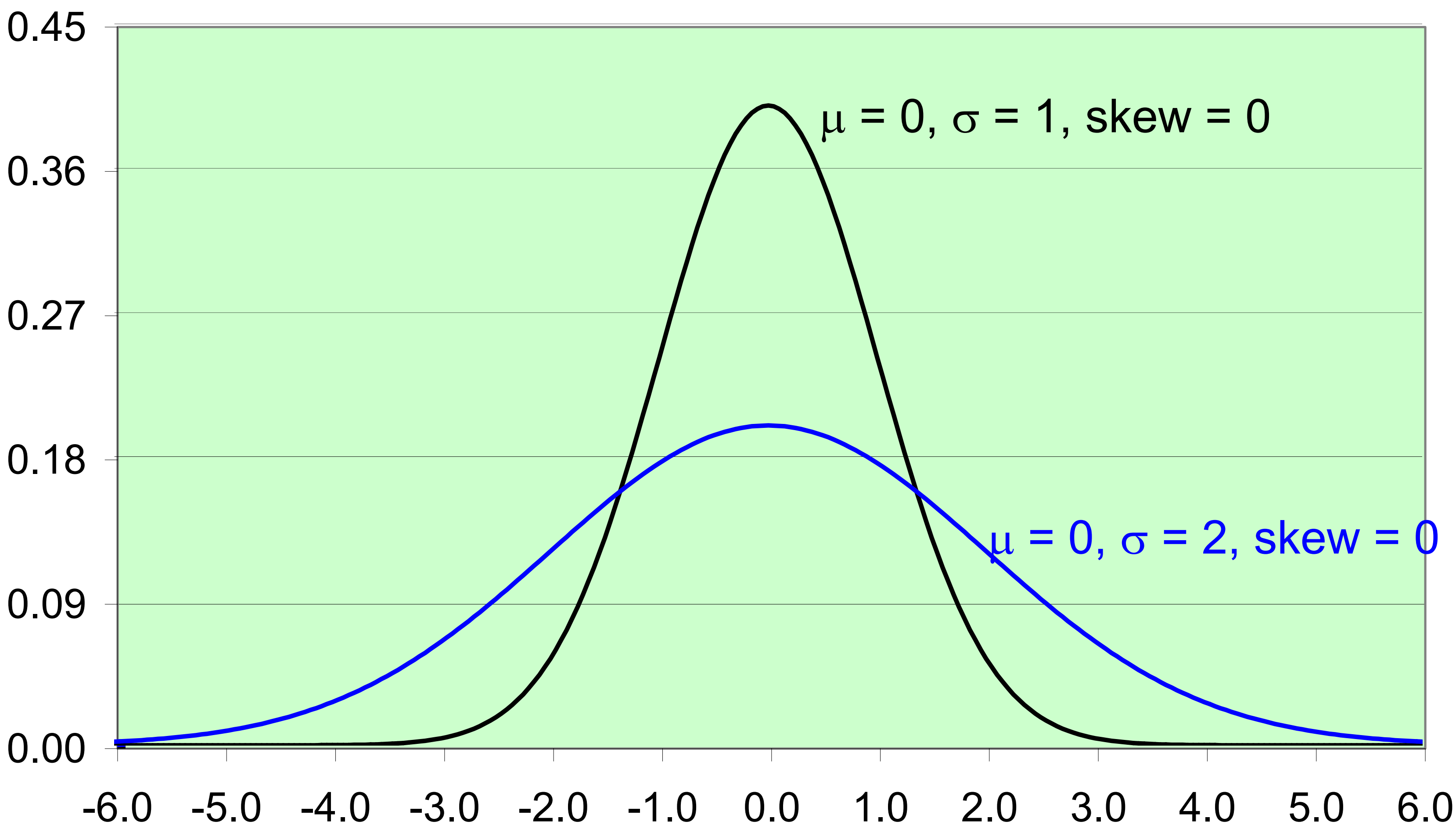
- **Sample of N observations**

sample mean = $\bar{x} = \frac{1}{N}\sum_i x_i$

sample variance = $s^2 = \frac{1}{N-1}\sum_i (x_i - \bar{x})^2$, standard deviation = s

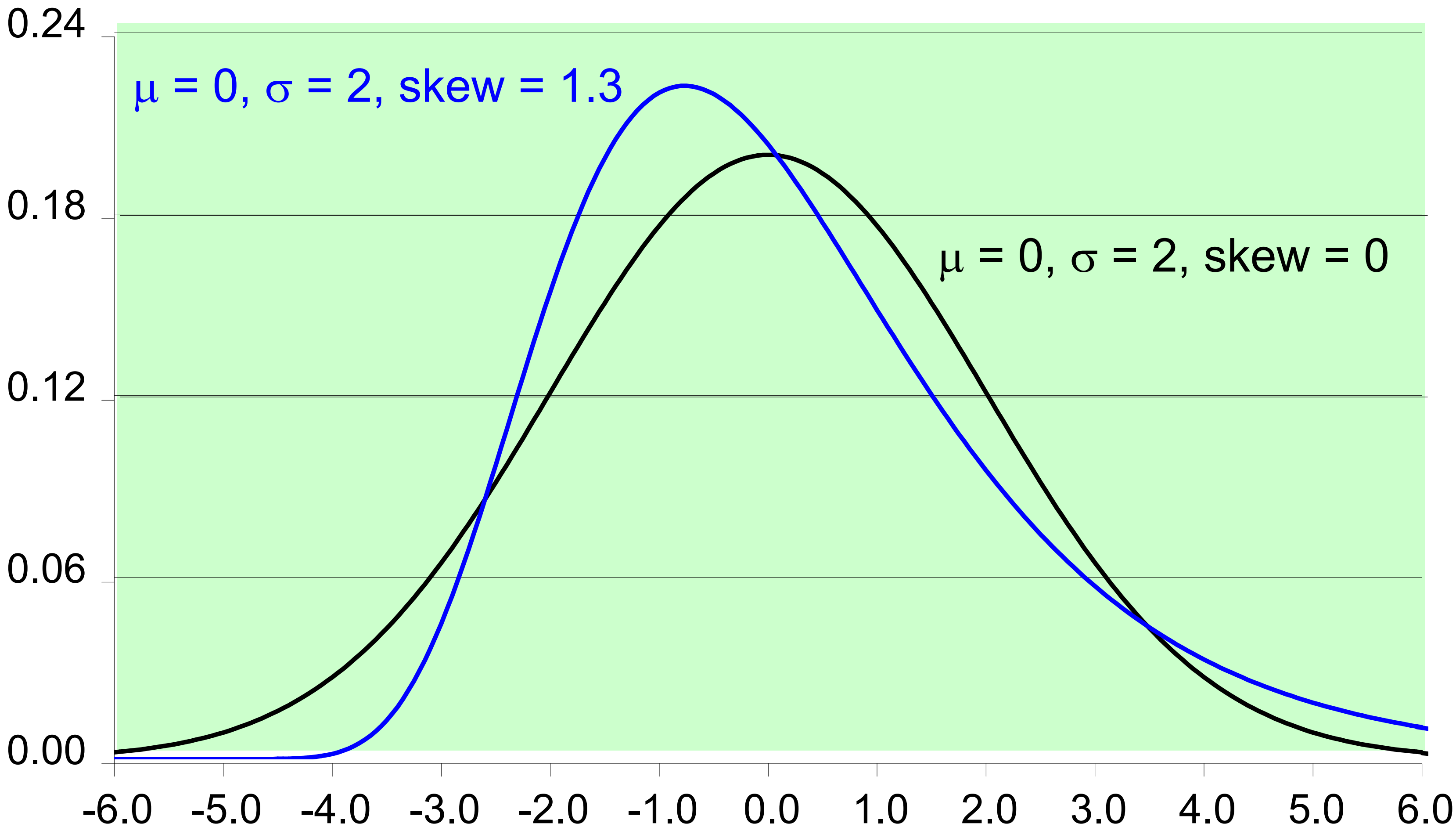
sample skewness

Example



[Probability density function: shows probability that x falls in an given range]

Example


0.24
0.18
0.12
0.06
0.00
μ = 0, σ = 2, skew = 1.3
μ = 0, σ = 2, skew = 0
-6.0
-5.0
-4.0
-3.0
-2.0
-1.0
0.0
1.0
2.0
3.0
4.0
5.0
6.0

