11.481J / 1.284J / ESD.192J Analyzing and Accounting for Regional Economic Growth
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# Multipliers in Input-Output Model

Presentation to 11.481J

Weifeng Li

# Introduction

- So far, we have learned the relationship between gross output and final demand based on Leontief Inverse Matrix:

$$X = (I - A)^{-1} Y$$

- We are more interested in assessing the effect on an economy of changes in final demand elements that are exogenous to the model of that economy.

$$\Delta X = (I - A)^{-1} \Delta Y$$

# General Structure of Multipliers: Output Multiplier

- The simple output multiplier

$$\frac{(\text{direct output} + \text{indirect output})}{\text{direct output}}$$

Direct and indirect impacts on gross output when the final demand for the $j^{th}$ sector changes by one unit, holding All other sectors constant.

$$O_j = \sum_{i=1}^{n} \alpha_{ij}$$

Leontief coefficient

- The total output multiplier

$$\frac{(\text{direct} + \text{indirect} + \text{induced output})}{\text{direct output}}$$

Leontief coefficient with households included in the input coefficient matrix

$$\overline{O}_j = \sum_{i=1}^{n+1} \overline{\alpha}_{ij}$$

# Example

$$A = \begin{pmatrix} .15 & .25 \\ .20 & .05 \end{pmatrix}$$

$$(I - A)^{-1} = \begin{pmatrix} 1.254 & .330 \\ .264 & 1.122 \end{pmatrix}$$

$$O_1 = 1.254 + .264 = 1.518$$

$$O_2 = .330 + 1.122 = 1.452$$

Simple Output Multiplier

Household

$$\overline{A} = \left(\begin{array}{cc:c} .15 & .25 & .05 \\ .20 & .05 & .40 \\ \hdashline .30 & .25 & .05 \end{array}\right)$$

Household

$$(I - \overline{A})^{-1} = \begin{pmatrix} 1.365 & .425 & .251 \\ .527 & 1.348 & .595 \\ .570 & .489 & 1.289 \end{pmatrix}$$

$$\overline{O}_1 = 1.365 + .527 + .570 = 2.462$$

$$\overline{O}_2 = .425 + 1.348 + .489 = 2.262$$

Total Output Multiplier

Source: Miller and Blair (1985).

# General Structure of Multipliers: Income Multiplier (1)

- The simple household income multiplier

  portion of simple output effect that is household income

$$H_j = \sum_{i=1}^{n} a_{n+1,i}\alpha_{ij}$$

- The total household income multiplier
  - ⇨ sum of income effects in each sector, or
  - ⇨ income row of the inverse matrix

$$\bar{H}_j = \sum_{i=1}^{n+1} a_{n+1,i}\,\bar{\alpha}_{ij} = \bar{\alpha}_{n+1,j}$$

# Example

Household

$$\bar{A} = \begin{pmatrix} .15 & .25 & .05 \\ .20 & .05 & .40 \\ .30 & .25 & .05 \end{pmatrix}$$

$$(I - A)^{-1} = \begin{pmatrix} 1.254 & .330 \\ .264 & 1.122 \end{pmatrix}$$

$$H_1 = (.3)(1.254) + (.25)(.264) = .442$$

$$H_2 = (.3)(.330) + (.25)(1.122) = .380$$

Simple Household Income Multiplier

$$\bar{A} = \begin{pmatrix} .15 & .25 & .05 \\ .20 & .05 & .40 \\ .30 & .25 & .05 \end{pmatrix}$$

Household

$$(I - \bar{A})^{-1} = \begin{pmatrix} 1.365 & .425 & .251 \\ .527 & 1.348 & .595 \\ .570 & .489 & 1.289 \end{pmatrix}$$

$$\bar{H}_1 = (.3)(1.365) + (.25)(.527) + (.05)(.570) = .570$$

$$\bar{H}_2 = (.3)(.425) + (.25)(1.348) + (.05)(.489) = .489$$

Total Household Income Multiplier

Source: Miller and Blair (1985).

