Topic 1

# Introduction to Nonlinear Analysis

**Contents:**

- **Introduction to the course**
- **The importance of nonlinear analysis**
- **Four illustrative films depicting actual and potential nonlinear analysis applications**
- **General recommendations for nonlinear analysis**
- **Modeling of problems**
- **Classification of nonlinear analyses**
- **Example analysis of a bracket, small and large deformations, elasto-plastic response**
- **Two computer-plotted animations**
  - **—elasto-plastic large deformation response of a plate with a hole**
  - **—large displacement response of a diamond-shaped frame**
- **The basic approach of an incremental solution**
- **Time as a variable in static and dynamic solutions**
- **The basic incremental/iterative equations**
- **A demonstrative static and dynamic nonlinear analysis of a shell**

**Textbook:** Section 6.1

**Examples:** 6.1, 6.2, 6.3, 6.4

**Reference:** The shell analysis is reported in

Ishizaki, T., and K. J. Bathe, "On Finite Element Large Displacement and Elastic-Plastic Dynamic Analysis of Shell Structures," *Computers & Structures, 12,* 309–318, 1980.

**FIELD OF NONLINEAR ANALYSIS**

- CONTINUUM MECHANICS
- FINITE ELEMENT DISCRETIZATIONS
- NUMERICAL ALGORITHMS
- SOFTWARE CONSIDERATIONS

**WE CONCENTRATE ON:**

- METHODS THAT ARE GENERALLY APPLICABLE
- MODERN TECHNIQUES
- PRACTICAL PROCEDURES

⇓

METHODS THAT ARE OR ARE NOW BECOMING AN INTEGRAL PART OF CAD/CAE SOFTWARE

**BRIEF OVERVIEW OF COURSE**

- GEOMETRIC AND MATERIAL NONLINEAR ANALYSIS
- STATIC AND DYNAMIC SOLUTIONS
- BASIC PRINCIPLES AND THEIR USE
- EXAMPLE SOLUTIONS

WILL BE OF INTEREST IN MANY BRANCHES OF ENGINEERING THROUGHOUT THE WORLD

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**IN THIS LECTURE**

- WE DISCUSS SOME INTRODUCTORY VIEWGRAPHS AND SHOW SOME SHORT MOVIES
- WE THEN CLASSIFY NONLINEAR ANALYSES
- WE DISCUSS THE BASIC APPROACH OF AN INCREMENTAL SOLUTION
- WE GIVE EXAMPLES

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**Transparency 1-1**

## FINITE ELEMENT NONLINEAR ANALYSIS

- Nonlinear analysis in engineering mechanics can be an art.
- Nonlinear analysis can be a frustration.
- It always is a great challenge.

**Transparency 1-2**

Some important engineering phenomena can only be assessed on the basis of a nonlinear analysis:

- Collapse or buckling of structures due to sudden overloads
- Progressive damage behavior due to long lasting severe loads
- For certain structures (e.g. cables), nonlinear phenomena need be included in the analysis even for service load calculations.

The need for nonlinear analysis has increased in recent years due to the need for

— use of optimized structures

— use of new materials

— addressing safety-related issues of structures more rigorously

The corresponding benefits can be most important.