Statistics review

Other statistics

- **Median**

  50th percentile: prob (x < median) = 0.50

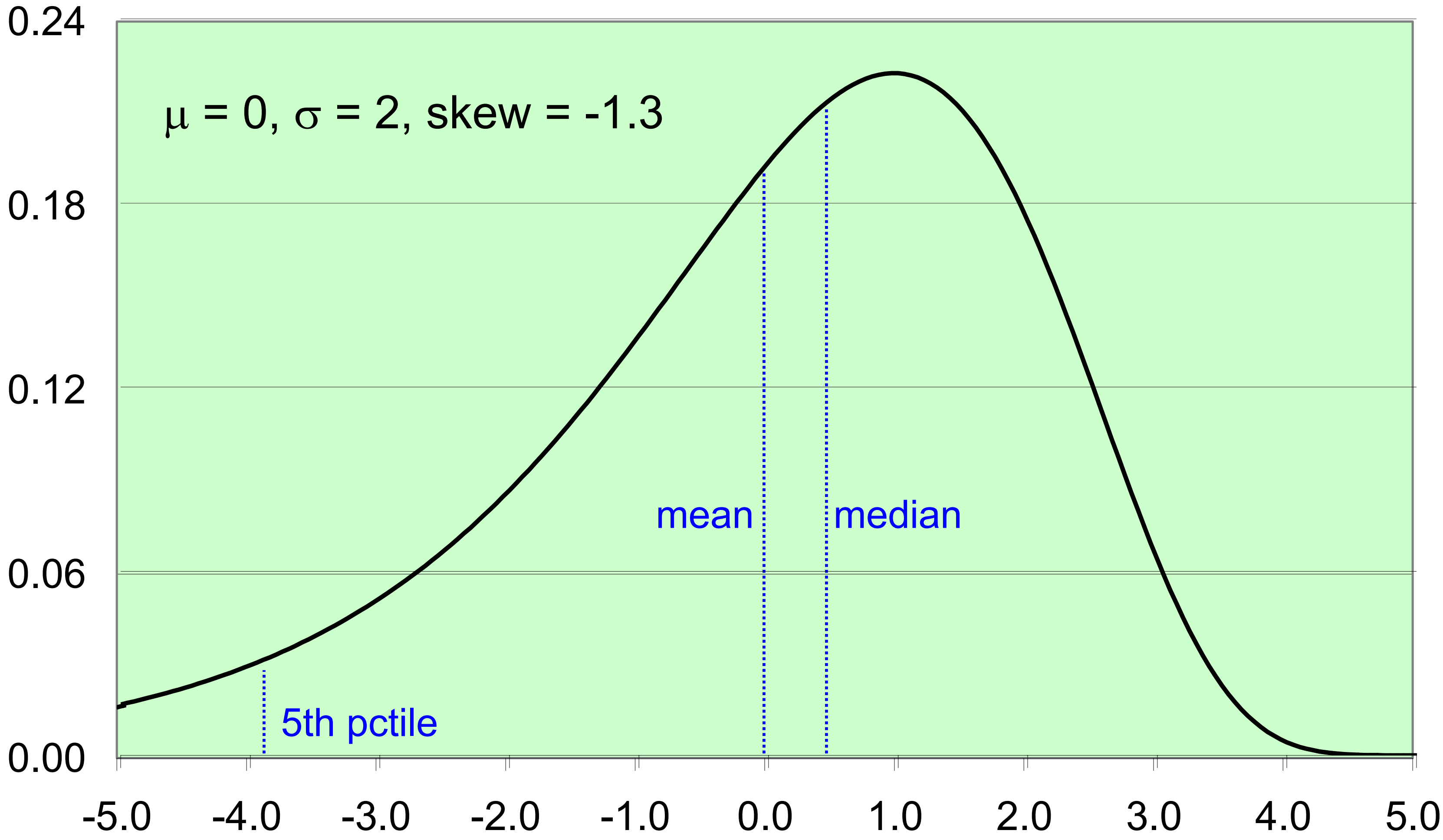
- **Value-at-Risk (VaR)**

  How bad can things get over the next day (or week)?

  1st or 5th percentile: prob (x < VaR) = 0.01 or 0.05

  'We are 99% certain that we won't lose more than $Y in the next 24 hours'

Example


μ = 0, σ = 2, skew = -1.3
0.24
0.18
0.12
0.06
0.00
mean
median
5th pctile
-5.0
-4.0
-3.0
-2.0
-1.0
0.0
1.0
2.0
3.0
4.0
5.0

Statistics review

Normal random variables

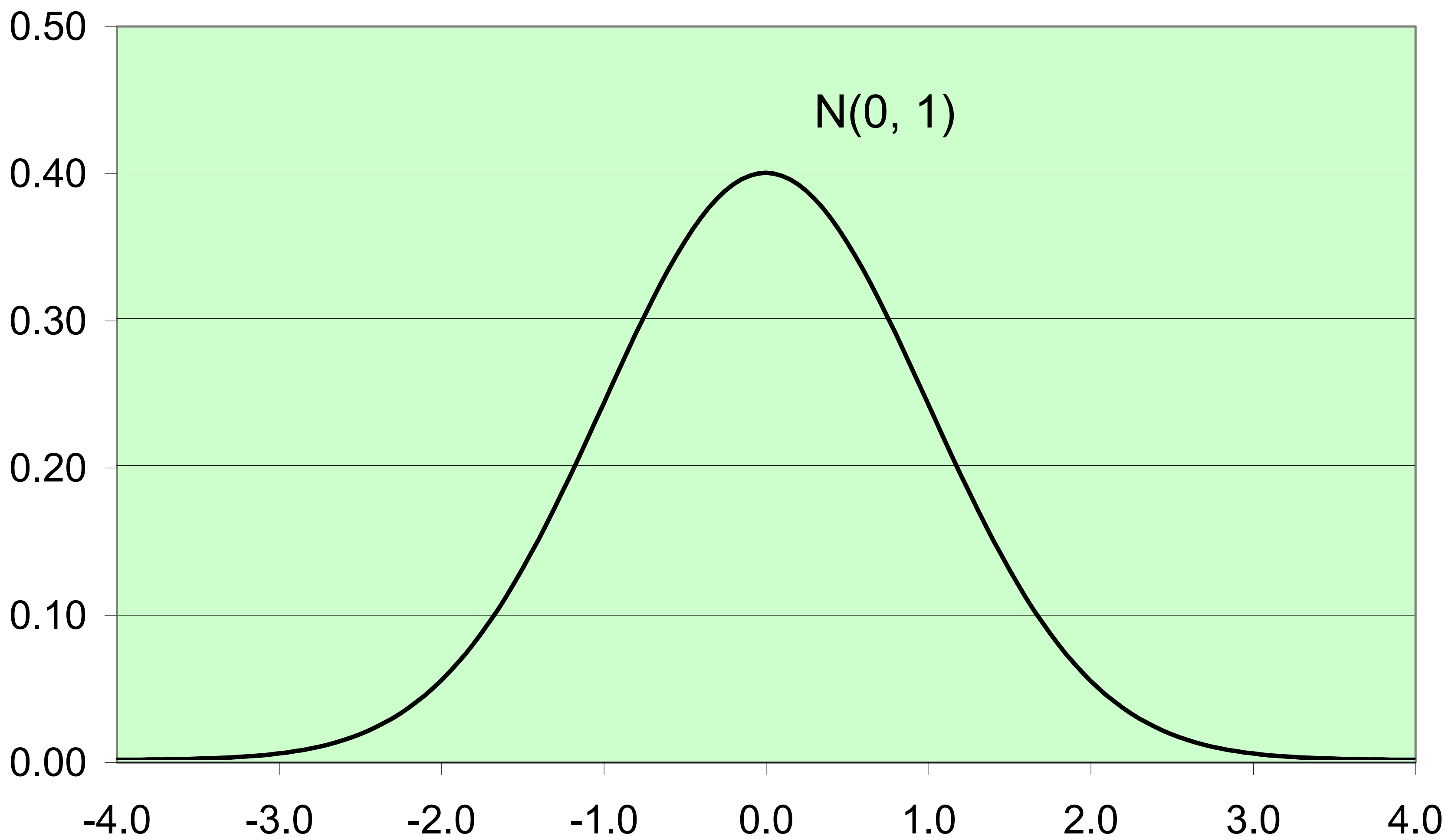
Bell-shaped, symmetric distribution

x ~ N(μ, σ^2)
x is normally distributed with mean μ and variance σ^2

'Standard normal'
mean 0 and variance 1 [or N(0, 1)]

Confidence intervals
68% of observations fall within +/–1 std. deviation from mean
95% of observations fall within +/–2 std. deviations from mean
99% of observations fall within +/–2.6 std. deviations from mean

Example



Statistics review

Estimating the mean

Given a sample $x_1, x_2, \ldots, x_N$

Don't know μ, σ^2 $\Rightarrow$ estimate μ by sample average $\bar{x}$
estimate σ^2 by sample variance s^2

How precise is $\bar{x}$?

std dev $(\bar{x}) \approx s / \sqrt{N}$

95% confidence interval for μ

$$\bar{x} - 2\frac{s}{\sqrt{N}} \qquad \bar{x} \qquad \bar{x} + 2\frac{s}{\sqrt{N}}$$

Application

From 1946 – 2001, the average return on the U.S. stock market was 0.63% monthly above the Tbill rate, and the standard deviation of monthly returns was 4.25%. Using these data, how precisely can we estimate the risk premium?

- **Sample:** $\bar{x} = 0.63\%$, $s = 4.25\%$, $N = 672$ months

- **Std dev ($\bar{x}$)** $= 4.25 / \sqrt{672} = 0.164\%$

- **95% confidence interval**

  Lower bound = $0.63 - 2 \times 0.164 = 0.30\%$
  Upper bound = $0.63 + 2 \times 0.164 = 0.96\%$

  Annual ($\times$ 12): $3.6\% < \mu < 11.5\%$

Statistics review

Two random variables

How do x and y covary? Do they typically move in the same direction or opposite each other?

