15.433 INVESTMENTS

Class 17: The Credit Market

Part 1: Modeling Default Risk

Spring 2003

The Corporate Bond Market

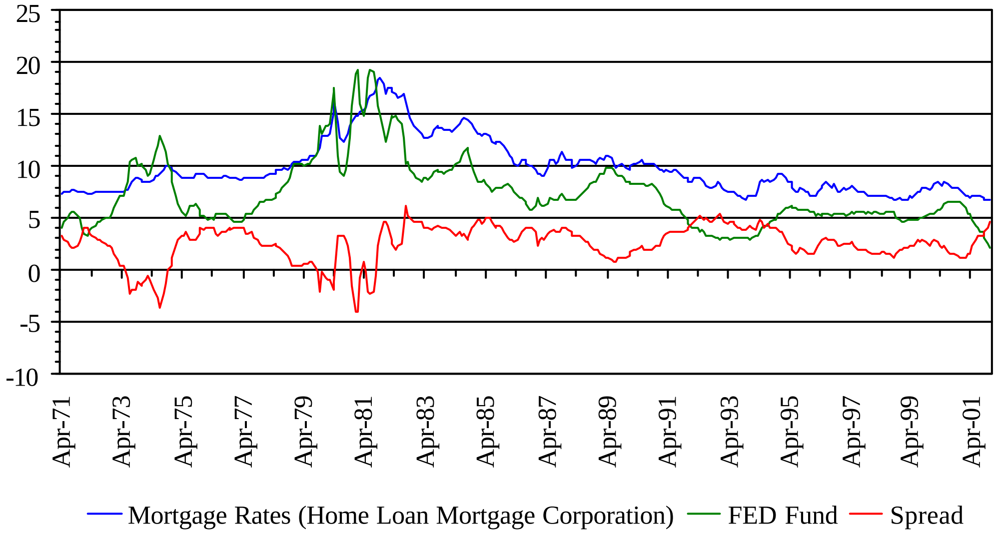

Figure 1: Mortgage and FED rates, Source : www.federalreserve.gov/releses/hr

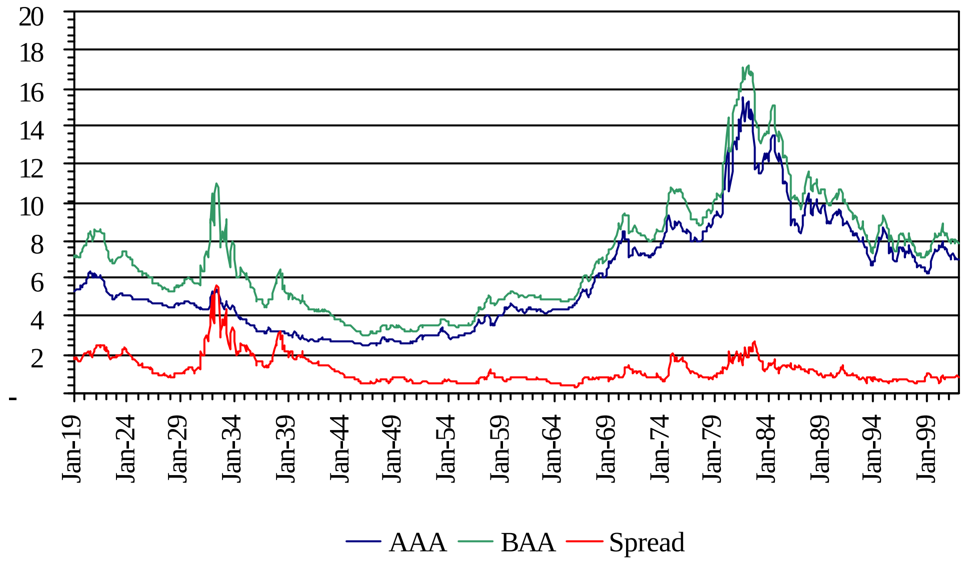