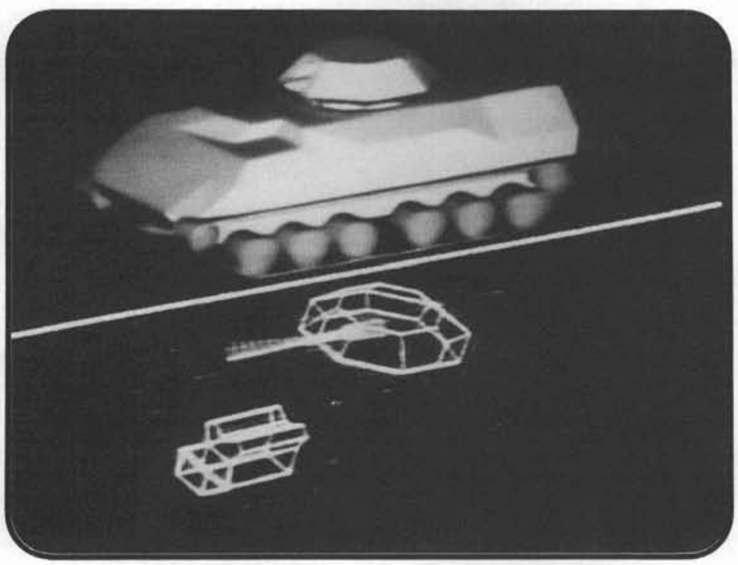
**Transparency 1-3**

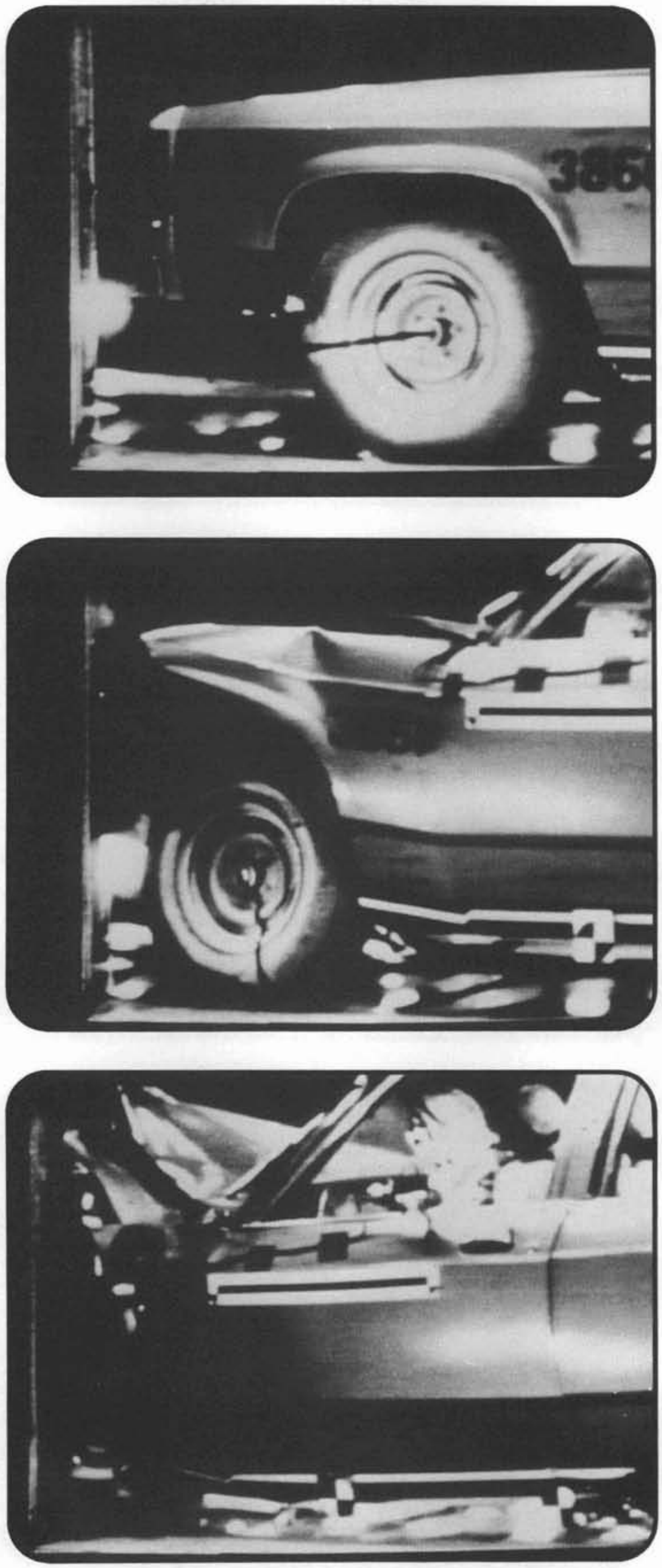
Problems to be addressed by a nonlinear finite element analysis are found in almost all branches of engineering, most notably in,

Nuclear Engineering
Earthquake Engineering
Automobile Industries
Defense Industries
Aeronautical Engineering
Mining Industries
Offshore Engineering
and so on

**Transparency 1-4**

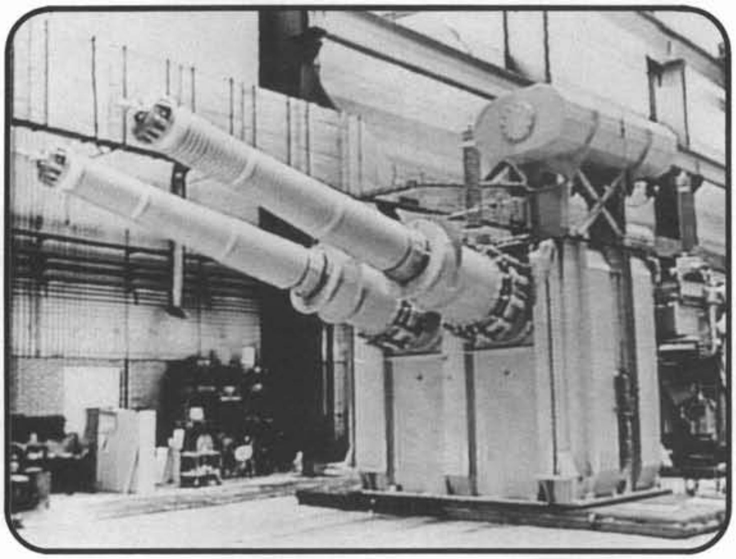
**Film Insert Armored Fighting Vehicle**
Courtesy of General Electric CAE International Inc.