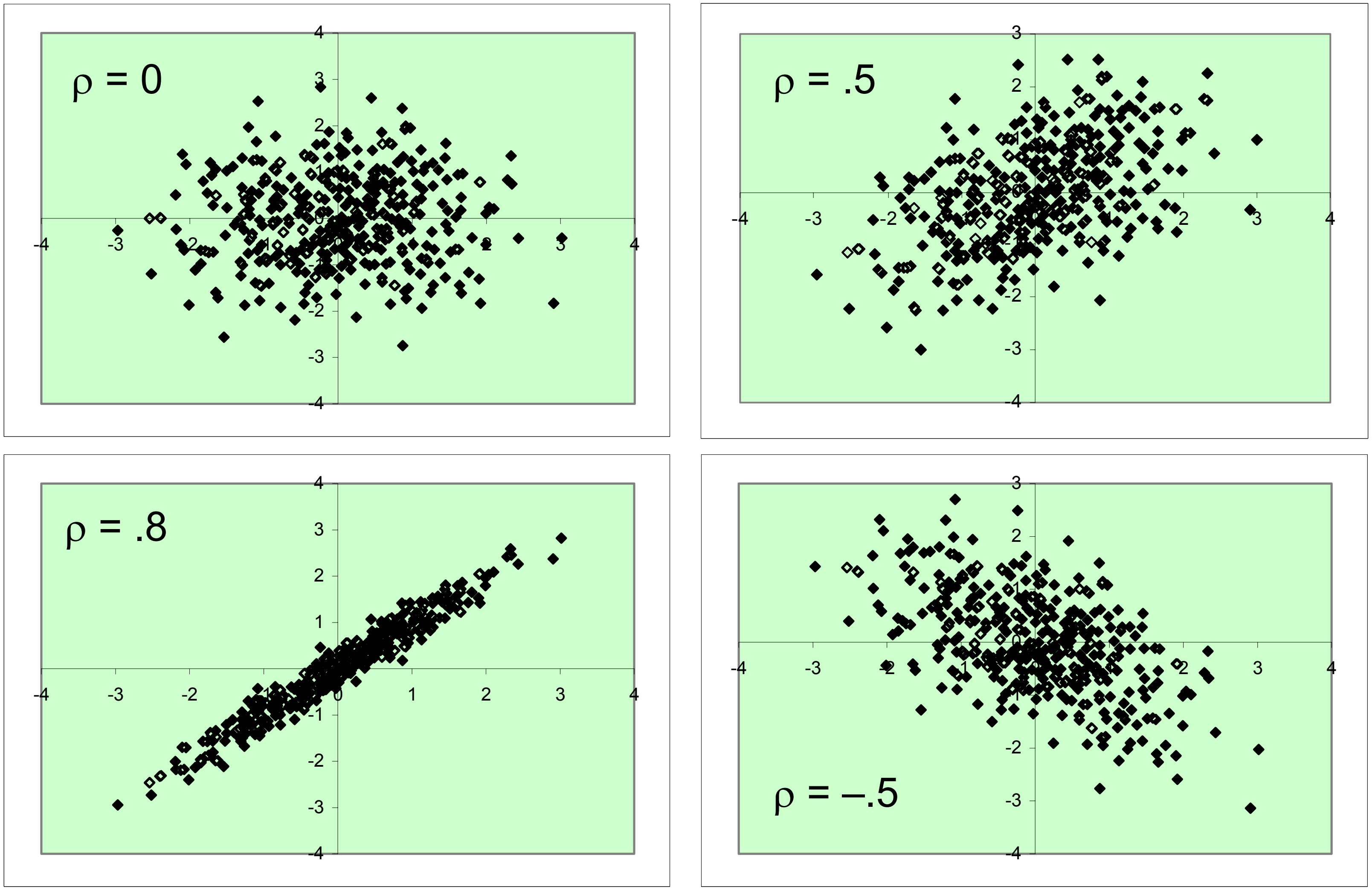
Covariance = $\sigma_{x,y} = E[(x - \mu_x)(y - \mu_y)]$

If $\sigma_{x,y} > 0$, then x and y tend to move in the same direction
If $\sigma_{x,y} < 0$, then x and y tend to move in opposite directions

Correlation = $\rho_{x,y} = \dfrac{\text{covariance}}{\text{stdev}_x \cdot \text{stdev}_y} = \dfrac{\sigma_{x,y}}{\sigma_x \sigma_y}$

$-1 \leq \rho_{x,y} \leq 1$

Correlation



Properties of stock prices

Time-series behavior

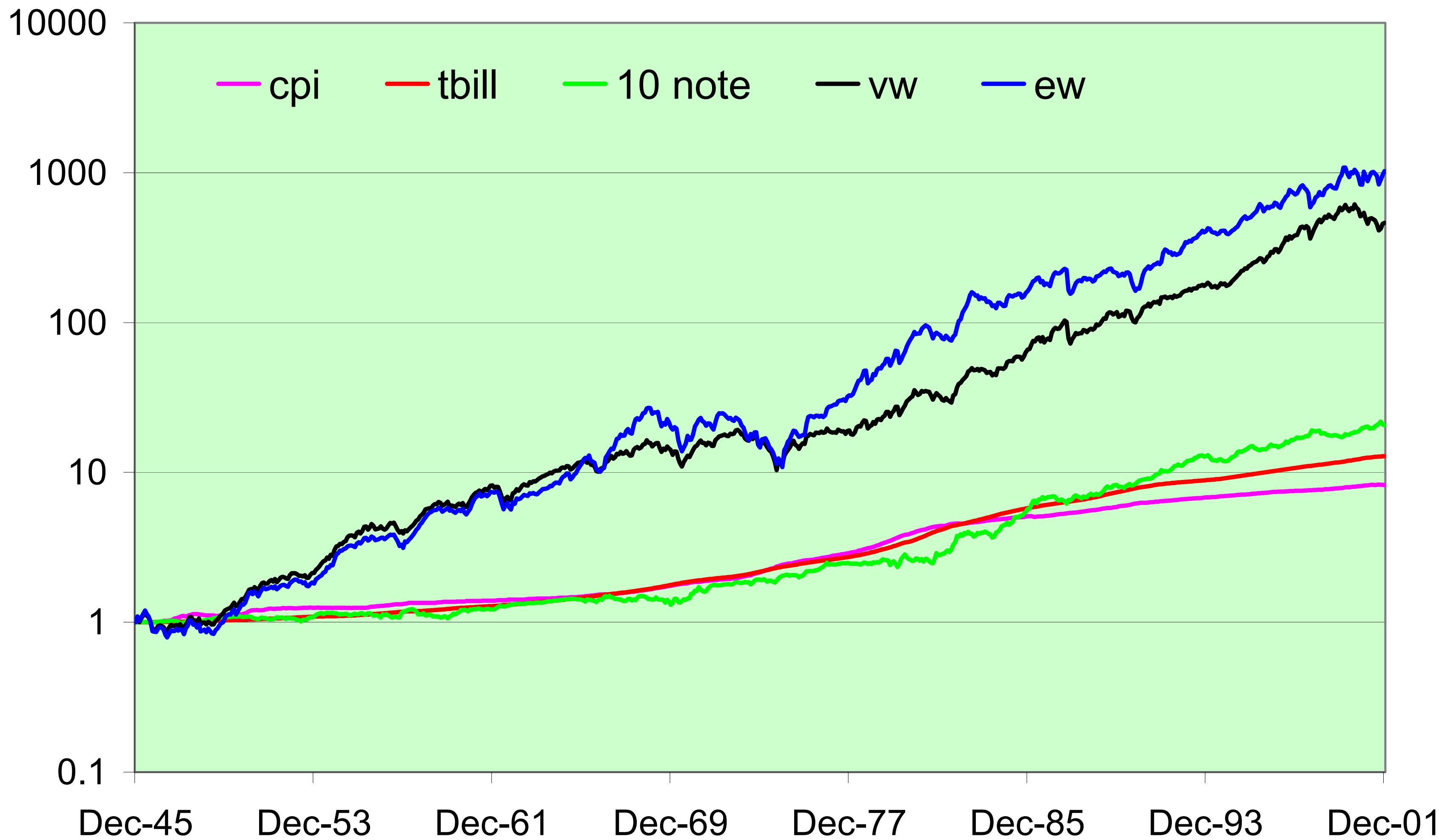
- How risky are stocks?
- How risky is the overall stock market?
- Can we predict stock returns?
- How does volatility change over time?