# General Structure of Multipliers: Income Multiplier (2)

- Type I income multiplier

$$\left(\frac{\text{direct} + \text{indirect income}}{\text{direct output}}\right) / \left(\frac{\text{direct income}}{\text{direct output}}\right)$$

$$Y_j = \frac{H_j}{a_{n+1,j}} = \sum_{i=1}^{n} \frac{a_{n+1,k}\alpha_{ij}}{a_{n+1,j}}$$

- Type II income multiplier

$$\bar{Y}_j = \frac{\bar{H}_j}{a_{n+1,j}} = \sum_{i=1}^{n+1} \frac{a_{n+1,k}\bar{\alpha}_{ij}}{a_{n+1,j}}$$

# Example

Household

$$\bar{A} = \begin{pmatrix} .15 & .25 & .05 \\ .20 & .05 & .40 \\ .30 & .25 & .05 \end{pmatrix}$$

$$\bar{A} = \begin{pmatrix} .15 & .25 & .05 \\ .20 & .05 & .40 \\ .30 & .25 & .05 \end{pmatrix}$$

Household

$$(I - A)^{-1} = \begin{pmatrix} 1.254 & .330 \\ .264 & 1.122 \end{pmatrix}$$

$$(I - \bar{A})^{-1} = \begin{pmatrix} 1.365 & .425 & .251 \\ .527 & 1.348 & .595 \\ .570 & .489 & 1.289 \end{pmatrix}$$

$$Y_1 = \frac{(.3)(1.254) + (.25)(.264)}{.3} = 1.47$$

$$Y_2 = \frac{(.3)(.330) + (.25)(1.122)}{.25} = .1.52$$

Type I Income Multiplier

$$\bar{Y}_1 = \frac{(.3)(1.365) + (.25)(.527) + (.05)(.570)}{.3} = 1.90$$

$$\bar{Y}_2 = \frac{(.3)(.425) + (.25)(1.348) + (.05)(.489)}{.25} = 1.96$$

Type II Income Multiplier

Source: Miller and Blair (1985).

# General Structure of Multipliers: Employment Multiplier

- The simple household employment multiplier

$$E_j = \sum_{i=1}^{n} w_{n+1,i}\alpha_{ij}$$

- The total household employment multiplier

- Type I employment multiplier

$$W_j = \frac{E_j}{w_{n+1,j}} = \sum_{i=1}^{n} \frac{w_{n+1,i}\alpha_{ij}}{w_{n+1,j}}$$

- Type II employment multiplier

# Multipliers in Regional Models

## Single-Region Input-Output Model

- Assumptions
  - The technology for each sector at the regional level is identical to the technology in that sector at the national level
  - The local input ratio for sector j

$$p_j^R = \frac{X_j^R - E_j^R}{(X_j^R - E_j^R + M_j^R)} <= 1$$

Local input ratio

Regional output in sector j

Regional exports of good j

Regional imports of good j

# Multipliers in Regional Models

- Major difference
  - Regional Input-Coefficient Matrix ($A^R$)

$$A^R = \hat{P} * A, \qquad \hat{P} = \begin{bmatrix} p_1^R & 0 \,......0 \\ 0 & p_2^R .....0 \\ ... & ... \\ 0 & 0.. \quad p_n^R \end{bmatrix}$$

$A$: National input coefficient matrix

$\hat{P}$: Local input ratio coefficient matrix

  - Impact of Final Demand

$$X^R = (I - A^R)^{-1} Y^R = (I - \hat{P}A)^{-1} Y^R$$

$X^R$: Regional output vector (n * 1)

$(I - A^R)^{-1}$: Regional Leontief inverse matrix (n*n)

$Y^R$: Regional exogenous final demand vector (n*1)

# Example Application: Regional Input-Output Modeling System (RIMS II)

- Question: What is the total impact on a region of building a new sports facility?

