

Growth Rates and Log Graphs

In order of fast growth as  $x$  gets large

<b>log <math>x</math></b>	<b><math>x, x^2, x^3</math></b>	<b><math>2^x, e^x, 10^x</math></b>	<b><math>x!, x^x</math></b>
logarithmic	polynomial	exponential	factorial

Choose  $x = 1000 = 10^3$  so that  $\log x = 3$  OK to use  $x! \approx \frac{x^x}{e^x}$

$\log 1000 = 3$     $10^3, 10^6, 10^9$     $10^{300}, 10^{434}, 10^{1000}$     $10^{2566}, 10^{3000}$

Why is  $1000^{1000} = 10^{3000}$ ?   Logarithms are best for big numbers

**Logarithms are exponents!**    $\log 10^9 = 9$     $\log \log x$  is VERY slow

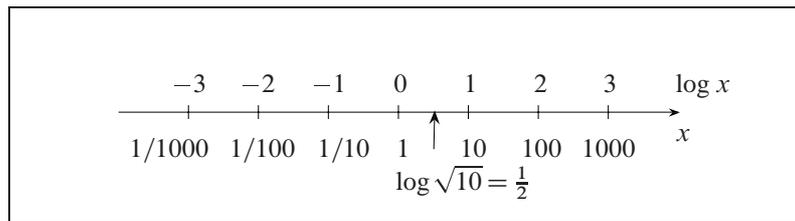
Logarithms   **3, 6, 9**   **300, 434, 1000**   **2566, 3000**

Polynomial growth  $\ll$  Exponential growth  $\ll$  Factorial growth

Decay to zero for NEGATIVE powers and exponents

$\frac{1}{x^2} = x^{-2}$  decays much more slowly than the exponential  $\frac{1}{e^x} = e^{-x}$

Logarithmic scale shows  $x = 1, 10, 100$  equally spaced. NO ZERO!



**Question** If  $x = 1, 2, 4, 8$  are plotted, what would you see?

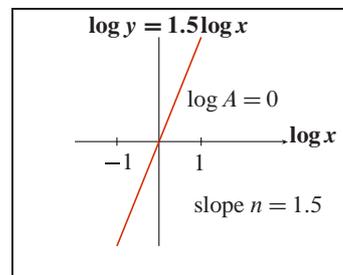
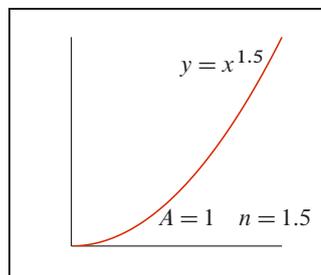
**Answer** THEY ARE EQUALLY SPACED TOO!

**log-log graphs** (log scale up and also across)

If  $y = Ax^n$ , how to see  $A$  and  $n$  on the graph?

Plot  $\log y$  versus  $\log x$  to get a straight line

**$\log y = \log A + n \log x$    Slope on a log-log graph is the exponent  $n$**



For  $y = Ab^x$  use **semilog** ( $x$  versus  $\log y$  is now a line)    $\log y = \log A + x \log b$

## Growth Rates and Log Graphs

*New type of question* How quickly does  $\frac{\Delta f}{\Delta x}$  approach  $\frac{df}{dx}$  as  $\Delta x \rightarrow 0$ ?

The error  $E = \frac{\Delta f}{\Delta x} - \frac{df}{dx}$  will be  $E \approx A(\Delta x)^n$  What is  $n$ ?

Usual one-sided  $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$  only has  $n = 1$

Centered difference  $\frac{f(x + \Delta x) - f(x - \Delta x)}{2\Delta x}$  has  $n = 2$

**Centered is much better than one-sided**  $E \approx (\Delta x)^2$  vs  $E \approx \Delta x$

[ IDEA FOR  $f(x) = e^x$  ] One-sided  $E$  vs centered  $E$   
 [ PROJECT at  $x = 0$  ] Graph  $\log E$  vs  $\log \Delta x$  Should see slope 1 or 2

## Practice Questions

- Does  $x^{100}$  grow faster or slower than  $e^x$  as  $x$  gets large?
- Does  $100 \ln x$  grow faster or slower than  $x$  as  $x$  gets large?
- Put these in increasing order for large  $n$ :

$$\frac{1}{n}, \quad n \log n, \quad n^{1.1}, \quad \frac{10^n}{n!}$$

- Put these in increasing order for large  $x$ :

$$2^{-x}, \quad e^{-x}, \quad \frac{1}{x^2}, \quad \frac{1}{x^{10}}$$

- Describe the log-log graph of  $y = 10x^5$  (graph  $\log y$  vs  $\log x$ )

Why don't we see  $y = 0$  at  $x = 0$  on this graph?

What is the slope of the straight line on the log-log graph?

The line crosses the vertical axis when  $x = \underline{\hspace{2cm}}$  and  $y = \underline{\hspace{2cm}}$

Then  $\log x = 0$  and  $\log y = \underline{\hspace{2cm}}$

The line crosses the horizontal axis when  $x = \underline{\hspace{2cm}}$  and  $y = 1$

Then  $\log x = \underline{\hspace{2cm}}$  and  $\log y = 0$

- Draw the semilog graph (a line) of  $y = 10e^x$  (graph  $\log y$  versus  $x$ )

- That line cross the  $x = 0$  axis at which  $\log y$ ? What is the slope?

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Resource: Highlights of Calculus  
Gilbert Strang

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