Stocks, bonds, bills, and inflation

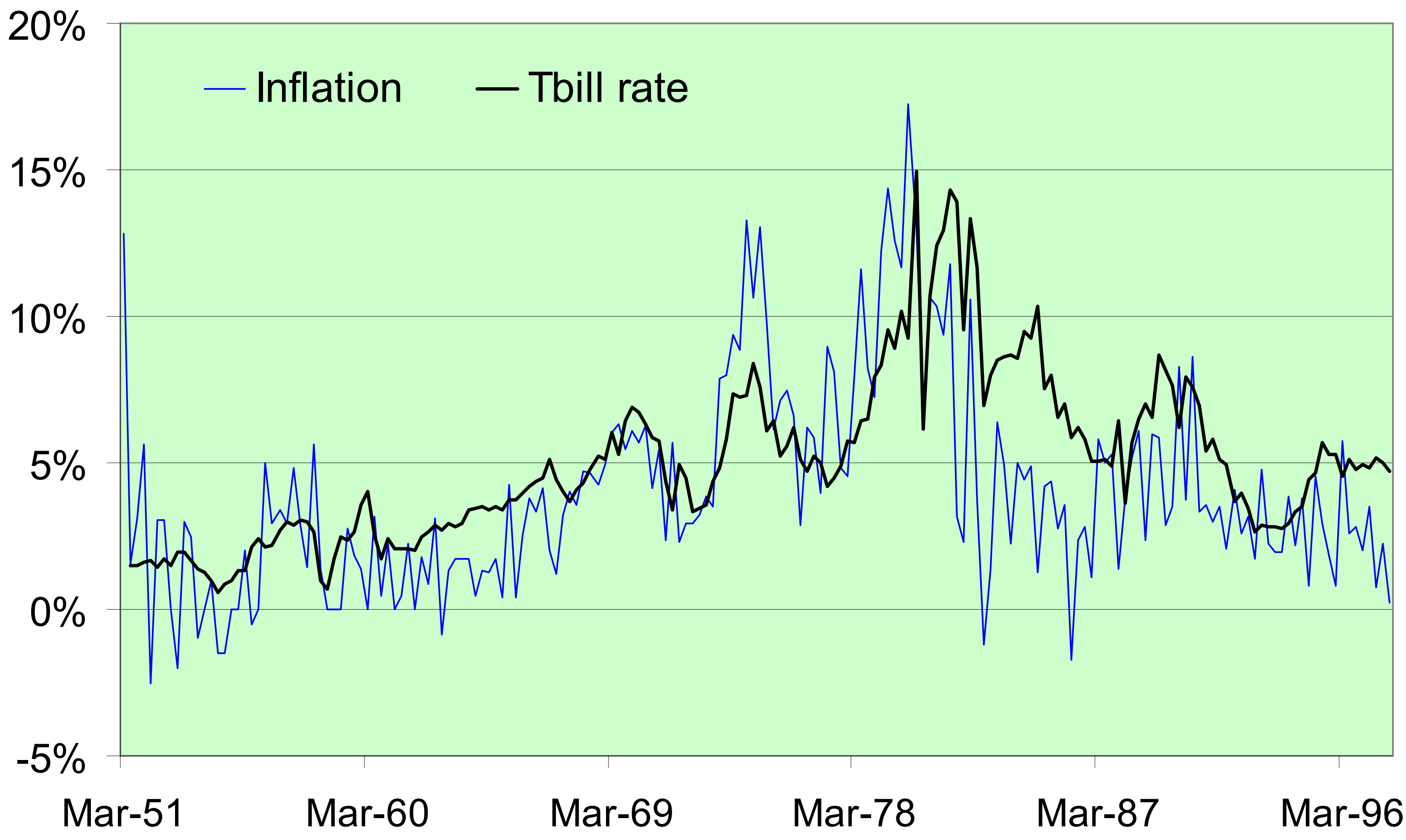
Basic statistics, 1946 – 2001
Monthly, %

Series	Avg	Stdev	Skew	Min	Max
Inflation	0.32	0.36	0.82	-0.84	1.85
Tbill (1 yr)	0.38	0.24	0.98	0.03	1.34
Tnote (10 yr)	0.46	2.63	0.61	-7.73	13.31
VW stock index	1.01	4.23	-0.47	-22.49	16.56
EW stock index	1.18	5.30	-0.17	-27.09	29.92
Motorola	1.66	10.02	0.01	-33.49	41.67

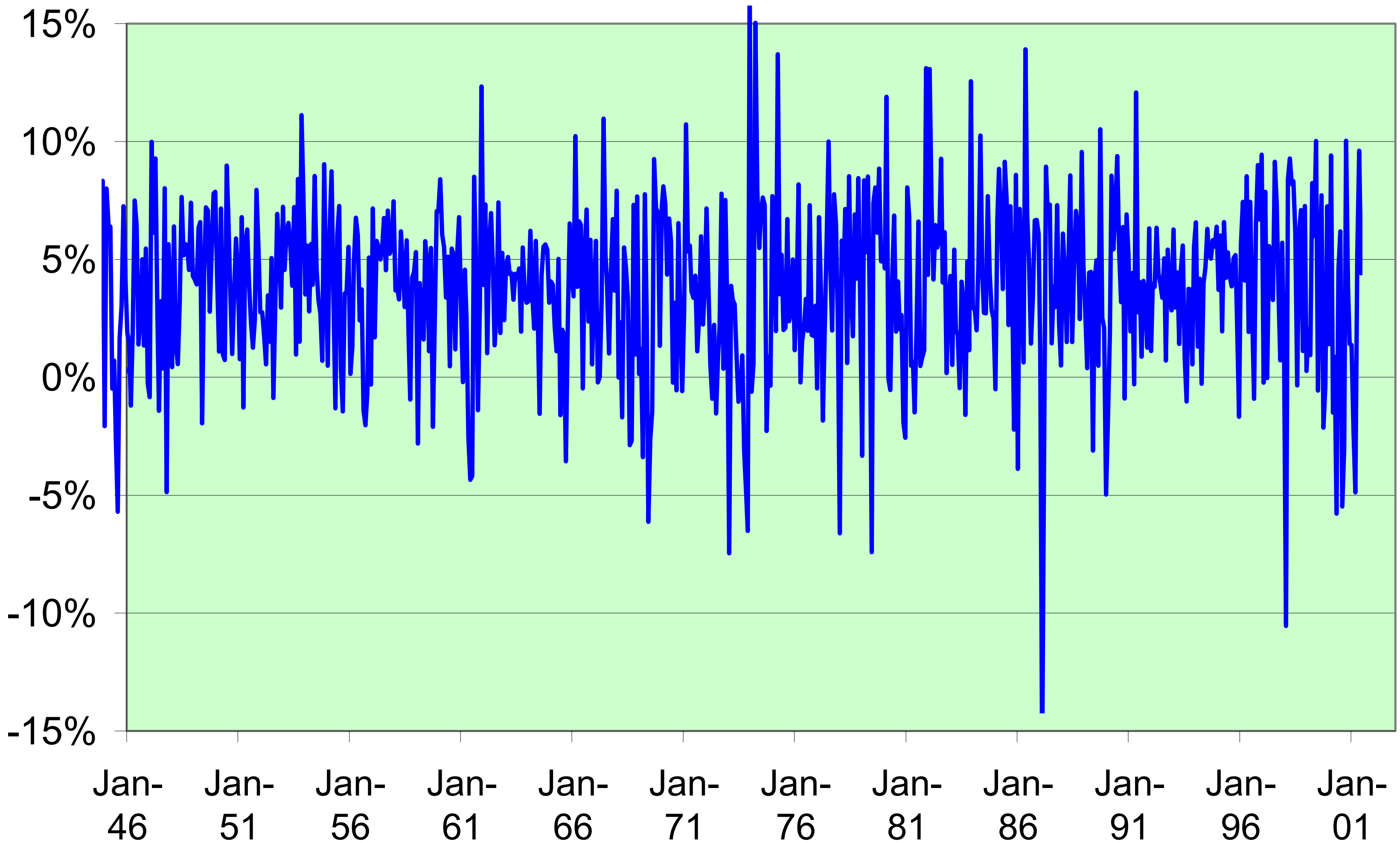
Stocks, bonds, bills, and inflation, $1 in 1946



