Probability Axioms, Conditional Probability

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HSSP – July 6, 2008

Administrative things

- Late registration
- Claroline class server

Review of last class

- What are the two types of probability?

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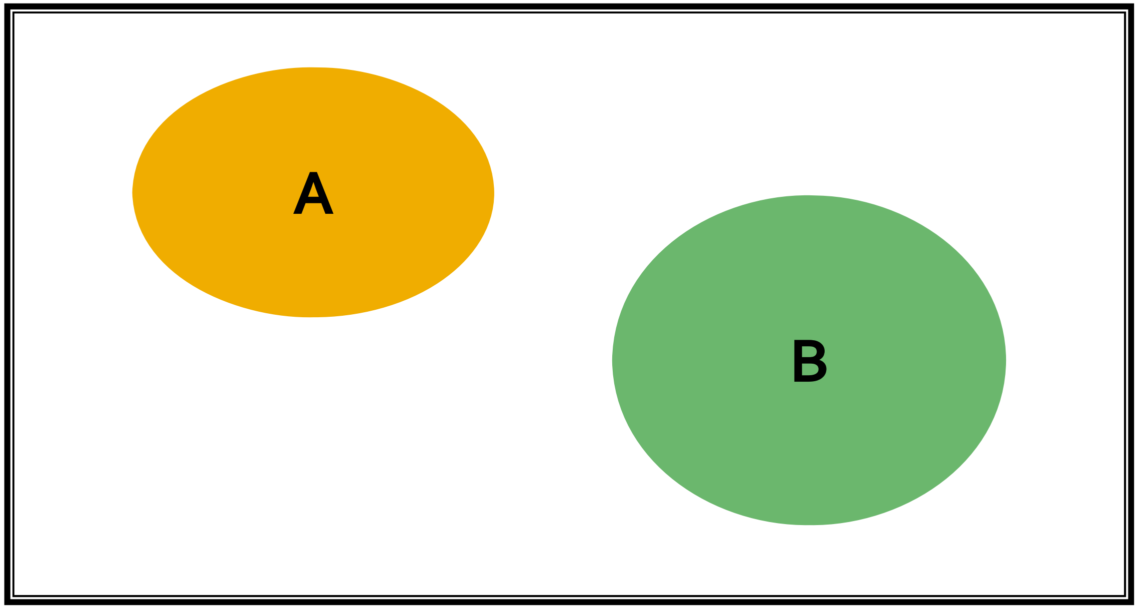
- What does "U" stand for?
- What does $P(A^C)$ mean?

