Combinatorics: The Fine Art of Counting

The Final Challenge – Part One

Whenever the question asks for a probability, enter your answer as either 0, 1, or the sum of the numerator and denominator of a reduced fraction equal to the probability. For example if the probability is 0.8 = 4/5, the answer would be 9.

1. A landscaper has 10 bushes that he is going to plant in 4 different sections of a yard. Assuming that he puts at least one bush in each section, how many different ways can he do this?

1 point				

2. Call a 7-digit phone number *good* if it contains 4 distinct non-zero digits, does not have a 1 in the first or second position, and both the first 3 digits or the last 4 digits is are palindromes. 262-1331, 343-5995, and 929-3113 are all examples of good phone numbers. How many good phone numbers are the altogether?

1 point				

3. Party favor bags for a party each contain either 3 lollipops which may be any of 5 flavors, or 6 gum-balls which may be any of 3 flavors, but not both. How many different bags can be assembled?

1 point				

4. Consider the 3-dimensional (x,y,z) space of points with integer coordinates. Any point can be reached from the origin (0,0,0) by taking steps of 1 unit in the positive or negative x, y, or z direction moving from point to point in the grid. A direct path from the origin to a point is a path which uses as few steps as possible, e.g. a direct path from the origin to (-3,2,-2) takes 7 steps. How many different direct paths are there from the origin to the point (-3,2,-2)?

1 point				

5. A grocery store display consists of a tetrahedral stack of cans of tomato soup. There is one can on top and each layer below is an equilateral triangle of cans which has one more can per side than the layer above it. If there are 17 cans per side in the bottom layer, how many cans are in the stack in total?

1 point				

6. How many ways can 4 women and 4 men be seated around a circular table so that the genders alternate?

1 point				

7. How many ways can 4 men and 4 women who are married couples be seated around a circular table so that genders alternate and no spouses are adjacent?

1 point				

8. As a follow-up to the previous question, how many different points can be reached from the origin by direct paths of exactly 7 steps?

2 points				

9. A space-diagonal connects two vertices of a polyhedron which do not lie on the same face. The truncated icosahedron (a.k.a. a soccer ball) is a semi-regular polyhedra with 12 pentagons and 20 hexagons. How many space-diagonals does it have?

2 points				

10. There is a 4x4 grid of intersections made up of the intersections of 1st through 4th Street and 1st through 4th Avenue in the center of a city which has been blanketed by snow. All the snow has been cleared by plows except the road sections between each of the intersections in this grid. A single remaining plow has been sent to clear this snow and can begin work at any intersection. How many different times must the plow interrupt work to drive on roads that have already been cleared in order to finish the job? (If the plow can clear all the snow in one continuous route, your answer should be 0). Note that after finishing one pass of plowing the plow can restart plowing at any intersection.

2 points				

11. A caterpillar graph is a connected graph with no cycles (i.e. a tree) in which the internal vertices all lie on a path (called the spine). Call a caterpillar “leggy” if every internal vertex has degree > 2. How many different (non-isomorphic) leggy caterpillars are there with 10 vertices.

2 points				

12. The cuboctahedron is a semi-regular polyhedron which can be constructed by placing a vertex at the mid-point of each edge of the cube and connecting vertices which lie on edges adjacent in the cube. It can also be constructed via the same process starting with an octahedron. Compute the number of faces this polyhedron has and determine the minimum number of colors

required to color the faces so that no two faces with a common edge have the same color. Multiply this number times the number of faces and enter the result.

3 points				

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