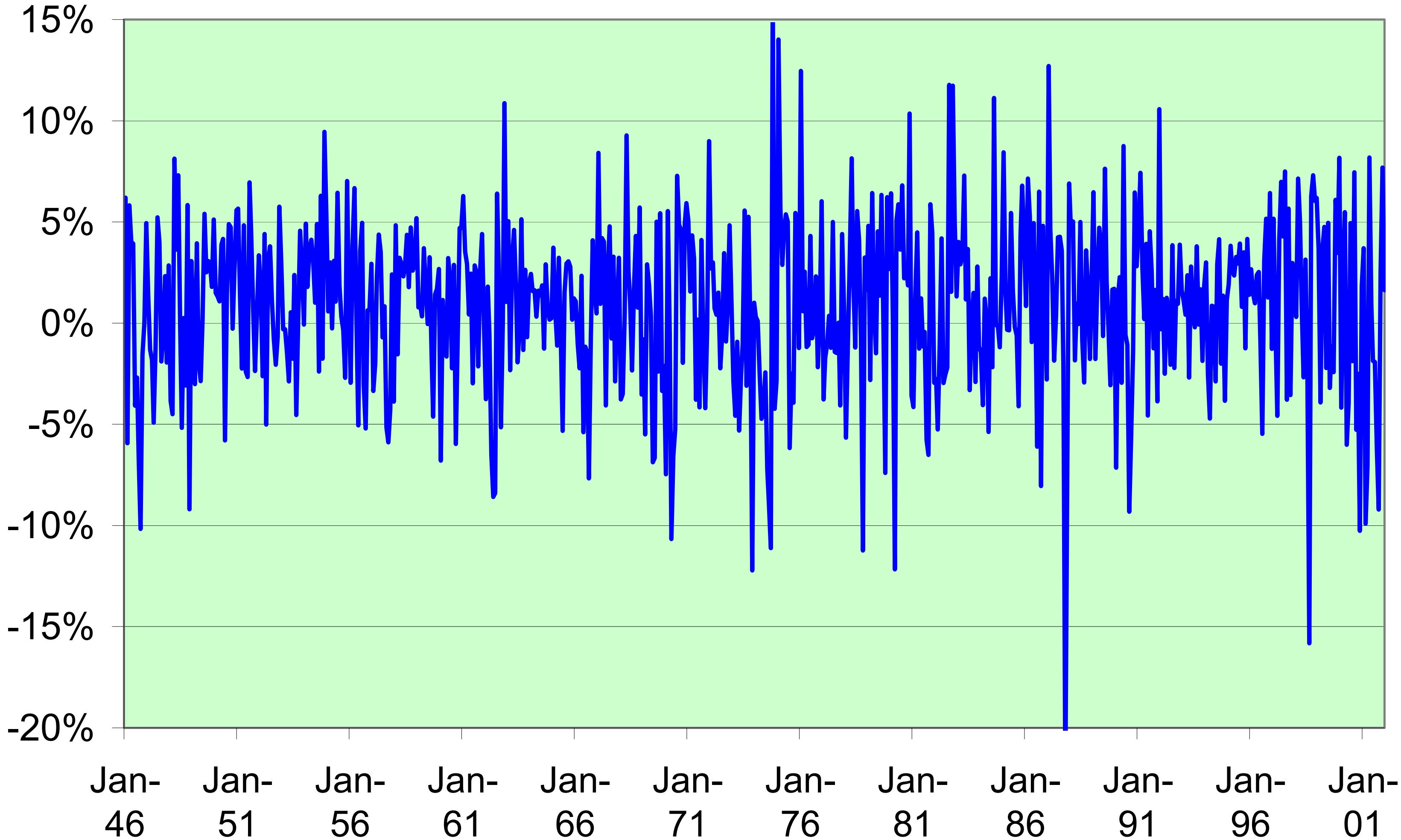
Tbill rates and inflation



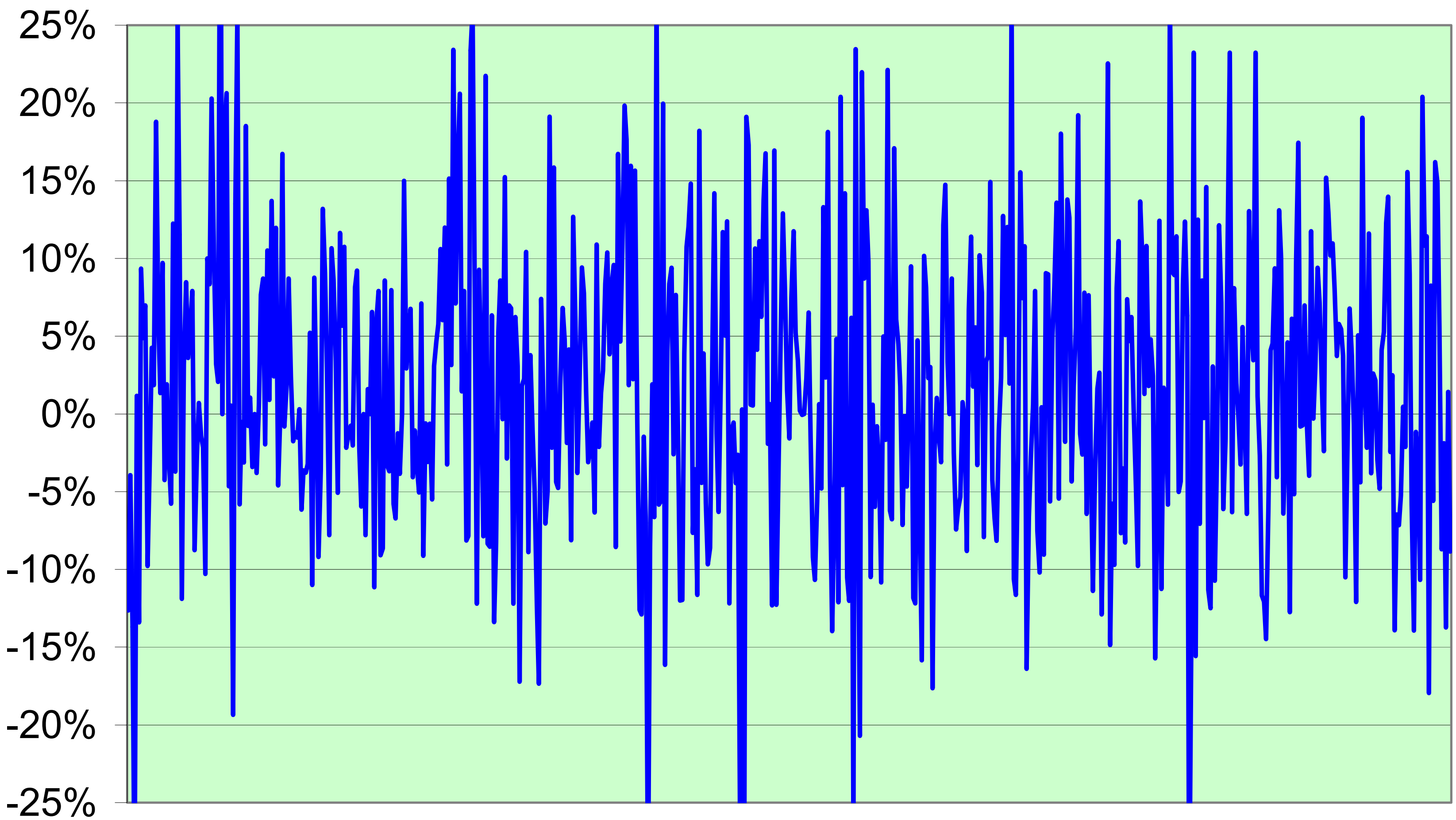
10-year Treasury note



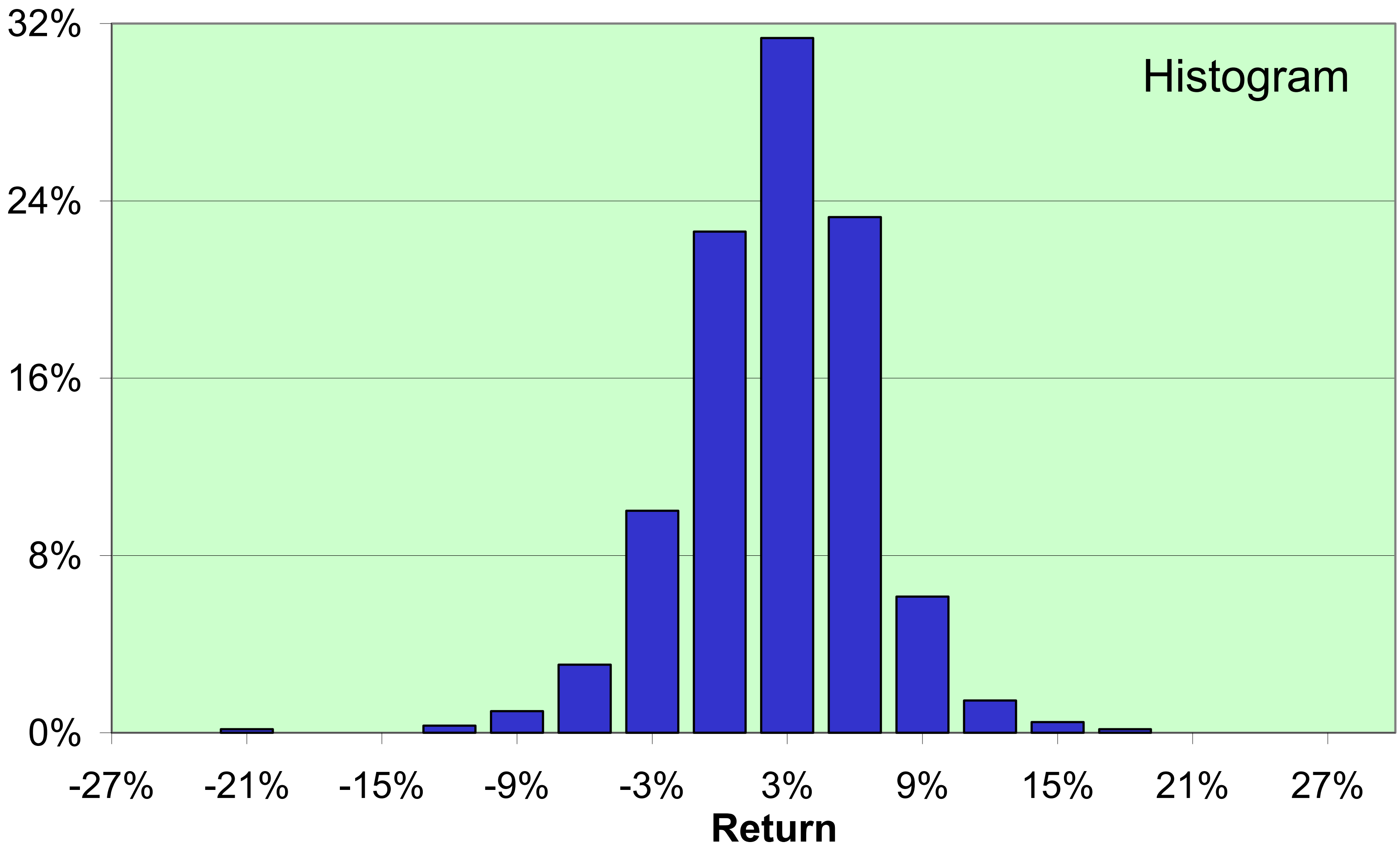
U.S. stock market returns, 1946 – 2001