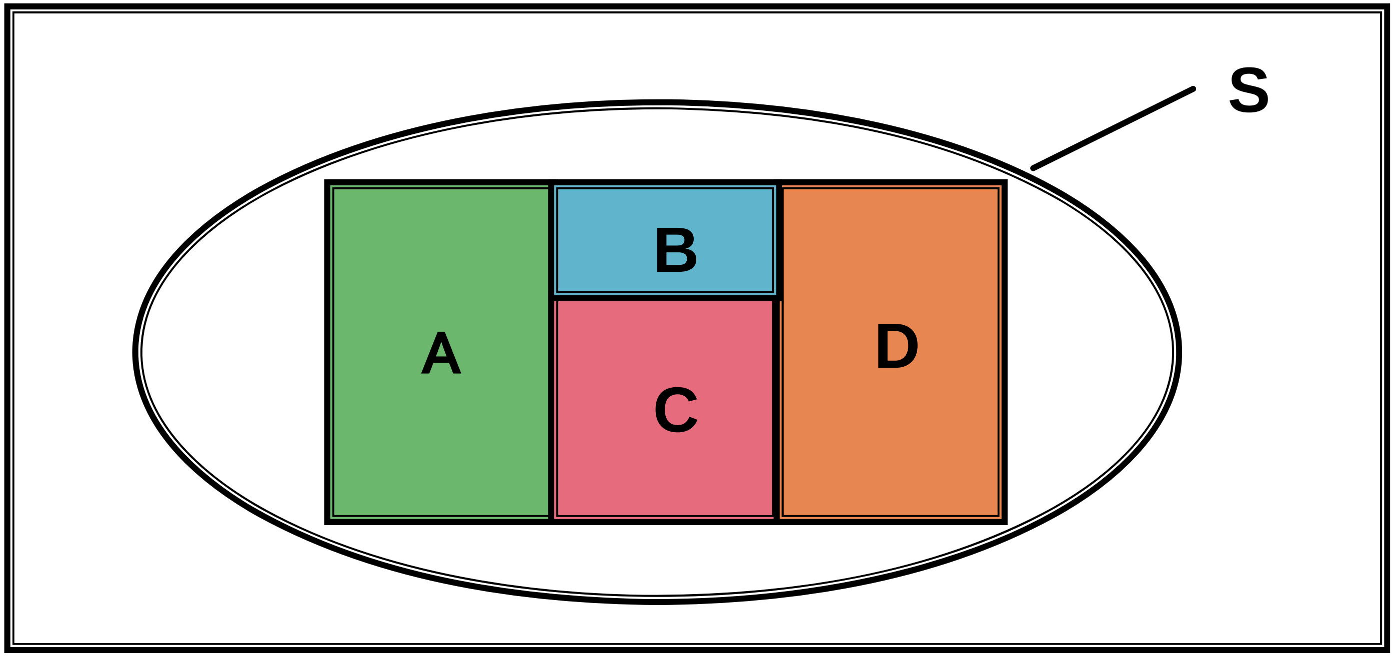
Two More Set Terms

- Disjoint sets
  - No common elements



Two More Set Terms

- Partition (of set S)
  - A collection of disjoint sets whose union is S



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  - $P(A \cup B \cup C \cup \ldots) = P(A) + P(B) + P(C) + \ldots$

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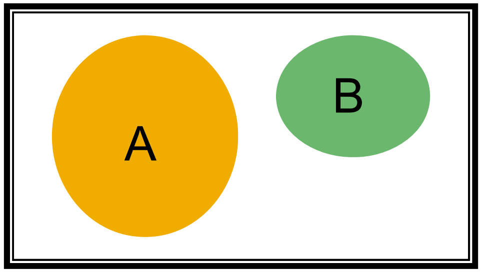
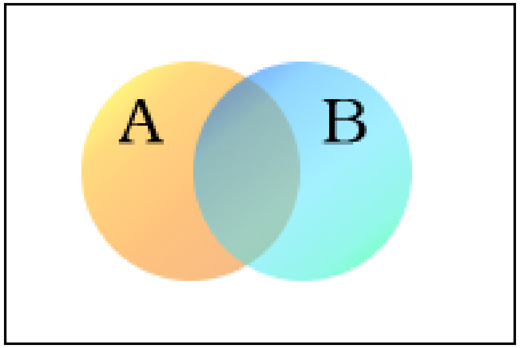
- Normalization
  - $P(\Omega) = 1$

What about overlapping events?

- If A and B are disjoint
  - $P(A \cup B) = P(A) + P(B)$



- What if A and B are not disjoint?
  - What is $P(A \cup B)$?



Second image from http://commons.wikimedia.org/wiki/Image:Venn-diagram-AB.png

Discrete vs. Continuous

- Discrete: **finite** number of possible outcomes
  - Number on a die roll
  - Possible letter grades on a test
- Continuous: **infinite** number of possible outcomes
  - How long you have to wait for a bus
  - How tall someone can be

Discrete Probability Laws

- The probability of any event $\{s_1, s_2, s_3, \ldots, s_n\}$ is the sum of the probabilities of its elements

$$P(\{s_1, s_2, \ldots, s_n\}) = P(s_1)+P(s_2)+\ldots+P(s_n)$$

Discrete Probability Laws

- If the sample space consists of n possible and equally likely outcomes, then the probability of any event A is

$$P(A) = \frac{\text{number of elements in A}}{n}$$

Conditional Probability

- Probability of an event based on partial information
- "Conditional probability of A given B"
- $P(A \mid B)$

Example: Die Roll

- Assume all six possible outcomes of a fair die are equally likely

- What is the probability that we rolled a 6, given that the outcome is even?

