

15.433 INVESTMENTS

Class 11: Equity Options Part 2:
Empirical Evidence

Spring 2003

The Black Scholes Model

Stock prices follow a geometric Brownian motion. Translating into discrete time, stock prices follow a Random Walk:

$$r_{t+\Delta t} = \mu \cdot \sqrt{\Delta t} + \sigma \sqrt{\Delta} \varepsilon_{t+\Delta t} \quad (1)$$

This model implies that, over time interval Δt stock returns are normally distributed with standard deviation $\sigma \sqrt{\Delta t}$.

The market is dynamically complete. Options are redundant, and can be replicated by a dynamic hedging strategy that involves the riskfree account r and the underlying stock S .

The Black-Scholes formula for a call option struck at K and expiring at T :

$$C_0^{BS} = BS(S_0, K, T, \sigma, r_f) \quad (2)$$

which depends on today's stock price S_0 , the volatility of the underlying stock σ , and the riskfree rate r_f .

Implications of the Black & Scholes

The Black-Scholes volatilities implied by options with different strike prices K and maturities T should be the same. They should also remain constant over time.

The volatility implied by options should be consistent with the level of volatility observed in the underlying stock market.

Time Series of Stocks and Options

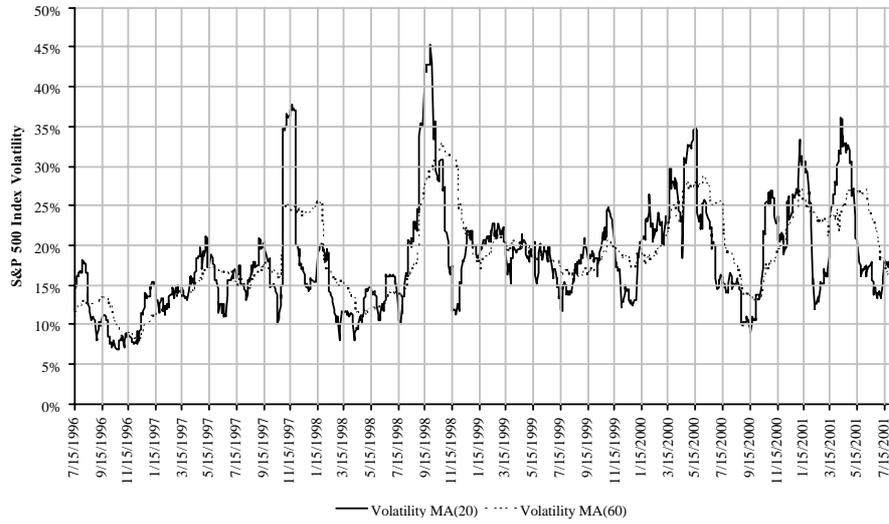


Figure 1: Which Volatility?

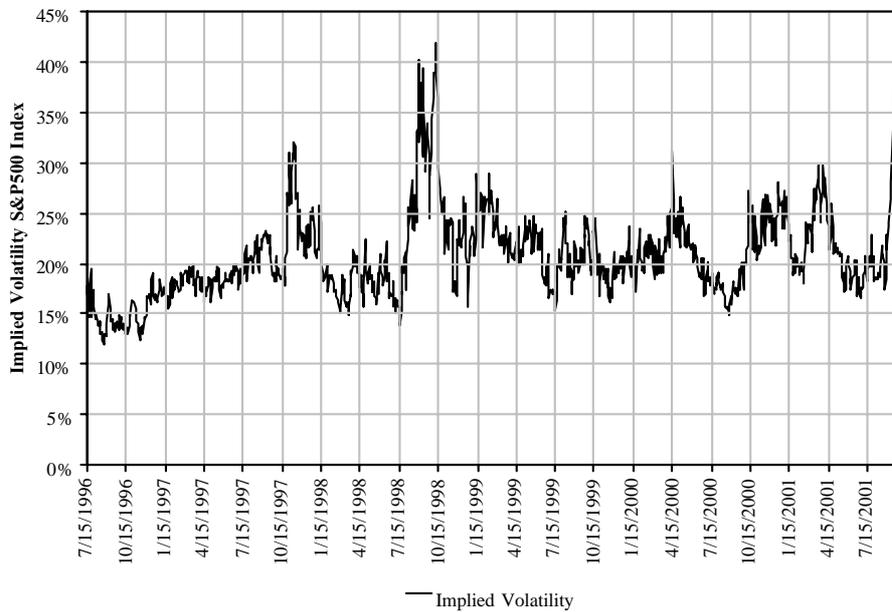


Figure 2: Which volatility?