Figure 2: Corporate rating spreads, Source : www.federalreserve.gov/releses/hr

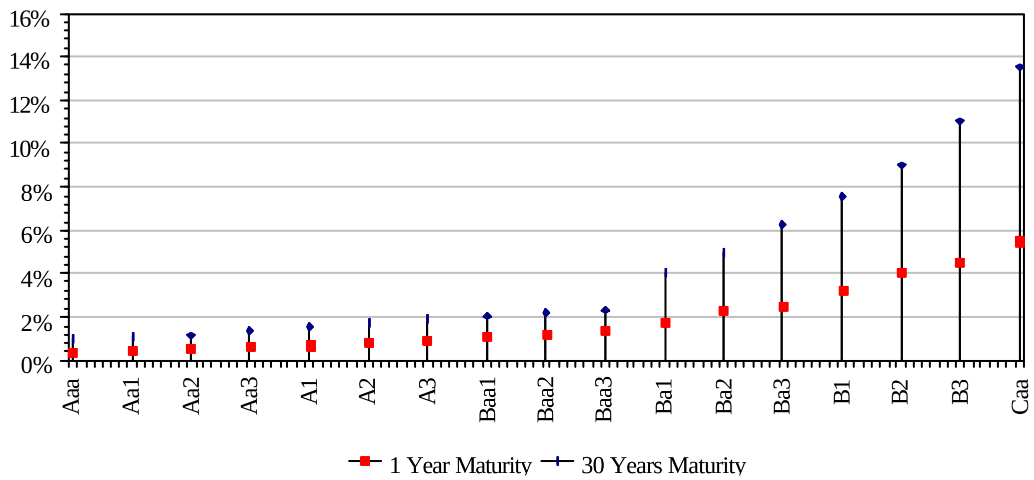

Figure 3: Corporate rating spreads, Source : Moody's

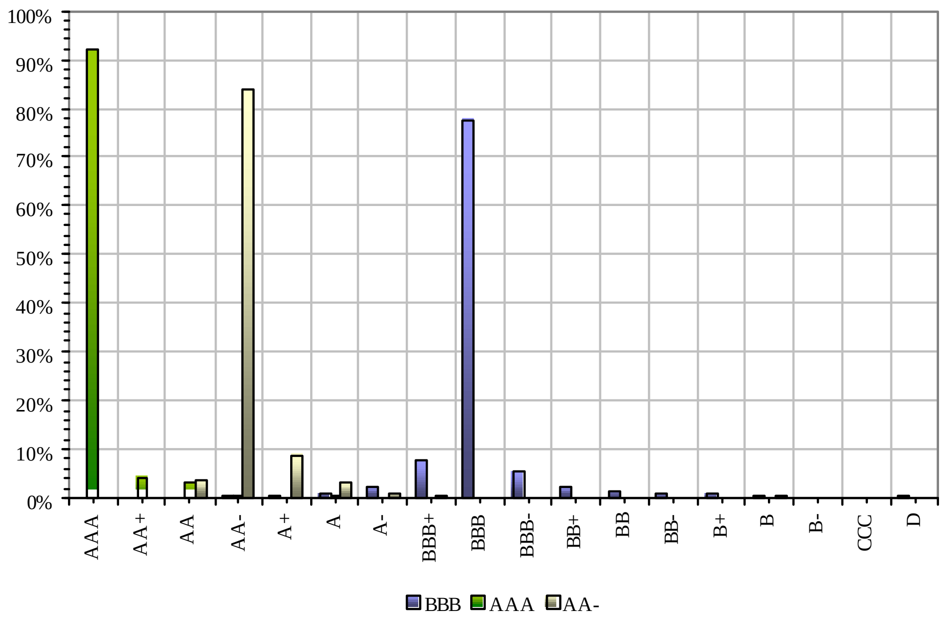

Figure 4: Corporate rating migration for industry-sector, Source : Standard Poor's.