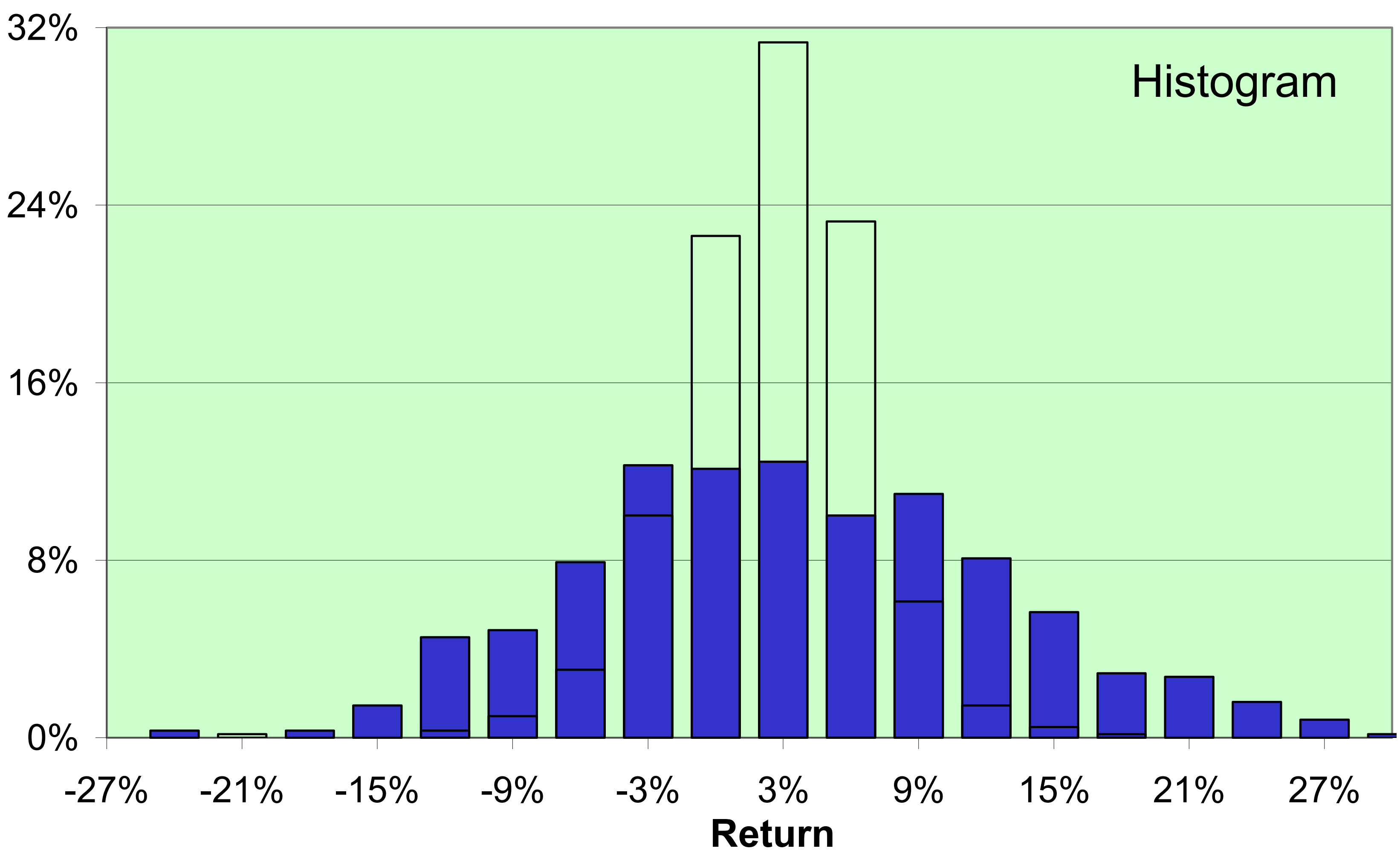
Motorola monthly returns, 1946 – 2001



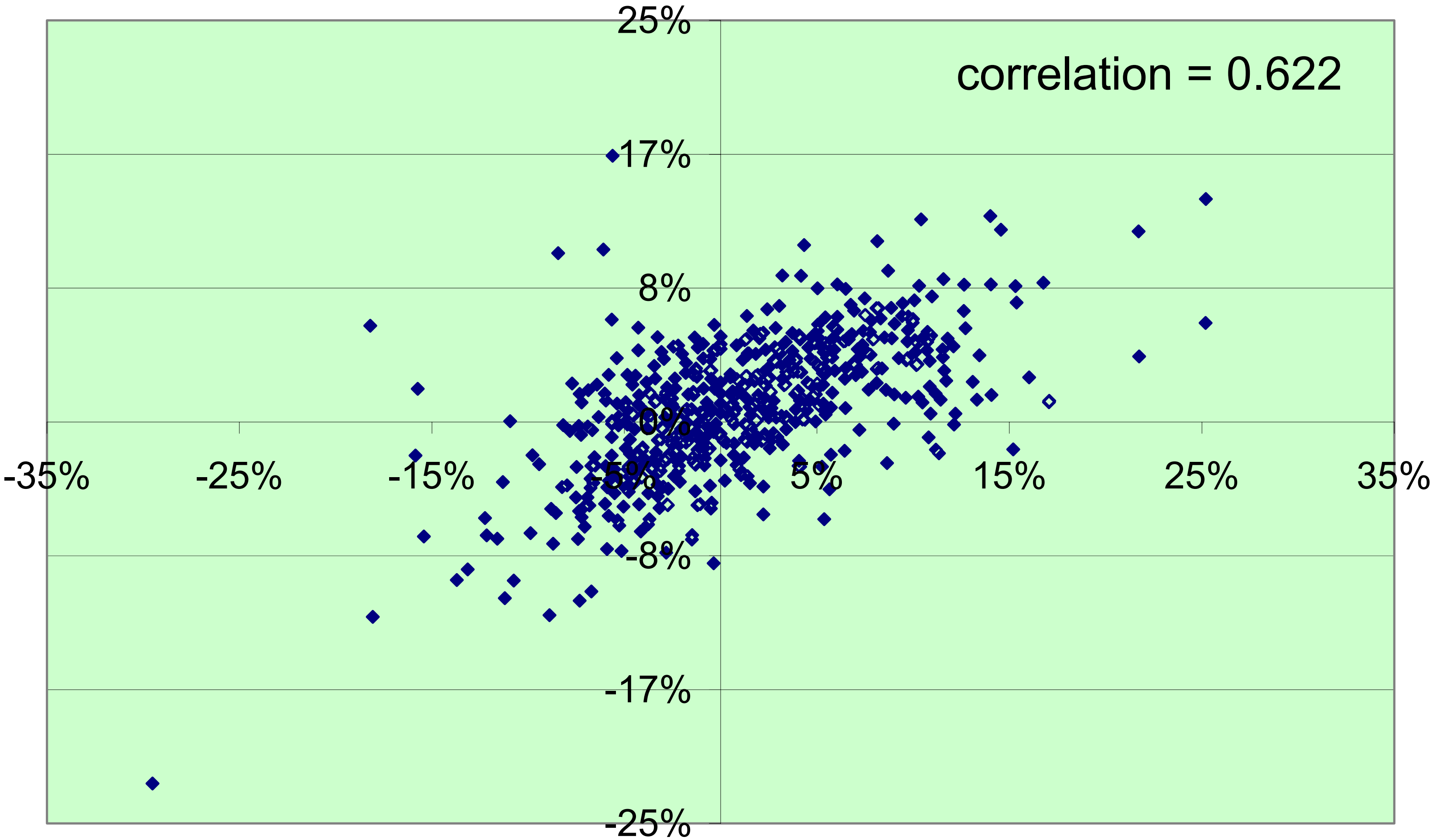
U.S. monthly stock returns



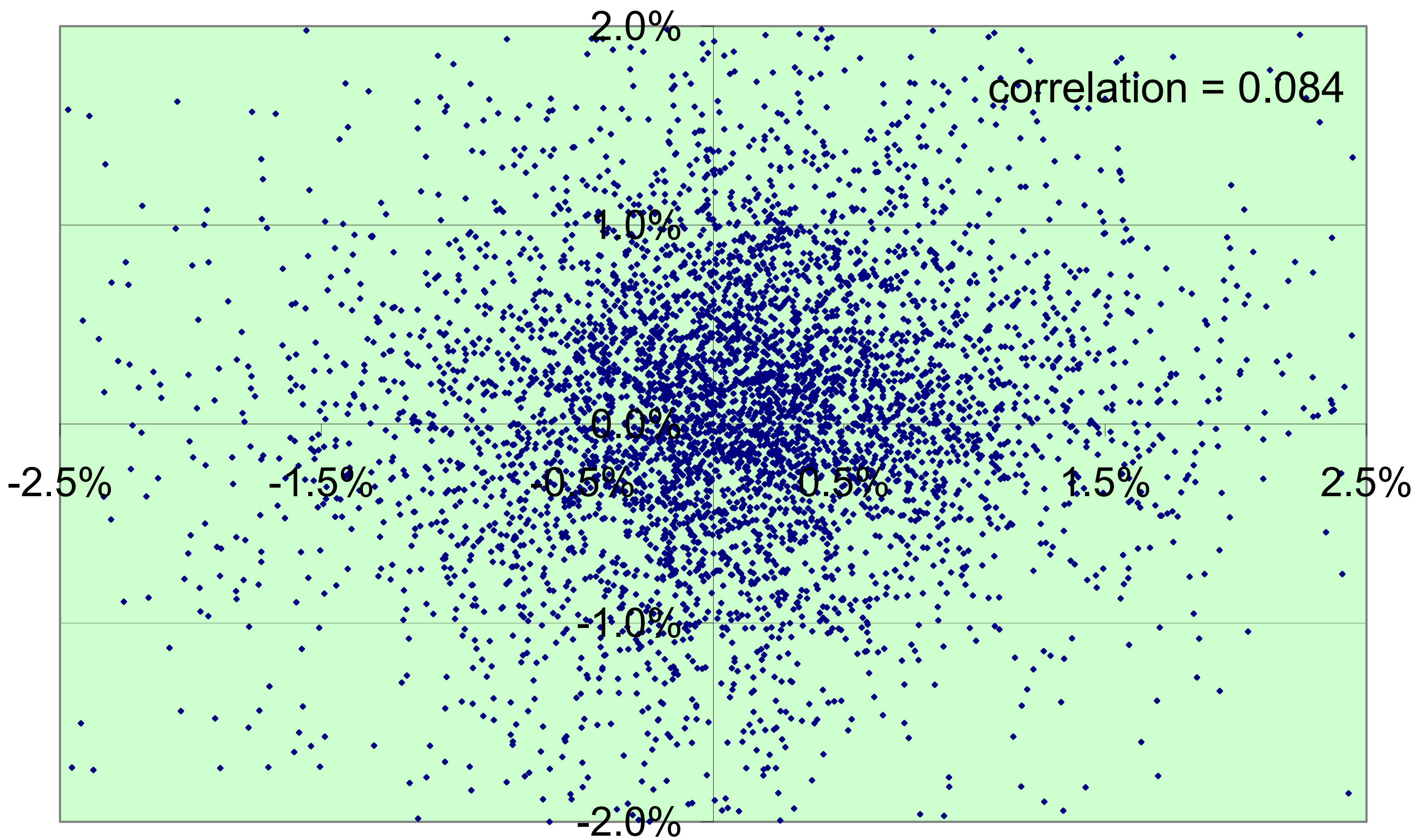