Evidence against the Black & Scholes

The Black-Scholes implied volatility does not stay constant over time.

The volatility implied by the options market is on average higher than that observed directly from the underlying stock market.

On any given day, options (both puts and calls) with different K and T exhibit a pattern of "smile" or "smirk":

- OTM puts have higher implied-vol than ATM options and OTM calls.
- This "smile" pattern is more pronounced in short-dated options.
- The volatility implied by long-dated options differs from that implied by short-dated options.

The Assumption of Constant Volatility

The fact that volatility is not constant can be seen from both the underlying stock market and the options market.

In fact, volatility is itself a stochastic process. For example, letting $V_t = \sigma_t^2$, a popular stochastic volatility model is:

$$dV_t = k (\bar{V} - V_t) dt + \eta \sqrt{V_t} dB_t^v \quad (3)$$

Some empirical "regularities":

1. volatility is stochastic (random);
2. volatility is persistent;
3. volatility is mean-reverting;
4. the volatility shocks are negatively correlated with the underlying price shocks.

For each regularity, can you provide a relevant empirical evidence from the options market?

The Assumption of No Jump

The fact that stock prices experience sudden jumps can be seen from both the underlying stock market and the options market (why?).

Price is continuous in the Black Scholes:

$$r_{t+\Delta t} = \mu \cdot \sqrt{\Delta t} + \sigma \sqrt{\Delta \varepsilon_{t+\Delta t}} \quad (4)$$

As we shrink the time interval Δt to zero, the volatility $\sigma\sqrt{\Delta t}$ of the surprise component also shrinks to zero.

In 1976, three years after the Black Scholes, Bob Merton proposed a jump extension.

$$r_{t+\Delta t} = \mu \cdot \sqrt{\Delta t} + \sigma \sqrt{\Delta \varepsilon_{t+\Delta t}} + J_{t+\Delta t} \quad (5)$$

Can you provide the relevant empirical evidence from the options market indicating negative price jumps in the underlying?

The Assumption of Complete Market

Although it is not an explicit assumption made by the Black-Scholes, market is dynamically complete with respect to the underlying stocks and the riskfree account.

The intuition is that there is only one source of uncertainty (associated with the Brownian motion), which can be fully reflected by the underlying stock prices.

If in addition to the Brownian shocks, there are jump risks in the underlying stock, then the market is no longer complete.

Similarly situation arises if the stock volatility has random shocks of its own.

In both cases, investors' aversion to the additional sources of uncertainty will be reflected in option prices.

Fear of Jumps

The options market, especially that of OTM puts, provides a unique opportunity for investors to express their aversion to negative price jumps.

The market prices of such OTM puts reflect not only the probability and magnitude of such jumps, but also investors' aversion to such jumps.

This aversion to negative jump risks is closely related to the aversion to negative skewness.

Just like backing out Black-Scholes implied volatility, one can back out the probability and magnitude of negative jumps from OTM puts.

It is found that option-implied jump risks are much exaggerated in comparison with those observed directly from the spot market (Jun Pan 2001), why?

Summary

The Black-Scholes model provides a powerful framework for us to formulate option pricing.

The Black-Scholes formula provides a powerful tool for us to measure the sensitivity of option prices with respect to the market condition.

To option traders, such sensitivity measures are as essential, if not more, as breathing air in and out.

There is mounting evidence, from both the underlying stock market and the options market, suggesting that the assumptions of the Black-Scholes are overly simplified.

In this class, we examined the effect of stochastic volatility, price jumps, and fear of jumps.

Focus:

BKM Chapters 21.

- p. 697-703,
- p. 708-711,
- p. 714-716,
- p. 718-722

type of potential questions: concept check question 1 to 8, table 21.3/21.4, p. 732 question 7, 10, 13, 14, 34, 22

Questions for the Next Class

Please read:

- BKM Chapter 14,
- Kritzman (1993)

What are the main differences between stocks and bonds?

Why do people believe that bonds are safer than stocks? Look up the Wall Street Journal, get a feeling of the daily percentage changes in stock prices and bond prices.