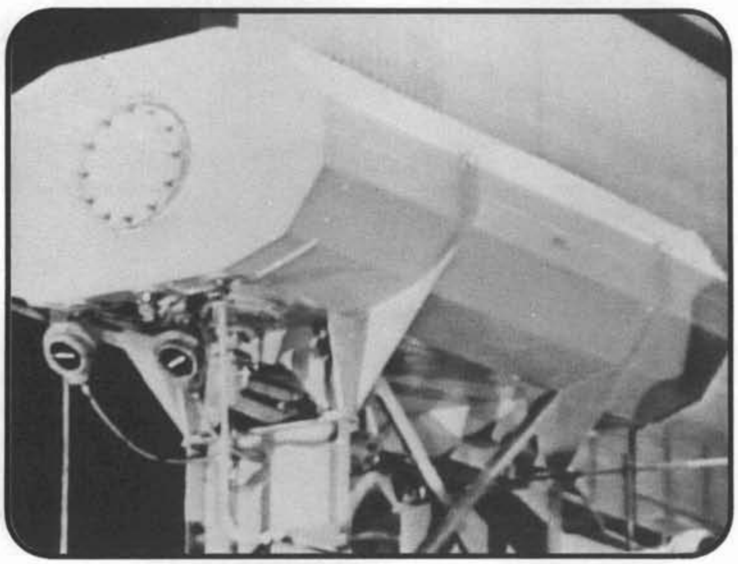


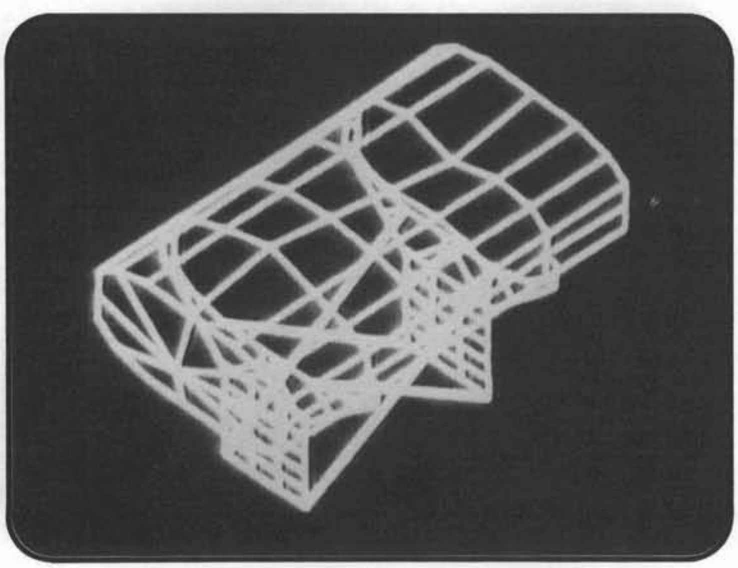


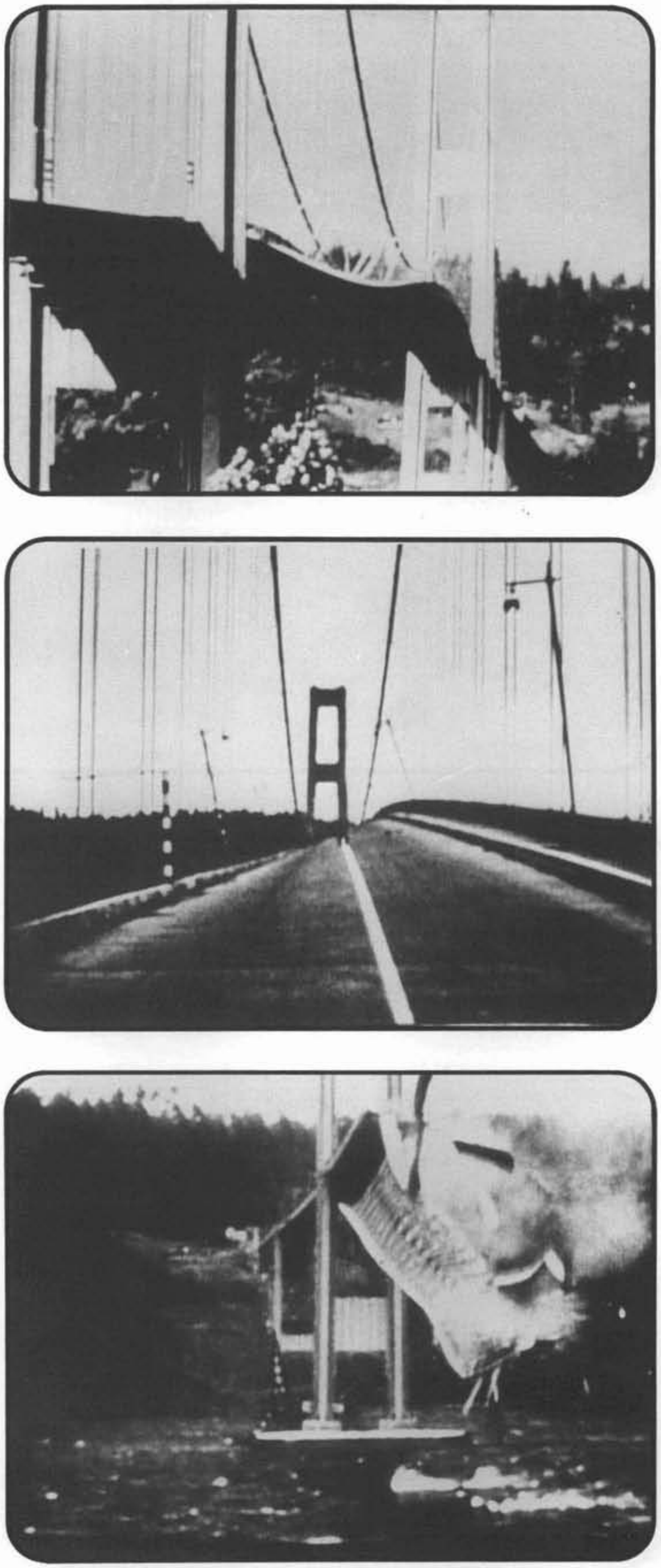
**Film Insert Automobile Crash Test**
Courtesy of Ford Occupant Protection Systems

**Film Insert**
**Earthquake**
**Analysis**
Courtesy of
ASEA Research
and Innovation-
Transformers
Division









**Film Insert Tacoma Narrows Bridge Collapse**
Courtesy of Barney D. Elliot

**Transparency 1-5**

For effective nonlinear analysis, a good physical and theoretical understanding is most important.