- P(outcome is 6 | outcome is even)

Example: Die Roll

- $P(\text{outcome} = 6 \mid \text{outcome is even}) = ?$

Conditional Probability

Can't divide by zero!

(Assuming P(B) > 0)

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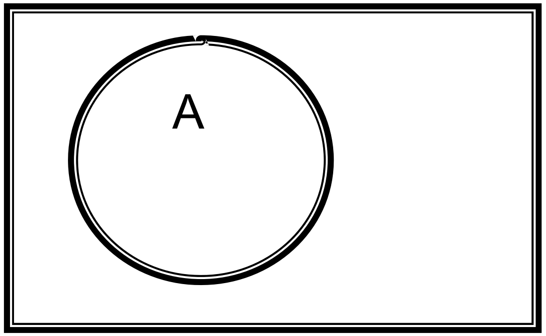
discrete!

(Assuming finite, equally likely outcomes)

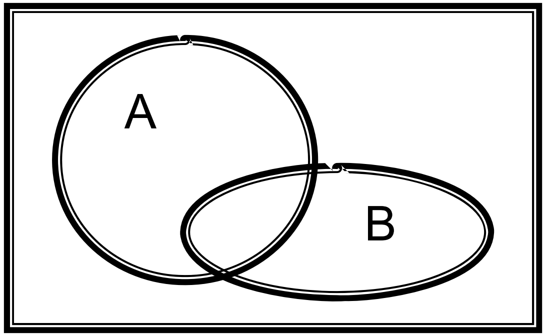
$$P(A|B) = \frac{\text{number of elements of } A \cap B}{\text{number of elements of } B}$$

Conditional Probability

- Probability
  - $P(A) = \frac{P(A \cap \Omega)}{P(\Omega)} = \frac{P(A)}{1} = P(A)$



- Conditional Probability
  - $P(A|B) = \frac{P(A \cap B)}{P(B)}$



Example: Radar Detection

- If an airplane is present in a certain area, the radar correctly registers its presence with 0.99 probability
- If it's not present, the radar falsely registers it anyway with 0.10 probability
- Assume the airplane is present with probability 0.05

Example: Radar Detection

- What is the probability of false alarm?
  - radar registers presence even though airplane is not there
- What is the probability of missed detection?
  - radar does not register, but airplane is there

Example: Radar Detection

- What is our sample space?
- How are we going to represent it?

Example: Radar Detection

- What are the probabilities?

Multiplication Rule

- P(sequence of events) =
  - P(event 1) x P(event 2 | event 1) x P(event 3 | event 1 and event 2) ....

- $P(A_{1-n}) = P(A_1)P(A_2|A_1)P(A_3|A_1 \cap A_2)....$

  [tree]

Problem #1

- Three cards are drawn from an ordinary 52-card decks without replacement (drawn cards do not go back into the deck).

- What's the probability that none of the three cards is a heart?

Problem #2

- There are 4 boys and 12 girls in a class. They are randomly divided into 4 groups of 4.
- What is the probability that each group includes 1 boy?

Monty Hall Problem

- Game show: there are three doors: one has $1 million behind it, the other two have nothing
- You pick one but it remains unclosed
- The host opens one door that reveals nothing (he knows which door has the prize)
- Before he opens your door (you only can pick one door), he gives you the choice of staying with your door or switching to the third door

Monty Hall Problem

Switch or Stay?

Summary

- More set terms: disjoint, partition
- Probability axioms
- Discrete vs. continuous
- Conditional probability
- Multiplication rule

Card Deck (for your reference)

Image removed due to copyright restrictions. To see an image of entire deck of cards, please click on the link below.
http://commons.wikimedia.org/wiki/Image:Cards.jpg

Probability: Random Isn't So Random
Summer 2008