

15.433 INVESTMENTS

Classes 8 & 9: The Equity Market

Cross Sectional Variation in Stock Returns

Spring 2003

Introduction

Equities are common stocks, representing ownership shares of a corporation. Two important characteristics:

- limited liability: non-negative stock prices,
- residual claim: equities are inherently more risky than fixed income securities.

In most countries, the equity market is perhaps the most popular venue of investments for individual investors. It also remains to be an important component of institutional investments. We will examine the equity market from two perspectives:

- cross-sectional (Classes 8 & 9), and
- time-series (Class 10).

”Cross-Section” vs. ”Time-Series”

These two concepts are empirically motivated. For a publicly traded firm i , the following information can be readily obtained.

- The stock price $P_{i,t}$ at any time t .
- The cash dividend $D_{i,t-1}$ paid between $t-1$ and t .

At any time t , we can calculate the realized stock return for $r_{i,t}$ for firm i :

- percentage returns: $r_{i,t} = \frac{P_{i,t} + D_{i,t} - P_{i,t-1}}{P_{i,t-1}}$
- log-returns: $r_{i,t} = \ln(P_{i,t} + D_{i,t}) - \ln(P_{i,t-1})$
- cross-section of stock returns: $r_{i,t}; \quad i = 1, 2, \dots, N$
- time series of stock returns: $r_{i,t}; \quad i = 1, 2, \dots, T$

The Cross-Sectional Distribution of Returns

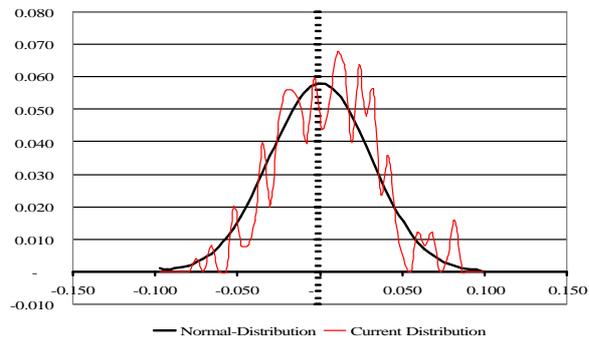


Figure 1: Distribution of the Nasdaq-index returns for the year 2000, source: Bloomberg Professional

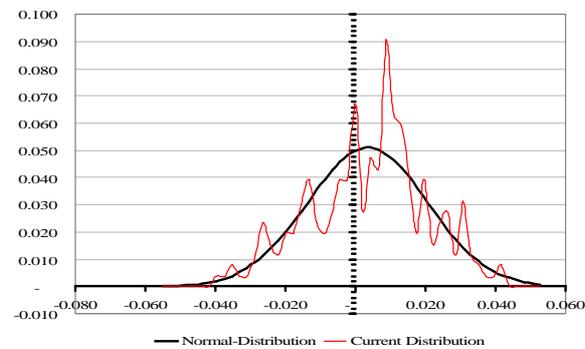


Figure 2: Distribution of the Nasdaq-index returns for the year 1999, source: Bloomberg Professional

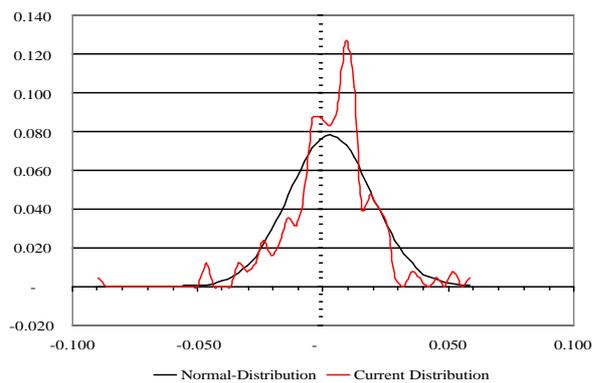


Figure 3: Distribution of the Nasdaq-index returns for the year 1998 (from left to right), source: Bloomberg Professional

What has changed, what has influenced the market?