PHYSICAL INSIGHT ⟷ MATHEMATICAL FORMULATION

(INTERACTION AND MUTUAL ENRICHMENT)

**Transparency 1-6**

## BEST APPROACH

- Use reliable and generally applicable finite elements.
- With such methods, we can establish models that we understand.
- Start with simple models (of nature) and refine these as need arises.

A "PHILOSOPHY" FOR PERFORMING A NONLINEAR ANALYSIS

Transparency 1-7

# TO PERFORM A NONLINEAR ANALYSIS

- Stay with relatively small and reliable models.
- Perform a linear analysis first.
- Refine the model by introducing nonlinearities as desired.
- Important:
  - Use reliable and well-understood models.
  - Obtain accurate solutions of the models.

NECESSARY FOR THE INTERPRETATION OF RESULTS

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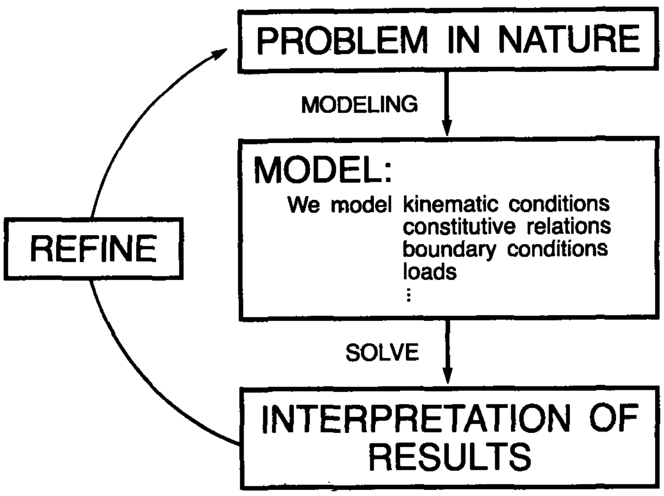

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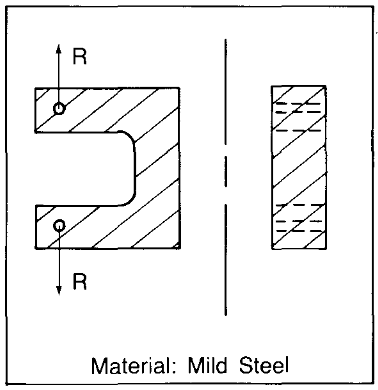
# A TYPICAL NONLINEAR PROBLEM




**POSSIBLE QUESTIONS:**

Yield Load?

Limit Load?

Plastic Zones?

Residual Stresses?

Yielding where Loads are Applied?

Creep Response?

Permanent Deflections?

⋮

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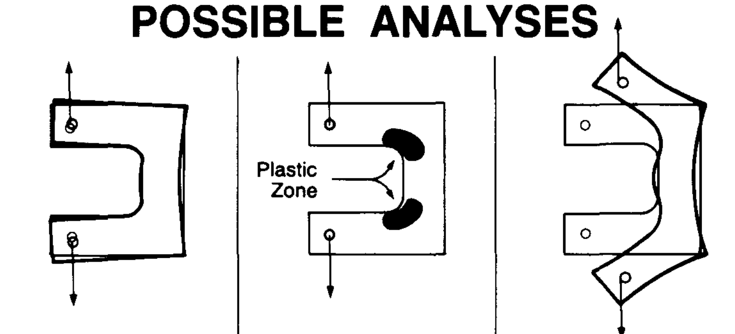
# POSSIBLE ANALYSES




| Linear elastic analysis | Plastic analysis (Small deformations) | Plastic analysis (Large deformations) |
|---|---|---|
| Determine: Total Stiffness; Yield Load | Determine: Sizes and Shapes of Plastic Zones | Determine: Ultimate Load Capacity |

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## CLASSIFICATION OF NONLINEAR ANALYSES

1) Materially-Nonlinear-Only (M.N.O.) analysis:

- Displacements are infinitesimal.
- Strains are infinitesimal.
- The stress-strain relationship is nonlinear.

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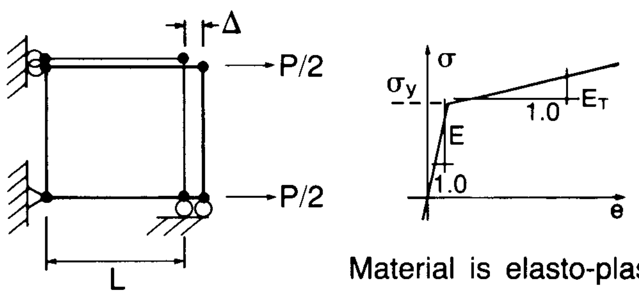
Example:




Material is elasto-plastic.

$$\frac{\Delta}{L} < 0.04$$

- As long as the yield point has not been reached, we have a linear analysis.