Bond Valuation with Default Risk

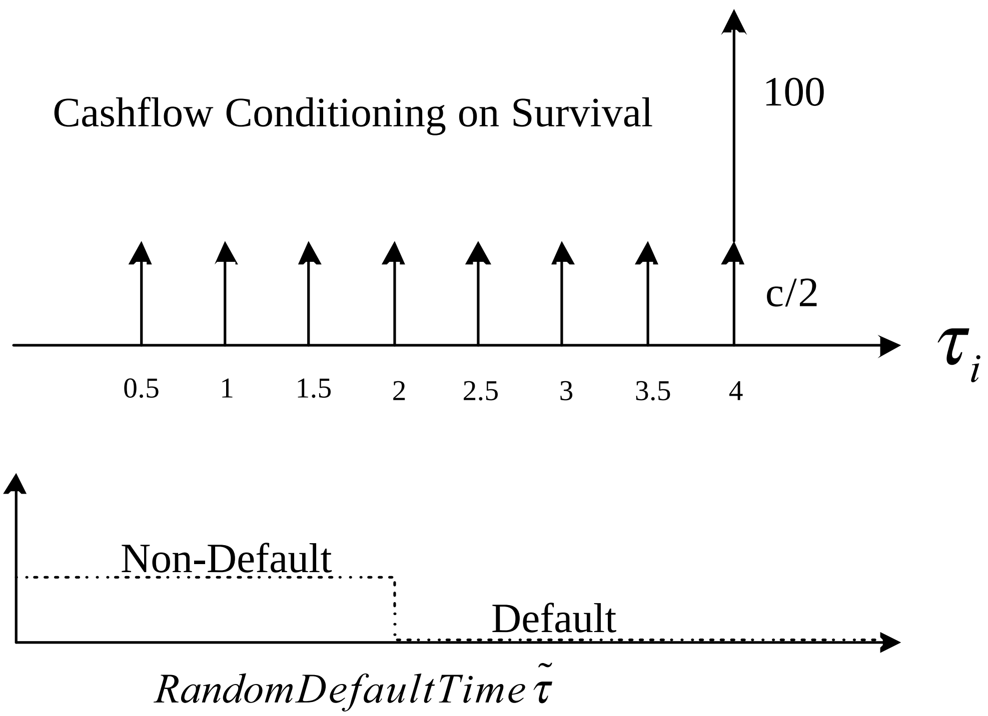

Figure 5: Chart cash flow Conditioning on survival.

Assuming no default risk,

$$P_0 = \sum_{i=1}^{8} e^{r \cdot t_i} + 100 \cdot e^{r \cdot 4} \tag{1}$$

How does the default risk affect the bond price?

Modelling Default Risk

Modelling default risk is central to the pricing and hedging of credit sensitive instruments.

Two approaches to modelling default risk:

- Structural approach, "first-passage": default happens when the total asset value of the firm falls below a threshold value (for example, the firm's book liability) for the first time.

- Reduced-form, "intensity-based": the random default time $\widetilde{\tau}$ is governed by an intensity process λ.

For pricing purpose, the reduced-form approach is adequate, and will be the focus of this class.

Modelling Random Default Times

The probability of survival up to time t:

$$Prob(\widetilde{\tau} \geq t) \tag{2}$$

The probability of default? before time t:

$$Prob\,(\widetilde{\tau} < 0) = 1 - Prob\,(\widetilde{\tau} \geq t) \tag{3}$$

We assume that $\widetilde{T}$ is exponentially distributed with constant default intensity λ:

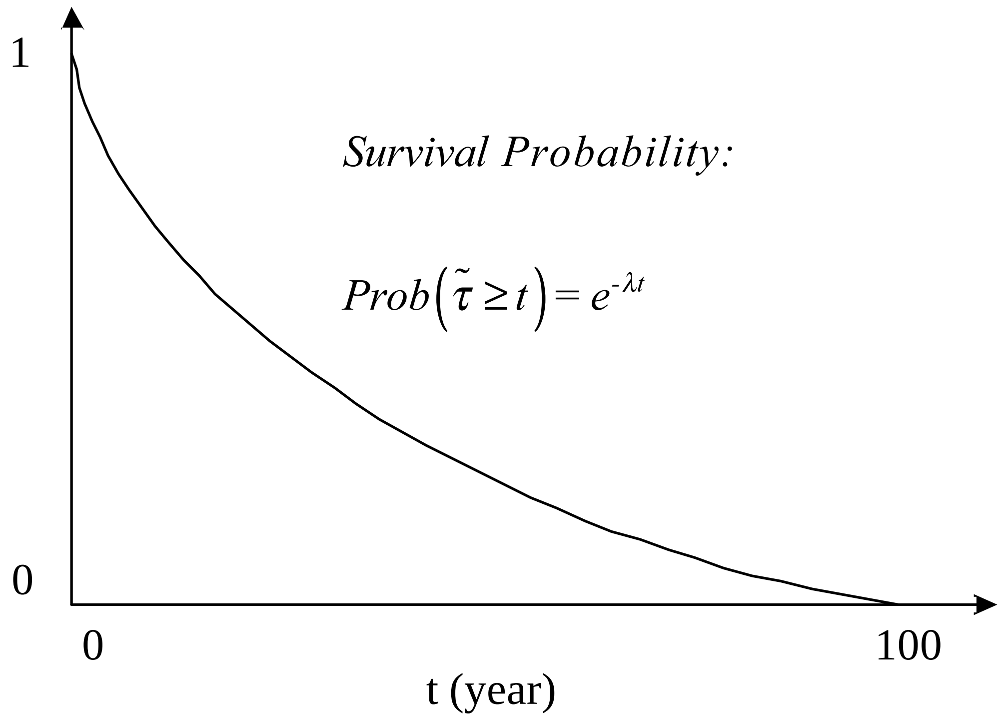

Figure 6: Survival Probability.

Default Probability and Credit Quality

One-Year default probability $= 1 - e^{\lambda}$

Default intensity $\lambda =?$

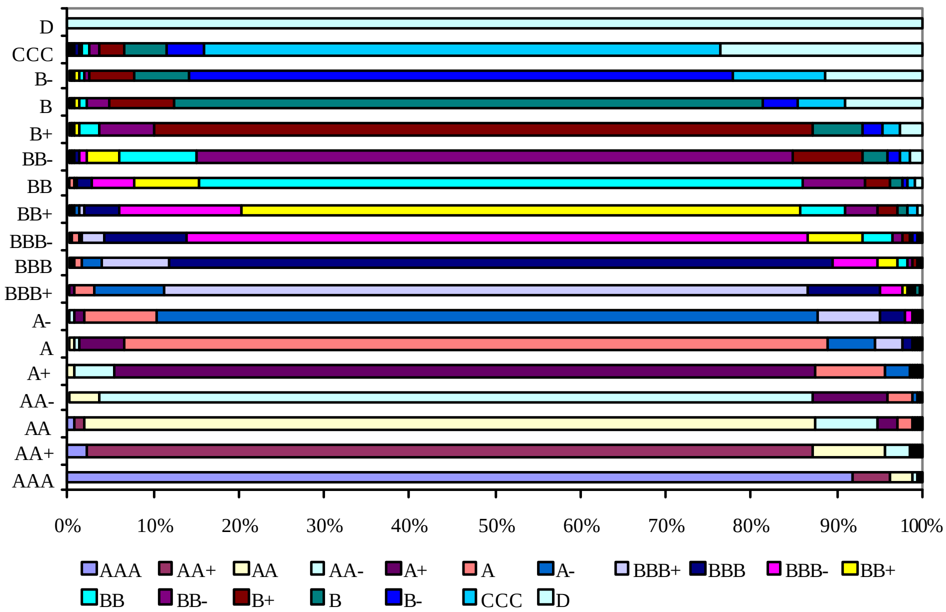

Figure 7: Survival Probability.

	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB
AAA	91.95%	4.11%	2.86%	0.48%	0.16%	0.20%	0.12%	0.04%	0.04%
AA+	2.31%	84.71%	8.75%	2.88%	0.19%	0.48%	0.10%	0.00%	0.38%
AA	0.62%	1.36%	85.42%	7.24%	2.60%	1.49%	0.25%	0.50%	0.22%
AA-	0.00%	0.15%	3.44%	83.67%	8.61%	3.02%	0.50%	0.23%	0.15%
A+	0.00%	0.03%	0.83%	4.47%	82.27%	8.08%	2.75%	0.46%	0.40%
A	0.08%	0.06%	0.49%	0.66%	5.25%	82.50%	5.44%	3.18%	1.11%
A-	0.14%	0.04%	0.11%	0.35%	1.13%	8.58%	77.39%	7.21%	3.00%
BBB+	0.00%	0.00%	0.08%	0.13%	0.59%	2.26%	8.32%	75.24%	8.36%
BBB	0.07%	0.03%	0.07%	0.17%	0.45%	0.93%	2.24%	7.83%	77.76%
BBB-	0.05%	0.00%	0.11%	0.21%	0.11%	0.69%	0.59%	2.67%	9.46%
BB+	0.17%	0.00%	0.00%	0.08%	0.08%	0.51%	0.34%	0.67%	4.21%
BB	0.00%	0.00%	0.12%	0.06%	0.06%	0.37%	0.18%	0.31%	1.59%
BB-	0.00%	0.00%	0.00%	0.05%	0.09%	0.05%	0.28%	0.33%	0.52%
B+	0.00%	0.03%	0.00%	0.10%	0.00%	0.03%	0.23%	0.10%	0.13%
B	0.00%	0.00%	0.07%	0.00%	0.00%	0.14%	0.21%	0.00%	0.14%
B-	0.00%	0.00%	0.00%	0.00%	0.18%	0.00%	0.00%	0.36%	0.00%
CCC	0.19%	0.00%	0.00%	0.00%	0.19%	0.00%	0.19%	0.19%	0.56%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

