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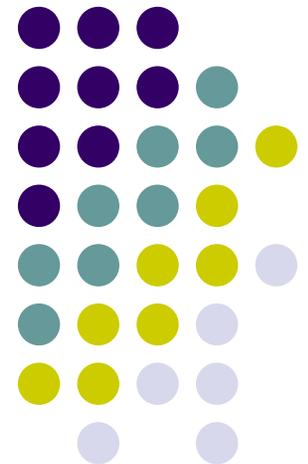
11.481J / 1.284J / ESD.192J Analyzing and Accounting for Regional Economic Growth  
Spring 2009

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# Multipliers in Input-Output Model

Presentation to 11.481J

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# Introduction

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- So far, we have learned the relationship between gross output and final demand based on Leontief Inverse Matrix:

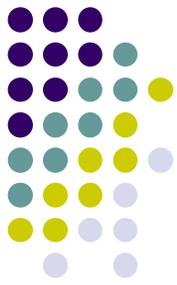
$$X = (I - A)^{-1} Y$$

- We are more interested in assessing the effect on an economy of changes in final demand elements that are exogenous to the model of that economy.

$$\Delta X = (I - A)^{-1} \Delta Y$$

# General Structure of Multipliers:

## Output Multiplier



- The simple output multiplier

$$\frac{(\text{direct output} + \text{indirect output})}{\text{direct output}}$$

Direct and indirect impacts on gross output when the final demand for the  $j^{\text{th}}$  sector changes by one unit, holding All other sectors constant.

$$O_j = \sum_{i=1}^n \alpha_{ij}$$

Leontief coefficient

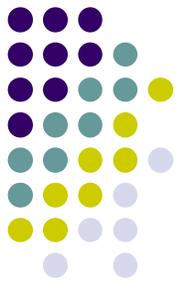
- The total output multiplier

$$\frac{(\text{direct} + \text{indirect} + \text{induced output})}{\text{direct output}}$$

$$\bar{O}_j = \sum_{i=1}^{n+1} \bar{\alpha}_{ij}$$

Leontief coefficient with households included in the input coefficient matrix

# Example



$$A = \begin{pmatrix} .15 & .25 \\ .20 & .05 \end{pmatrix}$$

$$(I - A)^{-1} = \begin{pmatrix} 1.254 & .330 \\ .264 & 1.122 \end{pmatrix}$$

$$O_1 = 1.254 + .264 = 1.518$$

$$O_2 = .330 + 1.122 = 1.452$$



Simple Output  
Multiplier

$$\bar{A} = \begin{pmatrix} .15 & .25 & .05 \\ .20 & .05 & .40 \\ .30 & .25 & .05 \end{pmatrix}$$

Household

Household

$$(I - \bar{A})^{-1} = \begin{pmatrix} 1.365 & .425 & .251 \\ .527 & 1.348 & .595 \\ .570 & .489 & 1.289 \end{pmatrix}$$

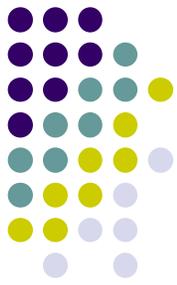
$$\bar{O}_1 = 1.365 + .527 + .570 = 2.462$$

$$\bar{O}_2 = .425 + 1.348 + .489 = 2.262$$

Total Output  
Multiplier

Source: Miller and Blair (1985).

# General Structure of Multipliers: Income Multiplier (1)



- The simple household income multiplier  
portion of simple output effect that is household income

$$H_j = \sum_{i=1}^n a_{n+1,i} \alpha_{ij}$$

- The total household income multiplier
  - ⇒ sum of income effects in each sector, or
  - ⇒ income row of the inverse matrix

$$\bar{H}_j = \sum_{i=1}^{n+1} a_{n+1,i} \bar{\alpha}_{ij} = \bar{\alpha}_{n+1,j}$$



# Example

$$\bar{A} = \begin{pmatrix} .15 & .25 & .05 \\ .20 & .05 & .40 \\ \color{red}{.30} & \color{red}{.25} & .05 \end{pmatrix}$$

$$(I - A)^{-1} = \begin{pmatrix} 1.254 & .330 \\ .264 & 1.122 \end{pmatrix}$$

$$H_1 = (.3)(1.254) + (.25)(.264) = .442$$

$$H_2 = (.3)(.330) + (.25)(1.122) = .380$$



Simple Household  
Income Multiplier

$$\bar{A} = \begin{pmatrix} .15 & .25 & .05 \\ .20 & .05 & .40 \\ \color{red}{.30} & \color{red}{.25} & \color{red}{.05} \end{pmatrix}$$

Household ←

$$(I - \bar{A})^{-1} = \begin{pmatrix} 1.365 & .425 & .251 \\ .527 & 1.348 & .595 \\ .570 & .489 & 1.289 \end{pmatrix}$$

$$\bar{H}_1 = (.3)(1.365) + (.25)(.527) + (.05)(.570) = .570$$

$$\bar{H}_2 = (.3)(.425) + (.25)(1.348) + (.05)(.489) = .489$$



Total Household  
Income Multiplier

Source: Miller and Blair (1985).