**Transparency 1-13**

2) Large displacements / large rotations but small strains:

- Displacements and rotations are large.
- Strains are small.
- Stress-strain relations are linear or nonlinear.

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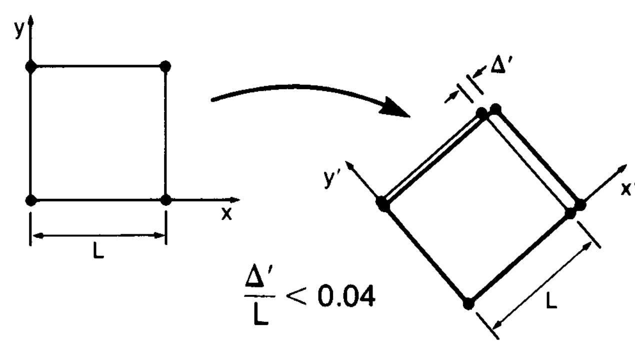
Example:




- As long as the displacements are very small, we have an M.N.O. analysis.

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3) Large displacements, large rotations, large strains:

- Displacements are large.
- Rotations are large.
- Strains are large.
- The stress-strain relation is probably nonlinear.

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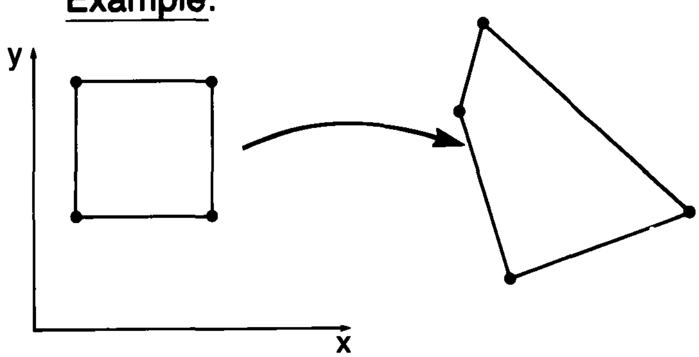
Example:




- This is the most general formulation of a problem, considering no nonlinearities in the boundary conditions.

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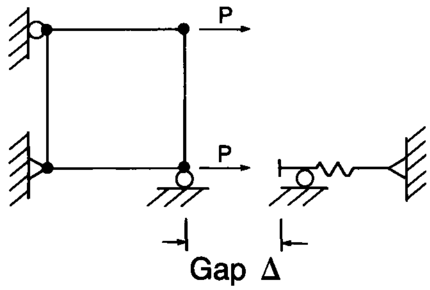
4) Nonlinearities in boundary conditions

Contact problems:




- Contact problems can arise with large displacements, large rotations, materially nonlinear conditions, . . .

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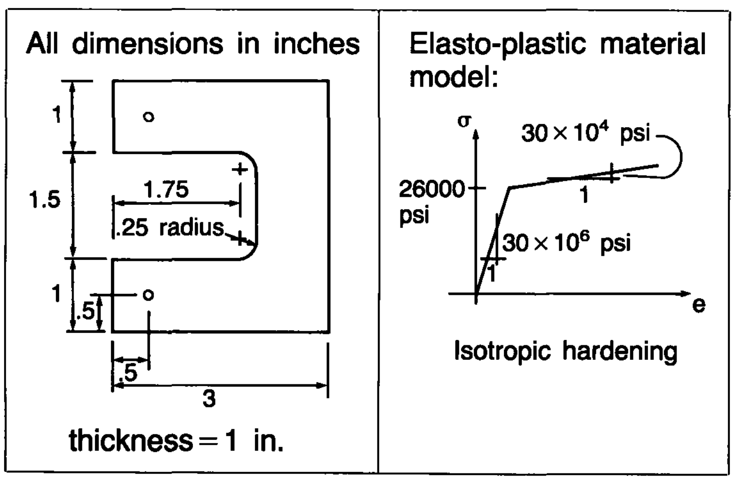
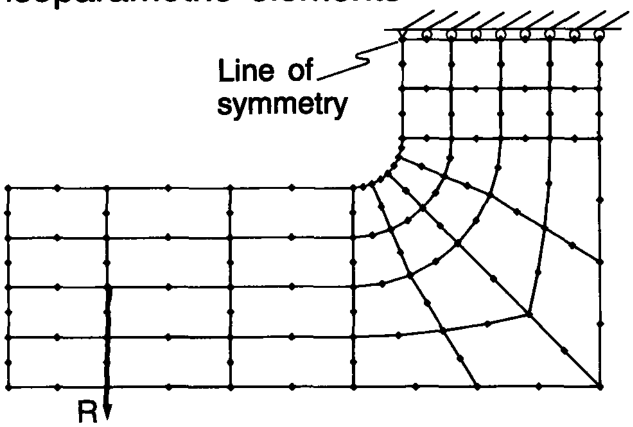
Example: Bracket analysis



Finite element model: 36 element mesh

- All elements are 8-node isoparametric elements

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Three *kinematic* formulations are used:

- Materially-nonlinear-only analysis (small displacements/small rotations and small strains)
- Total Lagrangian formulation (large displacements/large rotations and large strains)
- Updated Lagrangian formulation (large displacements/large rotations and large strains)

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However, different stress-strain laws are used with the total and updated Lagrangian formulations. In this case,

- The material law used in conjunction with the total Lagrangian formulation is actually not applicable to large strain situations (but only to large displ., rotation/ small strain conditions).
- The material law used in conjunction with the updated Lagrangian formulation is applicable to large strain situations.