Figure 8: Survival Probability, Migration table, Source: RiskMetrics^{TM}.

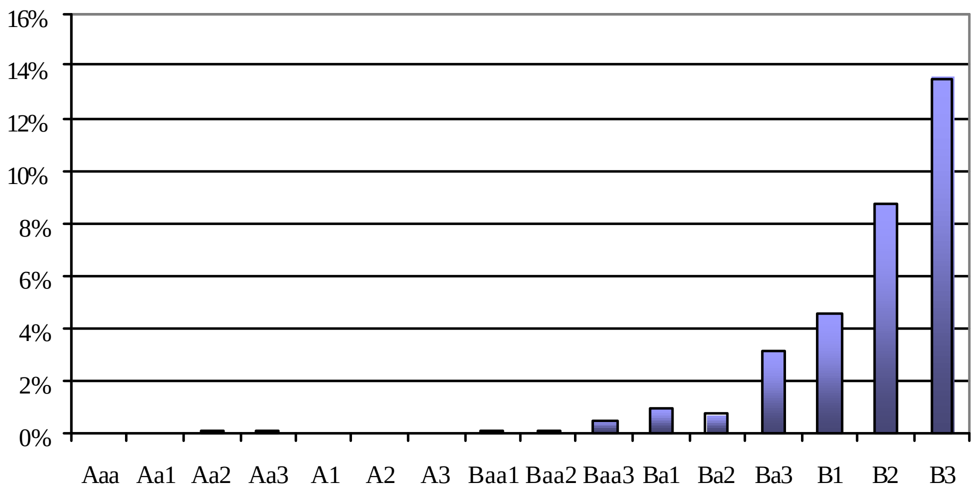

Figure 9: One-Year Default Rates by Modified Ratings, 1983-1995, Source: Moodys (1996).

Pricing A Defaultable Bond

For simplicity, let's first assume that the riskfree interest rate r is a constant. Consider a τ-year zero-coupon bond issued by a firm with default intensity λ:

$$P_0 = \$100 \cdot e^{-r\cdot\tau} \cdot Prob(\widetilde{\tau} \geq \tau) \tag{4}$$

$$P_0 = \$100 \cdot e^{-r\cdot\tau} \cdot e^{-\lambda\cdot\tau} \tag{5}$$

$$P_0 = \$100 \cdot e^{-(r+\lambda)\cdot\tau} \tag{6}$$

where we assume that conditioning on a default, the recovery value of the bond is 0 (we have also assumed risk-neutral pricing).

The yield on the defaultable bond is $r + \lambda$, resulting in a credit spread of λ.