# General Structure of Multipliers: Income Multiplier (2)



- Type I income multiplier

$$\left( \frac{\text{direct} + \text{indirect income}}{\text{direct output}} \right) / \left( \frac{\text{direct income}}{\text{direct output}} \right)$$

$$Y_j = \frac{H_j}{a_{n+1,j}} = \sum_{i=1}^n \frac{a_{n+1,k} \alpha_{ij}}{a_{n+1,j}}$$

- Type II income multiplier

$$\bar{Y}_j = \frac{\bar{H}_j}{a_{n+1,j}} = \sum_{i=1}^{n+1} \frac{a_{n+1,k} \bar{\alpha}_{ij}}{a_{n+1,j}}$$

# Example



$$\bar{A} = \begin{pmatrix} .15 & .25 & .05 \\ .20 & .05 & .40 \\ .30 & .25 & .05 \end{pmatrix}$$

$$(I - A)^{-1} = \begin{pmatrix} 1.254 & .330 \\ .264 & 1.122 \end{pmatrix}$$

$$Y_1 = \frac{(.3)(1.254) + (.25)(.264)}{.3} = 1.47$$

$$Y_2 = \frac{(.3)(.330) + (.25)(1.122)}{.25} = 1.52$$

↓  
Type I Income Multiplier

Household ↑

$$\bar{A} = \begin{pmatrix} .15 & .25 & .05 \\ .20 & .05 & .40 \\ .30 & .25 & .05 \end{pmatrix}$$

Household ←

$$(I - \bar{A})^{-1} = \begin{pmatrix} 1.365 & .425 & .251 \\ .527 & 1.348 & .595 \\ .570 & .489 & 1.289 \end{pmatrix}$$

$$\bar{Y}_1 = \frac{(.3)(1.365) + (.25)(.527) + (.05)(.570)}{.3} = 1.90$$

$$\bar{Y}_2 = \frac{(.3)(.425) + (.25)(1.348) + (.05)(.489)}{.25} = 1.96$$

↓  
Type II Income Multiplier

# General Structure of Multipliers: Employment Multiplier



- The simple household employment multiplier

$$E_j = \sum_{i=1}^n w_{n+1,i} \alpha_{ij}$$

- The total household employment multiplier

- Type I employment multiplier

$$W_j = \frac{E_j}{w_{n+1,j}} = \sum_{i=1}^n \frac{w_{n+1,i} \alpha_{ij}}{w_{n+1,j}}$$

- Type II employment multiplier

# Multipliers in Regional Models



## Single-Region Input-Output Model

- Assumptions

- The technology for each sector at the regional level is identical to the technology in that sector at the national level
- The local input ratio for sector  $j$

$$p_j^R = \frac{X_j^R - E_j^R}{(X_j^R - E_j^R + M_j^R)} \leq 1$$

Local input ratio

Regional output in sector  $j$

Regional exports of good  $j$

Regional imports of good  $j$

The diagram shows the equation for the local input ratio,  $p_j^R = \frac{X_j^R - E_j^R}{(X_j^R - E_j^R + M_j^R)} \leq 1$ . Red arrows point from the text labels to the corresponding terms in the equation: 'Local input ratio' points to  $p_j^R$ , 'Regional output in sector j' points to  $X_j^R$ , 'Regional exports of good j' points to  $E_j^R$ , and 'Regional imports of good j' points to  $M_j^R$ .



# Multipliers in Regional Models

- Major difference

- Regional Input-Coefficient Matrix ( $A^R$ )

$$A^R = \hat{P} * A,$$

National input coefficient matrix

$$\hat{P} = \begin{bmatrix} p_1^R & 0 & \dots & 0 \\ 0 & p_2^R & \dots & 0 \\ \dots & & \dots & \\ 0 & 0.. & & p_n^R \end{bmatrix}$$

→ Local input ratio coefficient matrix

- Impact of Final Demand

$$X^R = (I - A^R)^{-1} Y^R = (I - \hat{P}A)^{-1} Y^R$$

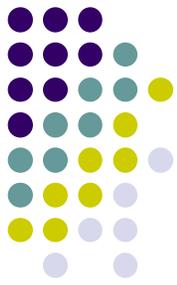
Regional Leontief inverse matrix (n\*n)

Regional exogenous final demand vector (n\*1)

Regional output vector (n \* 1)

# Example Application: Regional Input-Output Modeling System (RIMS II)

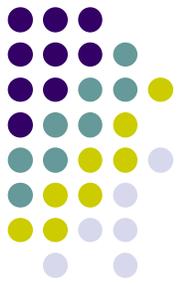
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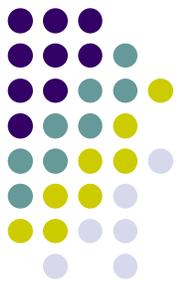
- Question: What is the total impact on a region of building a new sports facility?
- Analysis Process:
  - What is being studied?
  - What is the affected region?
  - What are the affected industries?
  - Is there more than one phase?
  - What are the initial changes (final demand, income and employment)?
  - How to separate the initial changes?