Multi-Factor Regressions

For each asset i , we use a multi-factor time-series regression to quantify the asset's tendency to move with multiple risk factors:

$$r_{i,t-1} - r_{f,t} = \alpha_i + \beta_i (r_{M,t} - r_{f,t}) + f_i F_t + \varepsilon_{i,t}$$

- 1. Systematic Factors:

$$r_{M,t} : \text{risk premium } \lambda_M = E(r_{M,t} - r_f)$$

$$F_t : \text{risk premium } \lambda_F = E(F_t)$$

- 2. Idiosyncratic Factors:

$$\varepsilon_{i,t} : \text{no risk premium } E(\varepsilon_{i,t})$$

- 3. Factor Loadings:

$$\beta_{i,t} : \text{firm's sensitivity to the market risk}$$

$$f_{i,t} : \text{firm's sensitivity to the factor risk}$$

See BKM
p. 559 - 572
and article
from Fama
(1992)

The Pricing Relation

Using the intuition of the CAPM, the reward for asset i should be related to its exposure to the market risk, as well as its exposure to the systematic risks.

Given the risk premia of the systematic factors, e.g., λ_m and λ_F , the determinants of expected returns:

$$E(r_{i,t-1} - r_{f,t}) = \beta_i \cdot \lambda_M + f_i \cdot \lambda_F \quad (1)$$

Without the systematic factors F , we are back to the CAPM.

What are the additional systematic factors?

The intuition of the CAPM: these factors should be proxies for the real, macroeconomic, aggregate, non-diversifiable risk.

Size: Small or Big

We can sort the cross-section of stocks by their Market Capitalizations:

"Share Price x Number of Shares Outstanding"

| Cap Decile | Market Cap(m\$) | NYSE Ticker | NYSE Stocks | AMEX Stocks | NASDAQ Stocks | Total |
|------------|-----------------|-------------|-------------|-------------|---------------|-------|
| 10 | 511,391 | GE | 172 | 5 | 80 | 257 |
| 9 | 10,486 | NSM | 172 | 3 | 81 | 256 |
| 8 | 4,428 | GLM | 172 | 5 | 136 | 313 |
| 7 | 2,237 | BLC | 172 | 5 | 166 | 343 |
| 6 | 1,387 | GES | 172 | 5 | 217 | 394 |
| 5 | 889 | SFG | 172 | 11 | 254 | 437 |
| 4 | 534 | PNK | 172 | 15 | 251 | 438 |
| 3 | 353 | FFD | 172 | 32 | 400 | 604 |
| 2 | 198 | SXI | 172 | 73 | 551 | 796 |
| 1 | 95 | AVS | 172 | 412 | 1,399 | 1,983 |

Size Range as of December 31, 2000. Source: www.dfafunds.com.

Value or Growth

We can also sort the cross-section of stocks by their Book-to-Market (BtM) ratios:

- Growth Stocks: Firms with low BtM ratios
- Value Stocks: Firms with high BtM ratios.

| Decile | BtM | NYSE Ticker | NYSE Stocks | AMEXS Stocks | NASDAQ Stocks | Total |
|--------|------|----------------|----------------|-----------------|------------------|-------|
| 1 | 0.01 | IN | 155 | 71 | 824 | 1,050 |
| 2 | 0.14 | SYJ | 155 | 31 | 362 | 548 |
| 3 | 0.25 | TDX | 155 | 36 | 223 | 414 |
| 4 | 0.36 | STJ | 155 | 24 | 177 | 356 |
| 5 | 0.45 | FLO | 155 | 37 | 229 | 421 |
| 6 | 0.58 | DOL | 155 | 43 | 248 | 446 |
| 7 | 0.72 | HCC | 155 | 56 | 274 | 485 |
| 8 | 0.92 | TWR | 155 | 51 | 251 | 457 |
| 9 | 1.19 | MTN | 155 | 71 | 279 | 505 |
| 10 | 1.81 | ZAP | 155 | 58 | 195 | 408 |

Value and Growth Definitions as of December 31, 2000. Source: www.dfafunds.com

Portfolios Formed on Size and BtM

Sort the stocks traded on the NYSE, NASDAQ, and AMEX by their size and BtM.

