Presentation based on:
Whitt, W. "Improving Service by Informing Customers About Anticipated Delays." *Management Science* 45 (2), 1999.

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This summary presentation is based on: Whitt, Ward. "Improving Service by Informing Customers About Anticipated Delays." ***Management Science*** **45, no. 2 (1999): 192-207.**

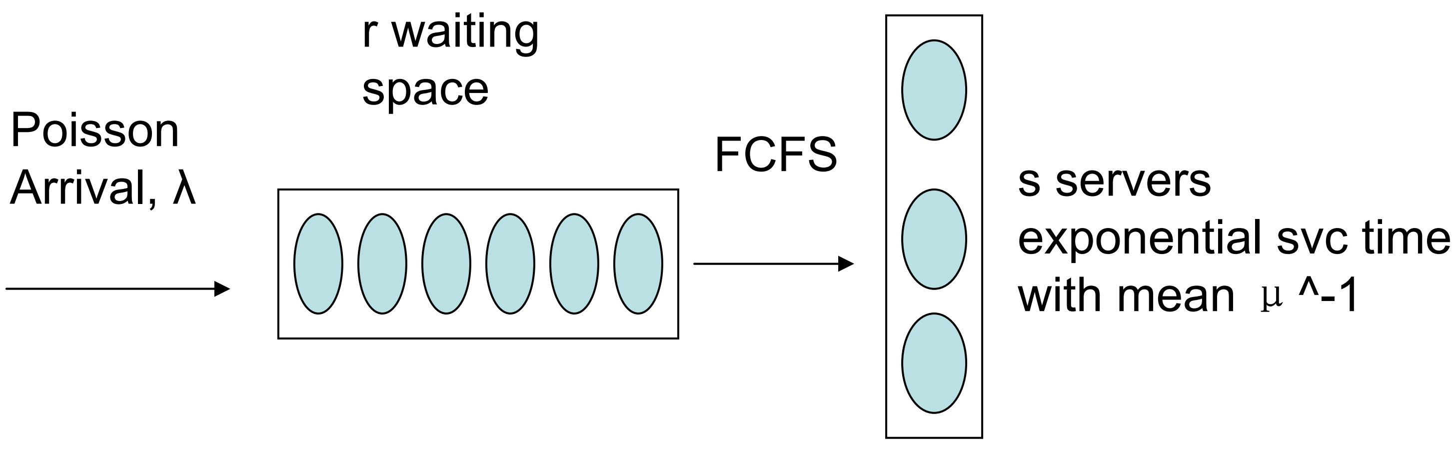
Motivation

- Investigate alternative ways to manage a service system, eg. Call Centers.
- Use Birth-and-Death (BD) stochastic process models to model 2 types of service systems
  - Conventional queues allowed with no info
  - Queues with delay or state information
- Of value to both customers and service providers

Two alternative queuing systems

- First: provide waiting room but no info on state or queuing time
  - No balking but customer may renege
- Second: provide waiting room but info on either state or queuing time
  - Higher balking rate relative to renege
  - Information about anticipated delays increases customer satisfaction, resulting in more repeat business
  - Increasing capability for service providers to provide delay info (Rappaport 1996)