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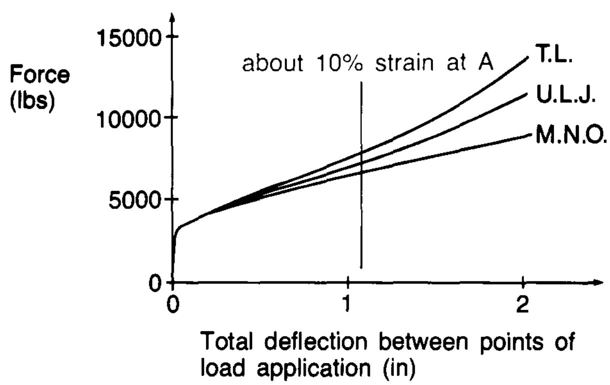
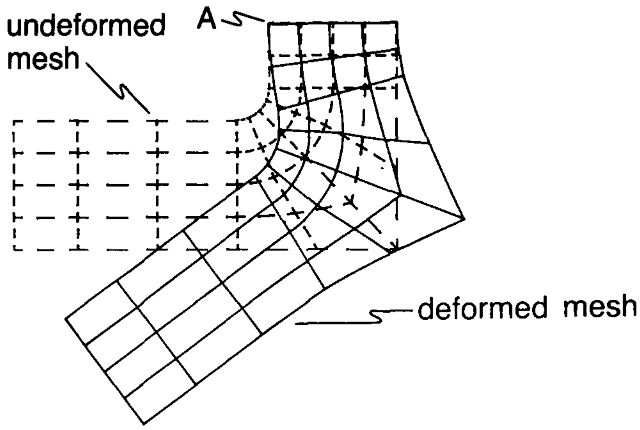
We present force-deflection curves computed using each of the three kinematic formulations and associated material laws:



The deformed mesh corresponding to a load level of 12000 lbs is shown below (the U.L.J. formulation is used).

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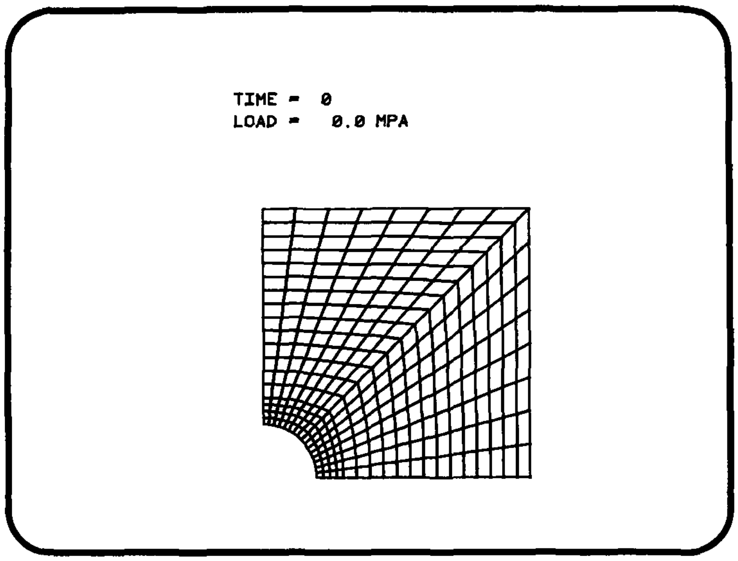
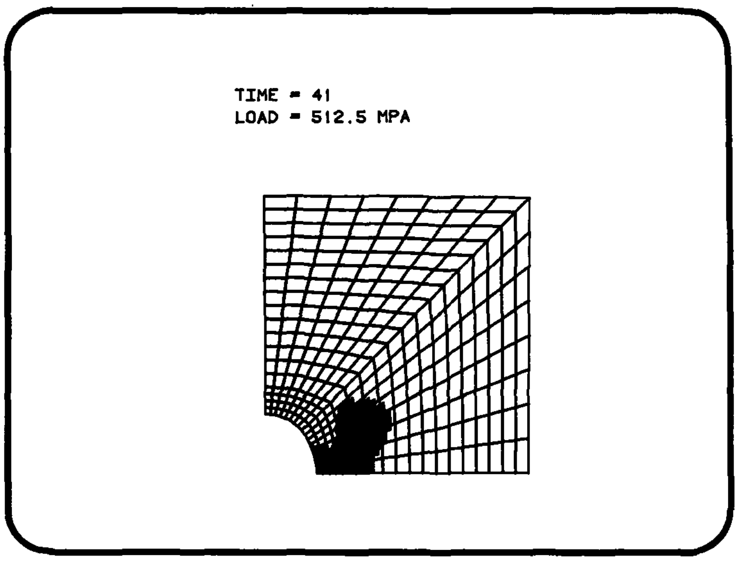
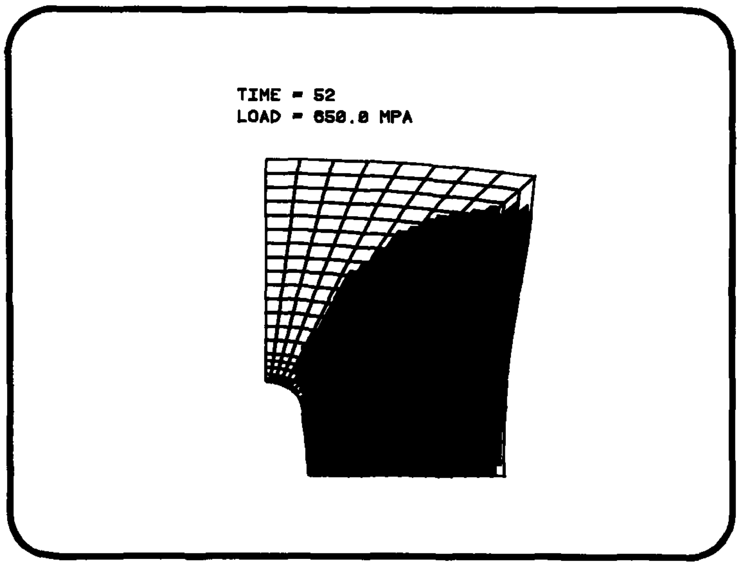
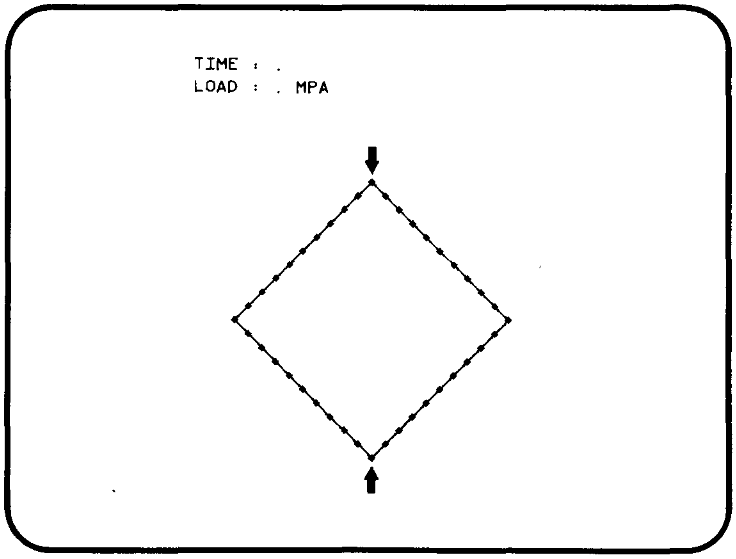
**Computer Animation**
Plate with hole