Size labels: A (small), B, C, D, and E (big).

BtM labels: 1 (low), 2, 3, 4, and 5 (high).

- low BtM growth stocks
- high BtM value stocks

The 25 Fama-French Portfolios:

| | A (s) | B | C | D | E (b) |
|-------|----------|---|---|---|----------|
| 1 (l) | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 (h) | | | | | |

Explain the Fama-French Portfolios

Start with the one-factor empirical model:

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i (r_{M,t} - r_{f,t}) + f_i F_t + \varepsilon_{i,t} \quad (2)$$

For each portfolio i , we perform the above regression and obtain an estimate $\hat{\beta}_i$ of the factor loading β_i . This regression procedure is equivalent to constructing sample estimates for β_i , why?

Estimate the market risk premium:

$$\hat{\lambda}_M = \frac{1}{T} \sum_{i=1}^T (r_{M,t} - r_{f,t}) \quad (3)$$

The mean excess return: $E(r_{i,t} - r_{f,t})$

- The model predicted: $\hat{\beta}_i \cdot \hat{\lambda}_M$
- Measured from the data: $\frac{1}{T} \sum_{i=1}^T (r_{i,t} - r_{f,t})$

A One-Factor Model (CAPM)

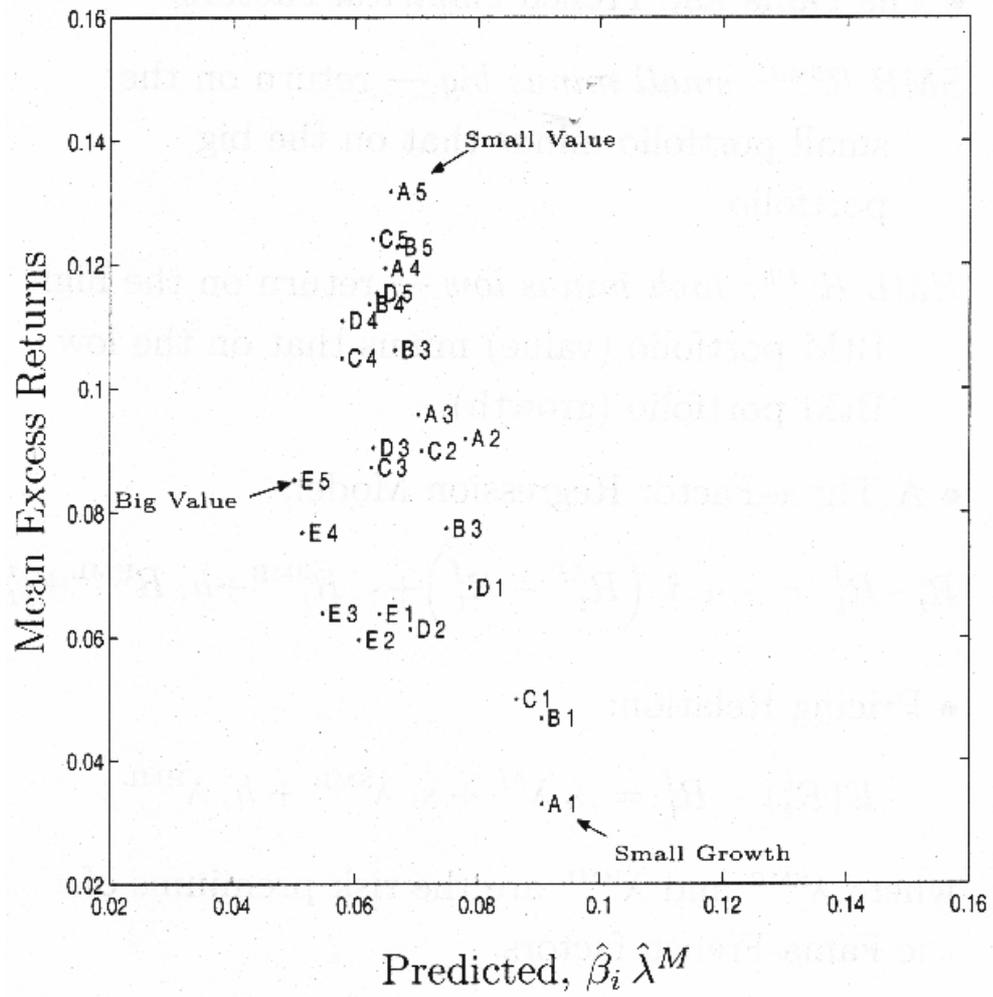


Figure 4: One-factor model, Source: Jun Pan, Investments 15.433 Spring 2001.

A Three-Factor Model

The Fama and French empirical Factors:

- SMB r_{smb} : small minus big - return on the small portfolio minus that on the big portfolio;
- HML r_{hml} : high minus low - return on the high BtM portfolio (value) minus that on the low BtM portfolio (growth).

A three-factor regression model:

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i (r_{M,t} - r_{f,t}) + s_i \cdot r_{smb,t} + h_i \cdot r_{hml,t} + \varepsilon_{i,t} \quad (4)$$

Pricing Relation:

$$E(r_{i,t}) - r_{f,t} = \beta_i \cdot \lambda_M + s_i \cdot \lambda_{smb,t} + h_i \cdot \lambda_{hml,t} \quad (5)$$

where λ_{smb} and λ_{hml} are the risk premiums of the Fama-French factors.