M/M/s/r Model 1



System state not known by customers

Independent α and β

If a server is not immediately available customer balk with probability β

Then, customer waits till T is reached before reneging

→ Model with time dependent reneging

Model 1

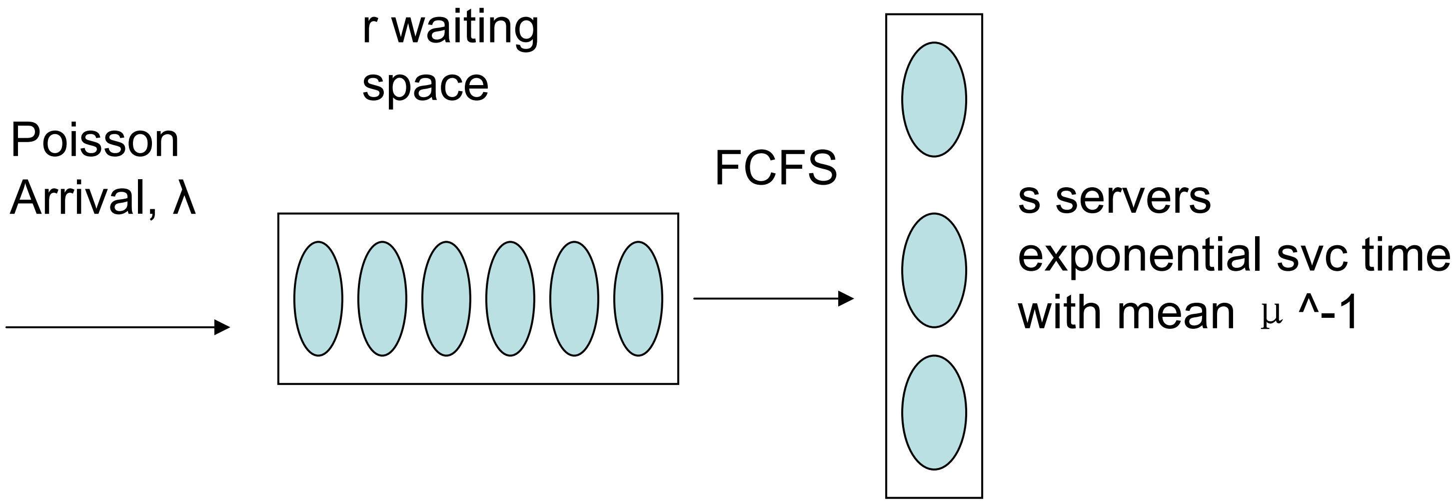
- (See explanation and variable definitions in section 2, page 194 of the Whitt paper.)
- Characterize by

$$\lambda_k = \begin{cases} \lambda & 0 \le k \le s-1 \\ \lambda(1-\beta) & s \le k \le s+r-1 \end{cases}$$

$$\mu_k = \begin{cases} k\mu & 1 \le k \le s-1 \\ s\mu + (k-s)\alpha & s \le k \le s+r \end{cases}$$

- Pk state probabilities are easy to calculate!

M/M/s/r Model 2



System state now communicated to customers upon arrival

Dependent α and β

Balking now depends on state of system

State dependent balking replaces reneging after waiting

→ Model with mainly state dependent balking plus some reneging

Model 2

- Case 1: Required waiting time is given as state information
- If waiting time > T, customer balks
- If not all servers are occupied, customer is served immediately
- If all servers are occupied, customer either balks or stays with probability

$$q_k \equiv P(T > S_k) \qquad 0 \leq k \leq r-1$$

Where Sk : time from arrival until first served where state at time is k.

Model 2

- To find the state dependent probability of joining in an exact manner,

$$q_k = \int_0^\infty e^{-\alpha t} g_k(t)dt = Ee^{-\alpha S_k} = \left(\frac{s\mu}{s\mu + \alpha} \right)^{k+1}$$

- To find a reasonable approximation of the state dependent probability of joining,

$$\overline{q}_k \equiv P(T > ES_k) = e^{-\alpha (k+1)/s\mu}, \quad k \geq 0$$

Model 2

- To add state dependent reneging to generalize model 2, define
- δ'j : renege rate of customer with j-1 customers ahead in queue
- Total renege rate, $\delta_k = \sum_{j=1}^{k} \delta'_j$
- BD process can be characterized by

$$\lambda_k = \begin{cases} \lambda & 0 \le k \le s-1 \\ \lambda(1-\beta)q_{k-s} & s \le k \le s+r-1 \end{cases}$$

$$\mu_k = \begin{cases} k\mu & 1 \le k \le s \\ s\mu + \delta_{k-s} & s+1 \le k \le s+r \end{cases}$$

Finding Performance Measures

- Step 1: Find the steady state distribution
- Step 2: Calculate probability of completing service and the mean, variance and full distribution of the conditional response time given that service is completed.
- Step 3: Calculate probability of customer reneging and the mean, variance and full distribution of the conditional time to renege given that customer reneges.

Stochastic Comparisons

- Consider Models 1 and 2 with all basic parameters fixed
- In reality parameters will change, as information increases customer satisfaction, arrival rates will increase, leading to increase in the number of servers, leading to higher service satisfaction
- Use existing tools for comparison (see Shaked and Shantikumar 1994)
  - Likelihood ratio ordering

Likelihood Ratio Ordering

(See section 4, pages 199-200 of the Whitt paper, particularly the explanation surrounding equations 4.1 and 4.2)

Stochastic Comparison

(See Theories 4.1, 4.2, 4.3, and 4.4 on pages 200-1 of the Whitt paper)

Numerical Example

- Economies of scale: All performance measures improve as s increases
- Two systems do not differ much, differences reduce as s gets larger

(See Table 1 on page 202 of the Whitt paper)

Critique

- No clear literature reviews and contributions
- Assume first paper?
- Use of k as system state and others..confusing