Time Variation of Default Probability

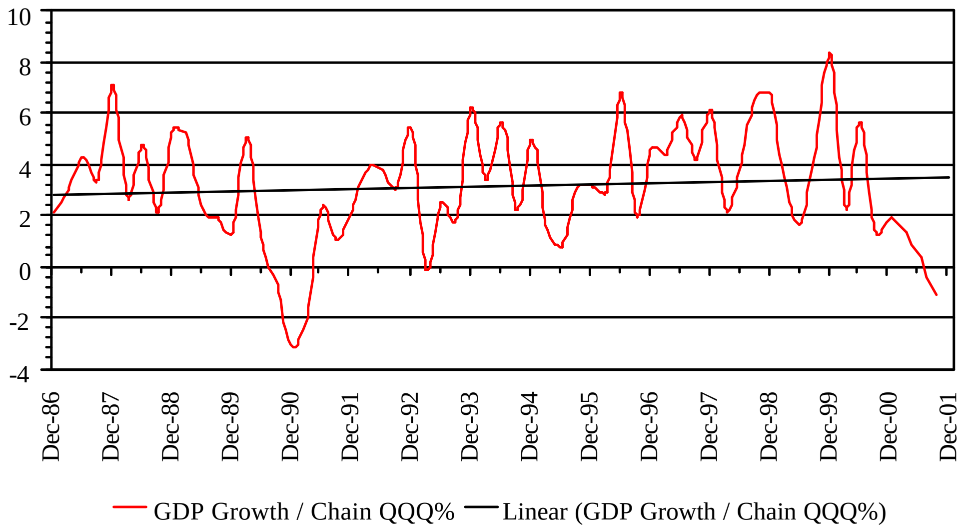

Figure 10: Chart Annual GDP Growth Rate, source: Bureau of Economic Analysis Stochastic

Default Intensity

In general, the credit quality of a firm changes over time.

A more realistic model is to treat the arrival intensity as a random process.

Suppose that intensities are updated with new information at the beginning of each year, and are constant during the year. Then the probability of survival for t years is

$$E\left(e^{-\lambda_0+\lambda_1+\cdots+\lambda_{t-1}}\right) \tag{7}$$

For example,

$$\lambda_{t+1}-\lambda_t=k\left(\bar{\lambda}-\lambda_t\right)+\varepsilon_{t+1} \tag{8}$$

Can you calculate the probability of survival for τ years? What is the price of a τ-year zero-coupon bond? What if the riskfree interest rate is also stochastic?

Example: A portfolio consists of two long assets \$100 each. The probability of default over the next year is 10% for the first asset, 20% for the second asset, and the joint probability of default is 3%. What is the expected loss on this portfolio due to credit risk over the next year assuming 40% recovery rate for both assets.

Probabilities:

$$0.1 \cdot (1 - 0.2) \quad - \quad default\ probability\ of\ A \tag{9}$$

$$0.2 \cdot (1 - 0.1) \quad - \quad default\ probability\ of\ B \tag{10}$$

$$0.03 \quad - \quad joint\ default\ probability \tag{11}$$

Expected losses:

$$0.1 \cdot (1 - 0.2) \cdot 100 \cdot (1 - 0.4) = 4.8 \tag{12}$$

$$0.2 \cdot (1 - 0.1) \cdot 100 \cdot (1 - 0.4) = 10.8 \tag{13}$$

$$0.03 \cdot 200 \cdot (1 - 0.4) = 3.6 \tag{14}$$

$$4.8 + 10.8 + 3.6 = \$19.2\ mio. \tag{15}$$

Example: Assume a 1-year US Treasury yield is 5.5% and a Eurodollar deposit rate is 6%. What is the probability of the Eurodollar deposit to default assuming zero recovery rate)?

$$\frac{1}{1.06} = \frac{1 - \pi}{1.055} \tag{16}$$

$$\pi = 0.5\% \tag{17}$$

Example: Assume a 1-year US Treasury yield is 5.5% and a and a default probability of a one year CP is 1%. What should be the yield on the CP assuming 50% recovery rate?

$$\frac{1}{1+x} = \frac{1-\pi}{1.055} + \frac{0.5\pi}{1.055} \quad (18)$$

$$= 6\% \quad (19)$$

Some Practitioner's Credit Risk Model

RiskMetrics: $CreditMetrics^{TM}$

http://riskmetrics.com/research

Credit Suisse Financial Products: CreditRisk+

http://www.csfb.com/creditrisk

KMV Corporation / $CreditMonitor^{TM}$

http://www.kmv.com

Focus:

BKM Chapter 14

- p. 415-422 (definitions of instruments, innovation in the bond market)
- p. 434-441 (determinants of bond safety, bond indentures)

Style of potential questions: Concept check questions, p. 448 ff. question 31

Questions for Next Class

Please read:

- Reyfman,
- Toft (2001), and
- Altman, Caouette, Narayanan (1998).