Motorola monthly returns



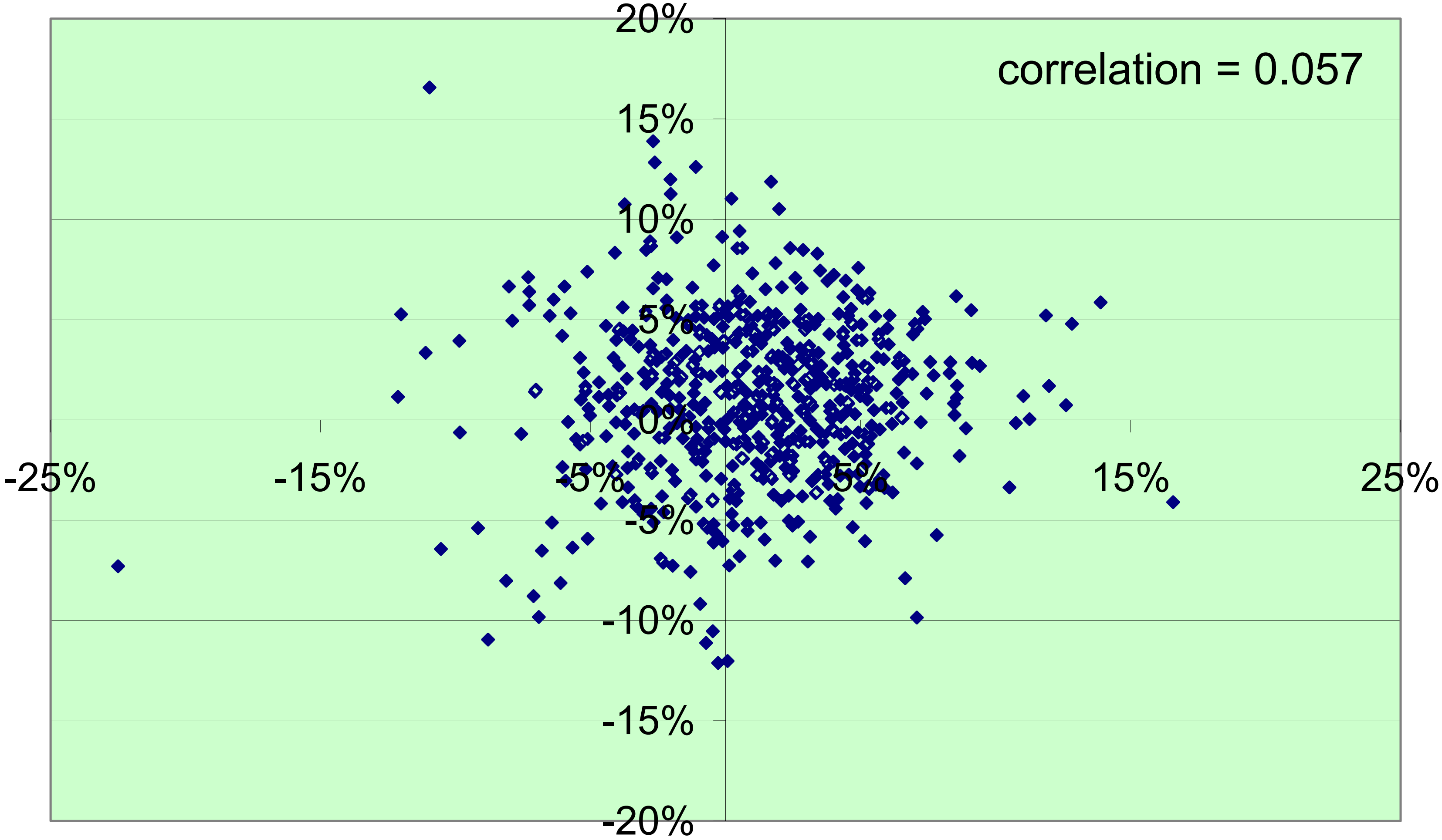
Scatter plot, GM vs. S&P 500 monthly returns



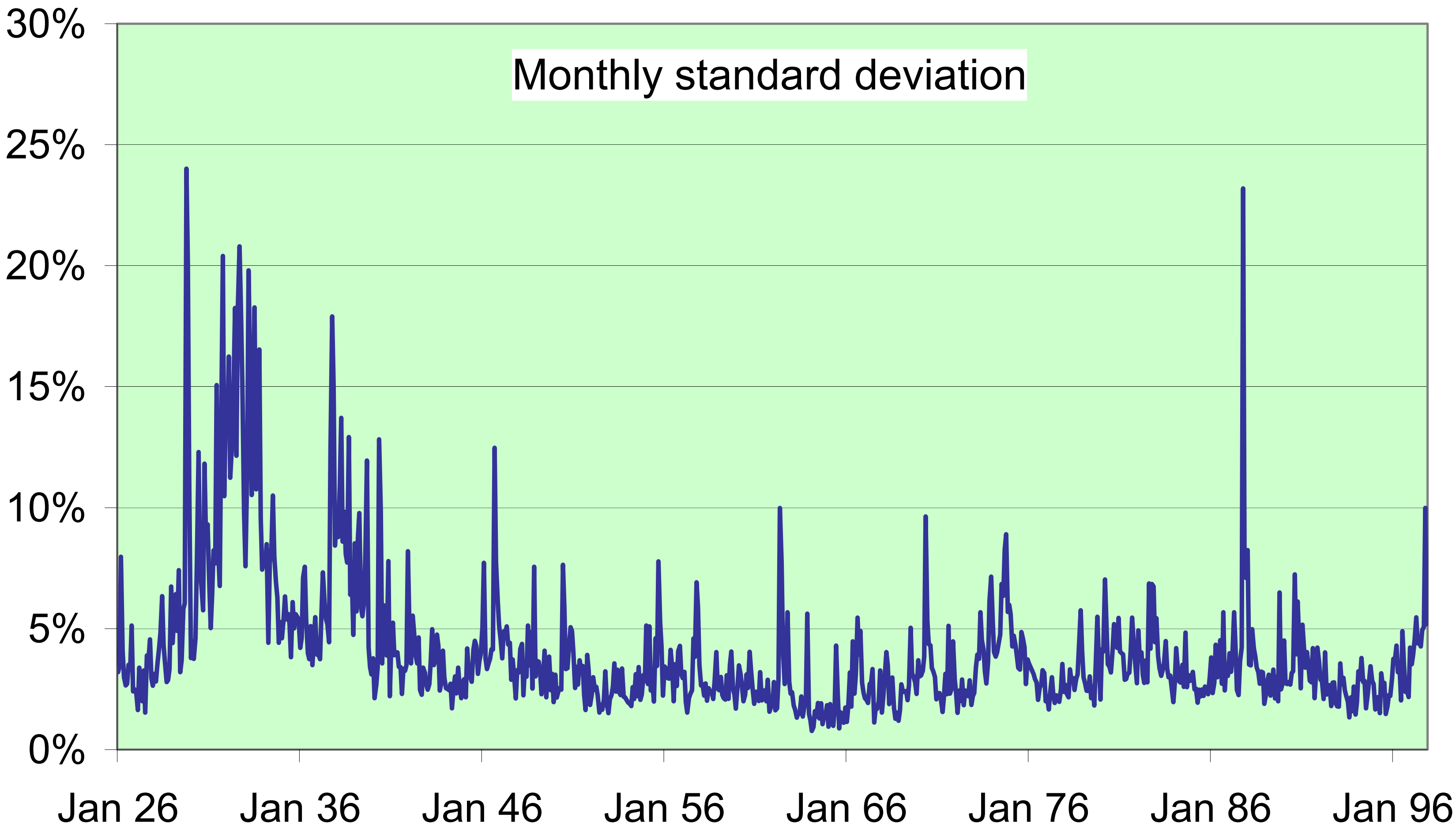
Scatter plot, $S\&P_t$ vs. $S\&P_{t-1}$ daily