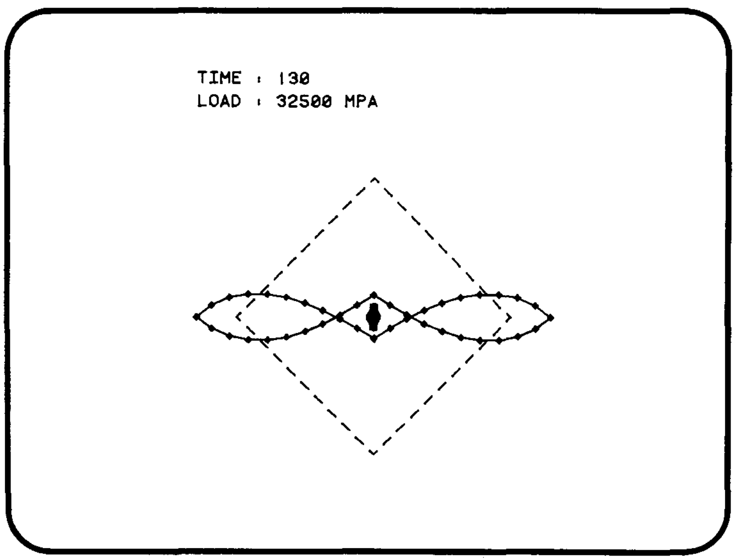


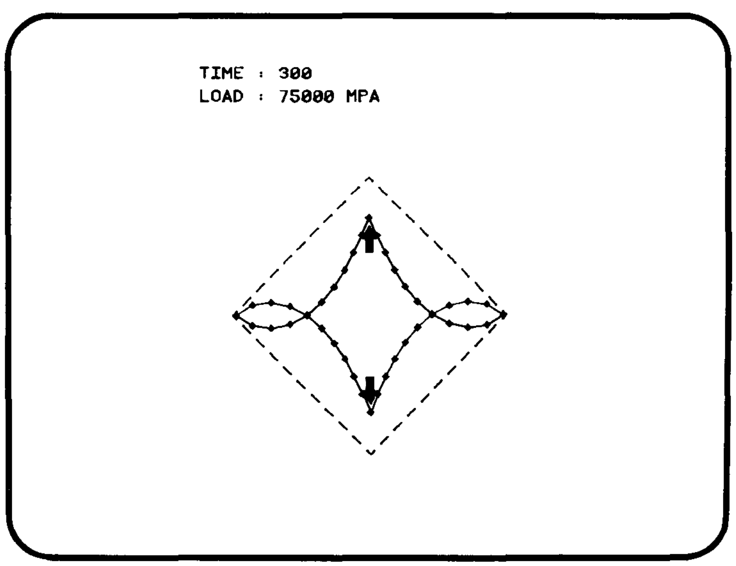






**Computer Animation**
Diamond shaped frame



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# THE BASIC APPROACH OF AN INCREMENTAL SOLUTION

- We consider a body (a structure or solid) subjected to force and displacement boundary conditions that are changing.