The Fama-French Three-Factor Model

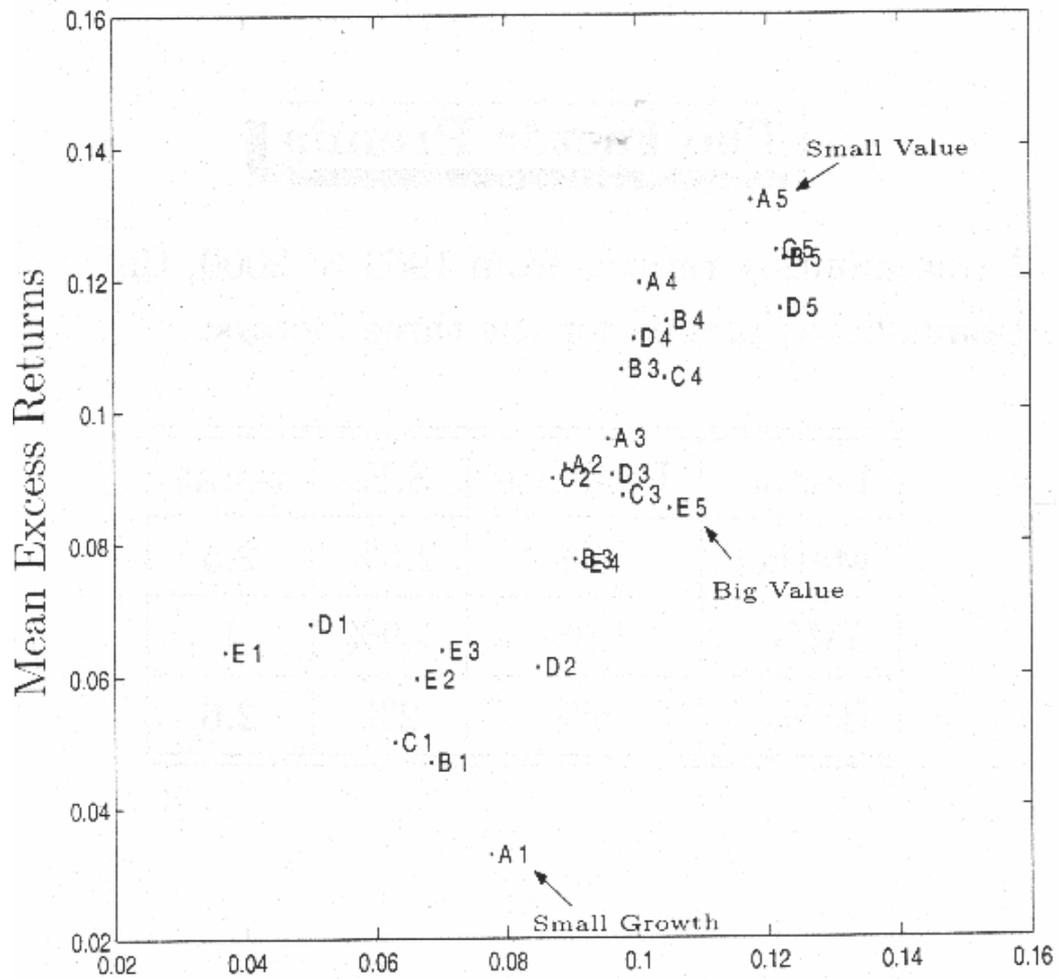


Figure 5: Fama-French three factor model, Source: Jun Pan, Investments 15.433 Spring 2001.

The Factor Premia

Using monthly returns from 1963 to 2000, the (annualized) premia for the three factors are:

| Factor | Estimate | S.E. | t-stat |
|--------|----------|------|--------|
| Market | 6% | 2.5% | 2.5% |
| SMB | 1.9% | 1.9% | 1% |
| HML | 5% | 2% | 2.6% |

The Market Risk Premium

The market risk premium has its foundation in the CAPM.

Investors are risk averse. They are worried about holding stocks that do badly at the times when the market does badly.

The market risk premium is a reward for holding the market risk.

What are the Size and Value Factors?

Unlike the market portfolio, the Size and Value portfolios are empirically motivated. Where do the size and value premia come from?

If we think of them as risk premia, then we need to understand the real, macroeconomic, aggregate, non-diversifiable risk that is proxied by the SMB and HML portfolios.

In particular, why are investors so concerned about holding stocks that do badly at the times that the hml and smb portfolios do badly, even though the market does not fall?

Some Explanations

Value: proxies for the "distress risk".

Size: proxies for the illiquidity of the stock.

HML and SMB contain information above and beyond that in the market return for forecasting GDP growth.

Proxies for variables that forecast time-varying investment opportunities or time-varying risk aversion.