Scatter plot, $S\&P_t$ vs. $S\&P_{t-1}$ monthly



Volatility of U.S. stock market



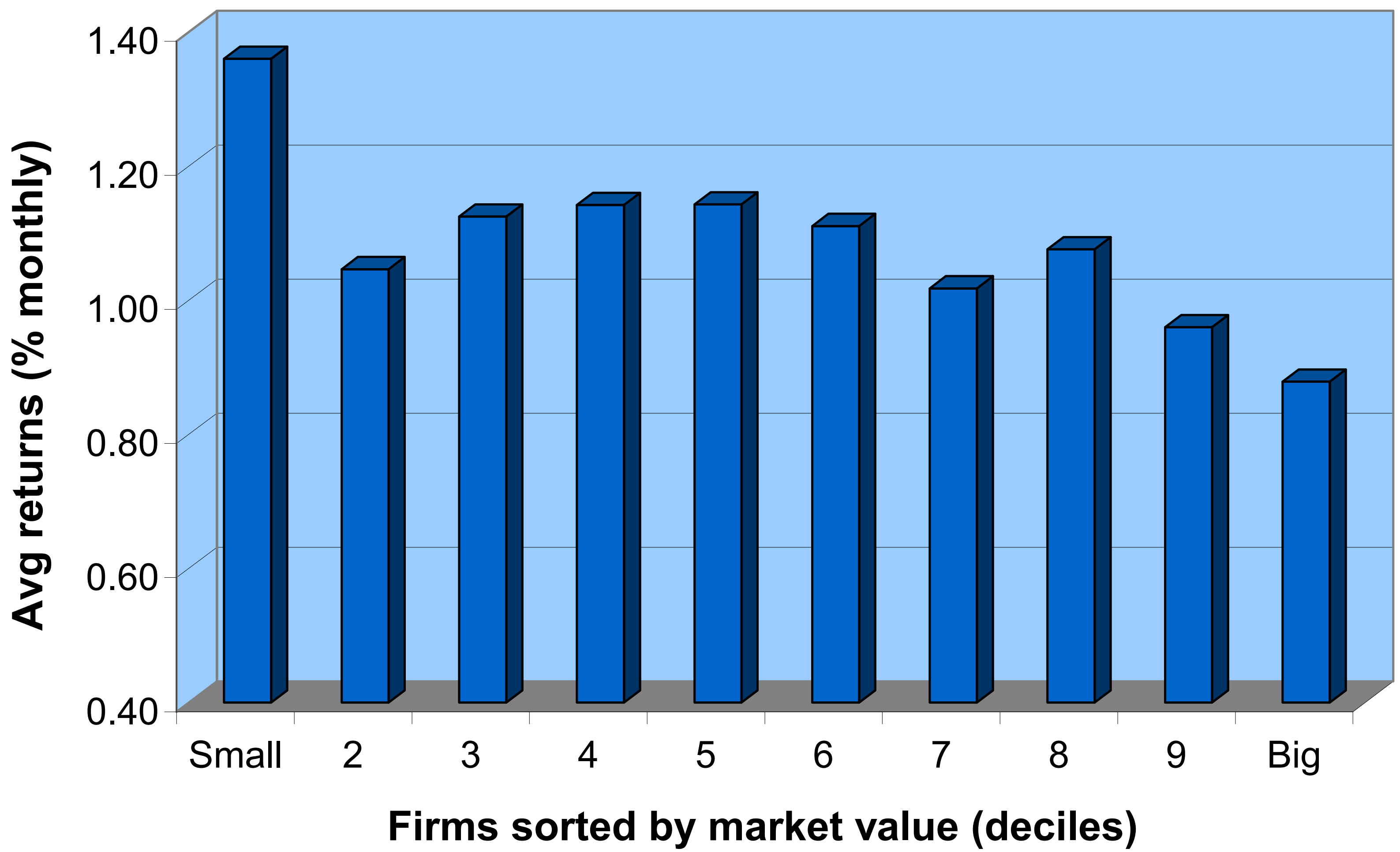
[Monthly std dev = std dev of daily returns during the month $\times\sqrt{21}$]

Properties of stock prices

Cross-sectional behavior

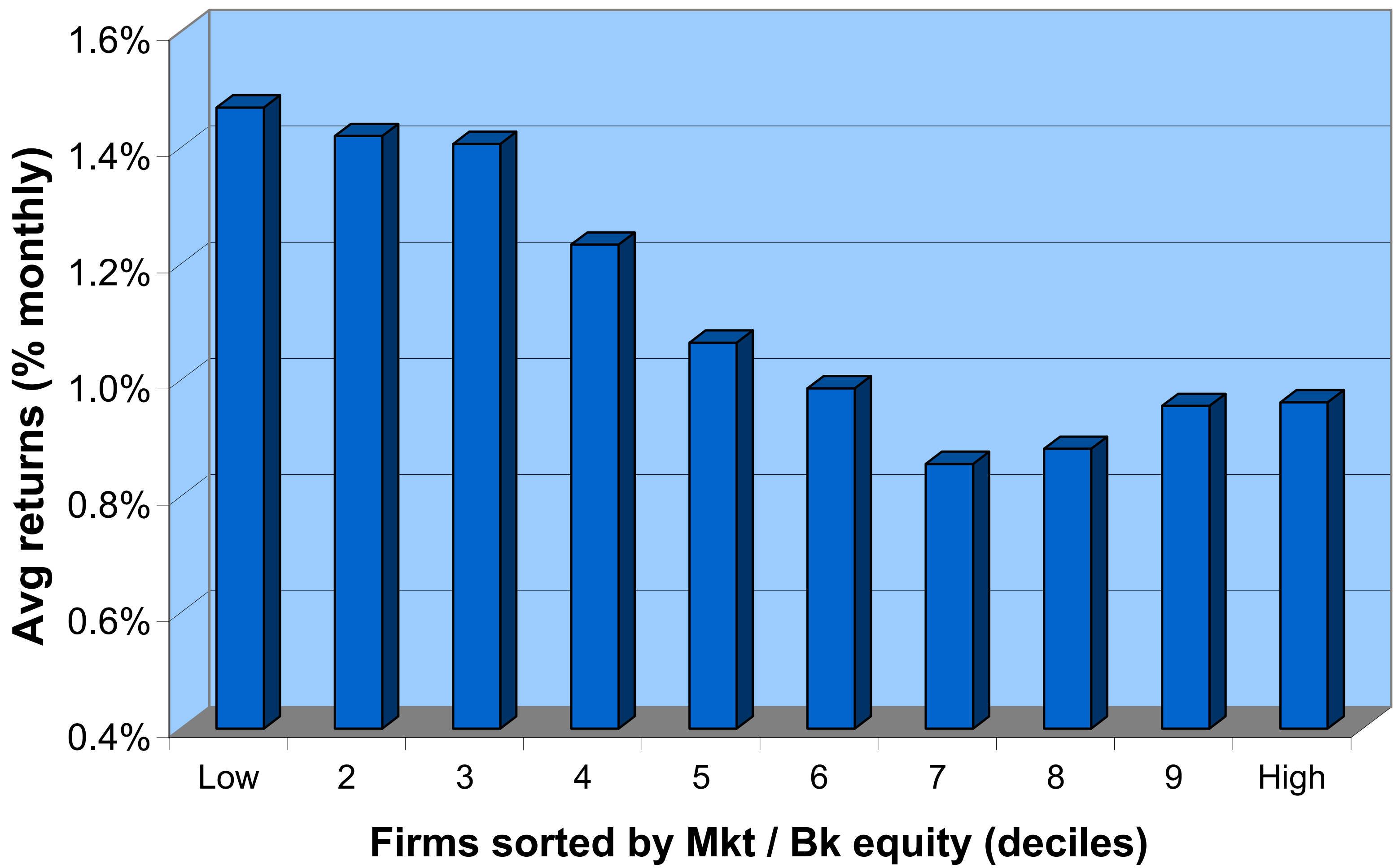
- What types of stocks have the highest returns?
- What types of stocks are riskiest?
- Can we predict which stocks will do well and which won't?

Size portfolios, monthly returns



Size portfolios in January

M/B portfolios, monthly returns



Momentum portfolios, monthly returns

Time-series properties

Observations

- **The average annual return on U.S. stocks from 1926 – 2001 was 11.6%.** The average risk premium was 7.9%.

- **Stocks are quite risky**. The standard deviation of monthly returns for the overall market is 4.5% (15.6% annually).

- **Individual stocks are much riskier**. The average monthly standard deviation of an individual stock is around 17% (or 50% annually).

- **Stocks tend to move together over time**: when one stock goes up, other stocks are likely to go up as well. The correlation is far from perfect.

Time-series properties

Observations

- **Stock returns are nearly unpredictable**. For example, knowing how a stock does this month tells you very little about what will happen next month.

- **Market volatility changes over time**. Prices are sometimes quite volatile. The standard deviation of monthly returns varies from roughly 2% to 20%.

- **Financial ratios like DY and P/E ratios vary widely over time**. DY hit a maximum of 13.8% in 1932 and a minimum of 1.17% in 1999. The P/E ratio hit a maximum of 33.4 in 1999 and a minimum of 5.3 in 1917.

Cross-sectional properties

Observations

- **Size effect**: Smaller stocks typically outperform larger stocks, especially in January.

- **January effect**: Average returns in January are higher than in other months.

- **M/B, or value, effect**: Low M/B (value) stocks typically outperform high M/B (growth) stocks.

- **Momentum effect**: Stocks with high returns over the past 3- to 12-months typically continue to outperform stocks with low past returns.