- Analysis Process:
  - What is being studied?
  - What is the affected region?
  - What are the affected industries?
  - Is there more than one phase?
  - What are the initial changes (final demand, income and employment)?
  - How to separate the initial changes?

# Example Application: Regional Input-Output Modeling System (RIMS II)

- Region
- Phase 1: Construction of new sports facility
- Initial Change:
  - $100 M investment in construction of the sports facility

# Calculating Total Output Impact

1.4 - BALTIMORE-TOWSON, MD MSA (2003 DEF.) 2002 regional data, 1997 US Annual I-O Table

| | Final-demand Output /1/ (dollars) | Final-demand Earnings /2/ (dollars) | Final-demand Employment /3/ (number of jobs) | Direct-effect Earnings /4/ (dollars) | Direct-effect Employment /5/ (number of jobs) |
|---|---|---|---|---|---|
| 230000 Construction | 2.1615 | 0.6671 | 18.2102 | 1.9481 | 2.1847 |

| **Final-demand output multiplier** (dollars) | **Output impact** (dollars) |
|---|---|
| From Table 1.4 | $100 m x final-demand output multiplier = |
| **2.1615** | **216,150,000** |

Source: http://www.bea.gov/regional/rims/

# Calculating Total Income Impact

- Income Multipliers
  - Simple household income multiplier
  - Total household income multiplier
  - Type I income multiplier
  - Type II income multiplier

# Calculating Total Income Impact

1.4 - BALTIMORE-TOWSON, MD MSA (2003 DEF.) 2002 regional data, 1997 US Annual I-O Table

| | Final-demand Output /1/ (dollars) | Final-demand Earnings /2/ (dollars) | Final-demand Employment /3/ (number of jobs) | Direct-effect Earnings /4/ (dollars) | Direct-effect Employment /5/ (number of jobs) |
|---|---|---|---|---|---|
| 230000 Construction | 2.1615 | 0.6671 | 18.2102 | 1.9481 | 2.1847 |

| **Final-demand earnings multiplier** (dollars) | **Earnings impact** (dollars) |
|---|---|
| From Table 1.4 | $100 m x final-demand earnings multiplier = |
| **0.6671** | **66,710,000** |

Source: http://www.bea.gov/regional/rims/

# Calculating Total Employment Impact

**1.4 - BALTIMORE-TOWSON, MD MSA (2003 DEF.) 2002 regional data, 1997 US Annual I-O Table**

| | Final-demand Output /1/ (dollars) | Final-demand Earnings /2/ (dollars) | Final-demand Employment /3/ (number of jobs) | Direct-effect Earnings /4/ (dollars) | Direct-effect Employment /5/ (number of jobs) |
|---|---|---|---|---|---|
| 230000 Construction | 2.1615 | 0.6671 | 18.2102 | 1.9481 | 2.1847 |

| **Final-demand employment multiplier** (jobs/million dollars of output) | **Employment impact** (jobs/million dollars of output) |
|---|---|
| From Table 1.4 | \$100 m x final-demand employment multiplier = |
| **18.2102** | **1,821.02** |

Source: http://www.bea.gov/regional/rims/

# Comments

- Multiplier effects are based on assumptions about the availability of un- or under-utilized resources and people to accommodate the effects (migration, unemployment etc.).
- ‘Large multipliers’ are NOT the same as ‘large multiplier impacts’.
- Multipliers developed for a region or an industry within a region are representing the region or industry as a whole but not individual sub-regions or establishments with an industry.

# References


- Bureau of Economic Analysis, 1997. *Regional Multipliers: A User Handbook of the Regional Input-Output Modeling System (RIMS II).* U.S. Department of Commerce; Bureau of Economic Analysis.

- Ronald E. Miller and Peter D. Blair. 1985. *Input-Output Analysis: Foundations and Extensions*. Englewood Cliffs, NJ: Prentice-Hall, Inc., pp. 45-97, 100-148, 236-265.