Over-reaction: earnings announcements.

Seasonal: the January effect.

Survival bias.

Data snooping

Other Factors

Empirical Factors: price-to-earning ratios, strategies based on five-year sales growth, etc.

Macroeconomic Factors: labor income, industrial production, inflation, investment growth, consumption wealth ratio, etc.

The Market Skewness Factor:

- If asset returns have systematic skewness, expected returns should include rewards for accepting this risk.
- Co-Skewness: the level of exposure to the systematic skewness.
- Harvey and Siddique (*Journal of Finance 2000*) report that systematic skewness is economically important and commands an average risk premium of 3.60 per year.

Long-Term Reversals

Firms whose three- and five-year returns are high (low) tend to have low (high) returns in subsequent years.

Firms with low (high) BE/ME, E/P, CF/P, D/P, and prior sales growth tend to have low (high) returns in subsequent years.

All these patterns seem to be manifestations of the same value vs. growth phenomenon.

This "reversal" effect makes sense given return predictability and mean-reversion, and is explained by the Fama-French three factor model.

Short-Term Momentum

Firms with high returns in the prior year tend to have high returns in the next few months.

Firms with low short-term returns tend to have low returns in subsequent months.

At the moment, the momentum effect is the most-studied anomaly in Finance. It cannot be explained by the Fama-French three factor model.