# Example Application: Regional Input-Output Modeling System (RIMS II)

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- Region
- Phase 1: Construction of new sports facility
- Initial Change:
  - \$100 M investment in construction of the sports facility



# Calculating Total Output Impact

1.4 - BALTIMORE-TOWSON, MD MSA (2003 DEF.) 2002 regional data, 1997 US Annual I-O Table

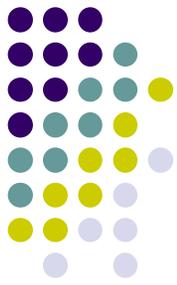
	Final-demand Output /1/ (dollars)	Final-demand Earnings /2/ (dollars)	Final-demand Employment /3/ (number of jobs)	Direct-effect Earnings /4/ (dollars)	Direct-effect Employment /5/ (number of jobs)
230000 Construction	2.1615	0.6671	18.2102	1.9481	2.1847

Final-demand output multiplier (dollars)	Output impact (dollars)
From Table 1.4	\$100 m × final-demand output multiplier =
<b>2.1615</b>	<b>216,150,000</b>

Source: <http://www.bea.gov/regional/rims/>

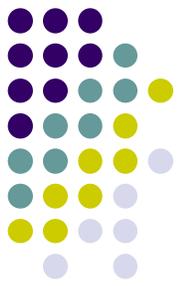
# Calculating Total Income Impact

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- Income Multipliers
  - Simple household income multiplier
  - Total household income multiplier
  - Type I income multiplier
  - Type II income multiplier

# Calculating Total Income Impact



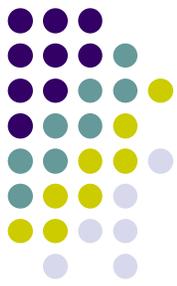
1.4 - BALTIMORE-TOWSON, MD MSA (2003 DEF.) 2002 regional data, 1997 US Annual I-O Table

	Final-demand Output /1/ (dollars)	Final-demand Earnings /2/ (dollars)	Final-demand Employment /3/ (number of jobs)	Direct-effect Earnings /4/ (dollars)	Direct-effect Employment /5/ (number of jobs)
230000 Construction	2.1615	0.6671	18.2102	1.9481	2.1847

Final-demand earnings multiplier (dollars)	Earnings impact (dollars)
From Table 1.4	\$100 m x final-demand earnings multiplier =
<b>0.6671</b>	<b>66,710,000</b>

Source: <http://www.bea.gov/regional/rims/>

# Calculating Total Employment Impact



1.4 - BALTIMORE-TOWSON, MD MSA (2003 DEF.) 2002 regional data, 1997 US Annual I-O Table

	Final-demand Output /1/ (dollars)	Final-demand Earnings /2/ (dollars)	Final-demand Employment /3/ (number of jobs)	Direct-effect Earnings /4/ (dollars)	Direct-effect Employment /5/ (number of jobs)
230000 Construction	2.1615	0.6671	18.2102	1.9481	2.1847

Final-demand employment multiplier (jobs/million dollars of output)	Employment impact (jobs/million dollars of output)
From Table 1.4	\$100 m x final-demand employment multiplier =
<b>18.2102</b>	<b>1,821.02</b>

Source: <http://www.bea.gov/regional/rims/>

# Comments

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- Multiplier effects are based on assumptions about the availability of un- or under-utilized resources and people to accommodate the effects (migration, unemployment etc.).
- ‘Large multipliers’ are NOT the same as ‘large multiplier impacts’.
- Multipliers developed for a region or an industry within a region are representing the region or industry as a whole but not individual sub-regions or establishments with an industry.

# References

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- Bureau of Economic Analysis, 1997. *Regional Multipliers: A User Handbook of the Regional Input-Output Modeling System (RIMS II)*. U.S. Department of Commerce; Bureau of Economic Analysis.
- Ronald E. Miller and Peter D. Blair. 1985. *Input-Output Analysis: Foundations and Extensions*. Englewood Cliffs, NJ: Prentice-Hall, Inc., pp. 45-97, 100-148, 236-265.