Risk-based stories: Proxy for systematic skewness: the low expected return momentum portfolios (losers) have higher skewness than high expected return portfolios (winners).

Behavioral (non-risk- based) stories:

- 1. underreaction: bad news travels slowly;
- 2. overreaction: positive feedback;
- 3. overconfidence.

Beta Hedging

Recall that:

$$E(r_i) = r_f + \beta_i(E(r_M) - r_f) \quad (6)$$

where

$$\beta_i = \frac{\text{cov}(r_M, r_i)}{\text{var}(r_M)} \quad (7)$$

Let's assume we are long-beta, e.g. $\beta = 1.3$, and we are not optimistic about the short-term outlook of the market. We anticipate some bade earnings numbers, which will send the market down ...and our portfolio even faster. Let's hedge our **Beta-Exposure!**

$$\frac{\Delta P}{P} \approx \beta \frac{\Delta M}{M} \quad (8)$$

A stock index futures contract worth:

$$\frac{\Delta P}{P} \approx \beta \frac{\Delta M}{M} \quad (9)$$

The generated change in portfolio value ΔV due to an adverse change in the market ΔM is:

$$\Delta V = \Delta P + N \Delta F \quad (10)$$

$$= \beta \cdot P \frac{\Delta M}{M} + N \cdot F \frac{\Delta M}{M} \quad (11)$$

The optimal $N^* = -\frac{\beta \cdot P}{F}$ (12) The optimal hedge with a stock index futures is given by beta of the portfolio times its value divided by the notional of the futures contract.

Example: A portfolio manager holds a stock portfolio worth \$10 mio., with a beta of 1.5 relative to S&P 500. The current S&P 500 index futures price is 1400, with a multiplier of \$250.

Compute:

1. The notional of the futures contract
2. the optimal number of contracts to hedge the beta-exposure against adverse market movements.

Solution:

1. The notional of the futures contract is:

$$\$250 \cdot 1'400 = \$350'000 \quad (13)$$

2. the optimal number of contracts to hedge the beta-exposure against adverse market movements is:

$$N^* = -\frac{\beta \cdot P}{F} = -\frac{1.5 \cdot \$10'000'000}{1 \cdot \$350'000} = -42.9 \quad (14)$$

However:

A typical US stock has a correlation of 50% with the S&P 500-index. Using the *regression effectiveness* we find that the volatility of the hedged portfolio is still about $(1 - 0.5^2 = 87\%)$ of the unhedged volatility for a typical stock.

If we wish to hedge an industry index with S&P futures, the correlation is about 75% and the unhedged volatility is 66% of its original level.

The lower number shows that stock market hedging is more effective for diversified portfolios.

Summary

The cross-sectional variation in stock returns cannot be fully explained by beta. Adding additional factors (size and value) helps.

While the risk premium associated with the beta risk has its theoretical foundation in the CAPM, the premia associated with the size and value factors do not.

There are many other puzzling patterns in stock returns, some of which are hard to reconcile with market efficiency.

Focus: Patterns in the Cross-Section of Stock Returns

- Reader: Fama and French (1992)

type of potential questions: how is value / growth style etc. defined? What was the general setup of the style analysis?

Focus: More Patterns in the Cross-Section of Stock Returns

- Cochrane (1999),
- Kritzman (1991a), and
- Kritzman (1991b)

type of potential questions: Cochrane: p. 39 to 43, 50 to 51, Kritzman: historical vs. implied volatility, normal assumption of volatility vs. nonlinearity

Questions for Next Class

Please read for class 9:

- BKM 13.4 and 13.5
- Cochrane (1999), Kritzman (1991a) and Kritzman (1991b)

In the next class, we will examine stock returns from a time-series perspective.

Our focus is on dynamic models that allow for:

- 1. time varying expected return μ_t
- 2. and time-varying volatility σ_t

Why are these issues important?

Please read for class 10: BKM Chapters 20.