- We describe the externally applied forces and the displacement boundary conditions as functions of time.

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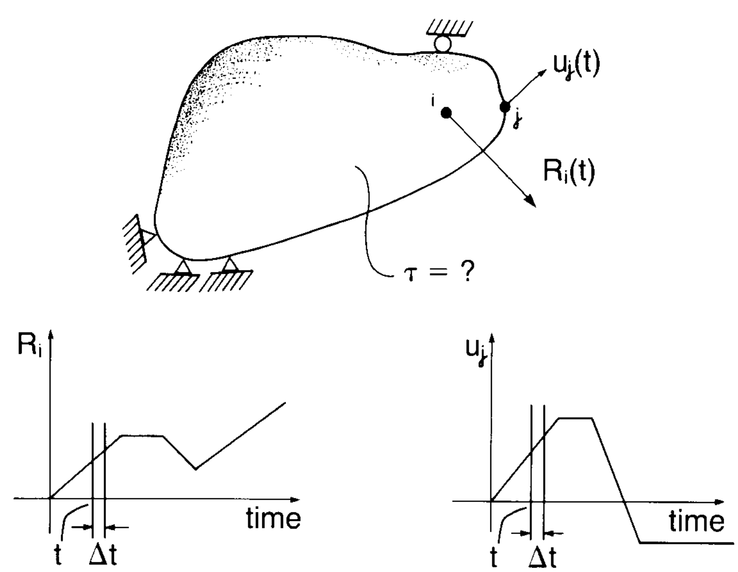
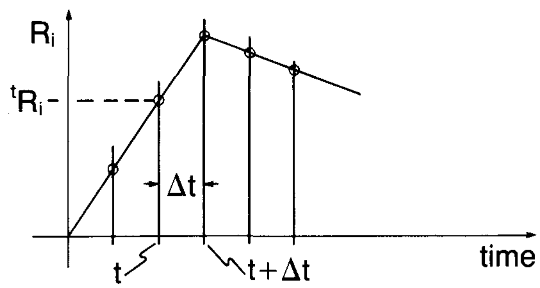


Since we anticipate nonlinearities, we use an incremental approach, measured in load steps or time steps




**Transparency 1-26**

When the applied forces and displacements vary

— slowly, meaning that the frequencies of the loads are much smaller than the natural frequencies of the structure, we have a static analysis;

— fast, meaning that the frequencies of the loads are in the range of the natural frequencies of the structure, we have a dynamic analysis.

**Transparency 1-27**

**Transparency 1-28**

Meaning of time variable

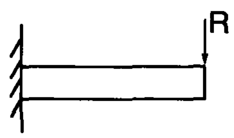
- Time is a pseudo-variable, only denoting the load level

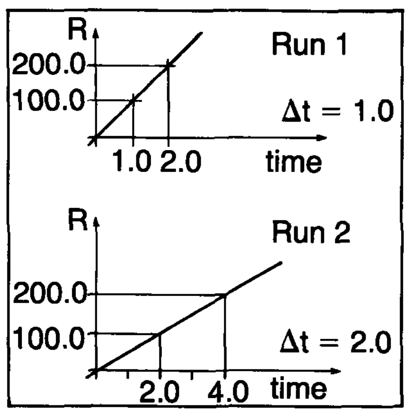
  in

  Nonlinear static analysis with time-independent material properties

**Transparency 1-29**

Example:




Identically the same results are obtained in Run 1 and Run 2

Time is an actual variable

— in dynamic analysis

— in nonlinear static analysis with time-dependent material properties (creep)

Now $\Delta t$ must be chosen carefully with respect to the physics of the problem, the numerical technique used and the costs involved.

**Transparency 1-30**

At the end of each load (or time) step, we need to satisfy the three basic requirements of mechanics:

- Equilibrium
- Compatibility
- The stress-strain law

This is achieved – in an approximate manner using finite elements – by the application of the principle of virtual work.

**Transparency 1-31**

**Transparency 1-32**

We idealize the body as an assemblage of finite elements and apply the principle of virtual work to the unknown state at time $t+\Delta t$.

$$^{t+\Delta t}\underline{R} = {}^{t+\Delta t}\underline{F}$$

$^{t+\Delta t}\underline{R}$: vector of externally applied nodal point forces (these include the inertia forces in dynamic analysis)

$^{t+\Delta t}\underline{F}$: vector of nodal point forces equivalent to the internal element stresses

**Transparency 1-33**

- Now assume that the solution at time t is known. Hence ${}^t\tau_{ij}$, ${}^tV$, ... are known.
- We want to obtain the solution corresponding to time $t+\Delta t$ (i.e., for the loads applied at time $t+\Delta t$).
- For this purpose, we solve in static analysis

$$^{t}\underline{K}\,\Delta\underline{U} = {}^{t+\Delta t}\underline{R} - {}^{t}\underline{F}$$

$$^{t+\Delta t}\underline{U} \doteq {}^{t}\underline{U} + \Delta\underline{U}$$

Transparency 1-34

More generally, we solve

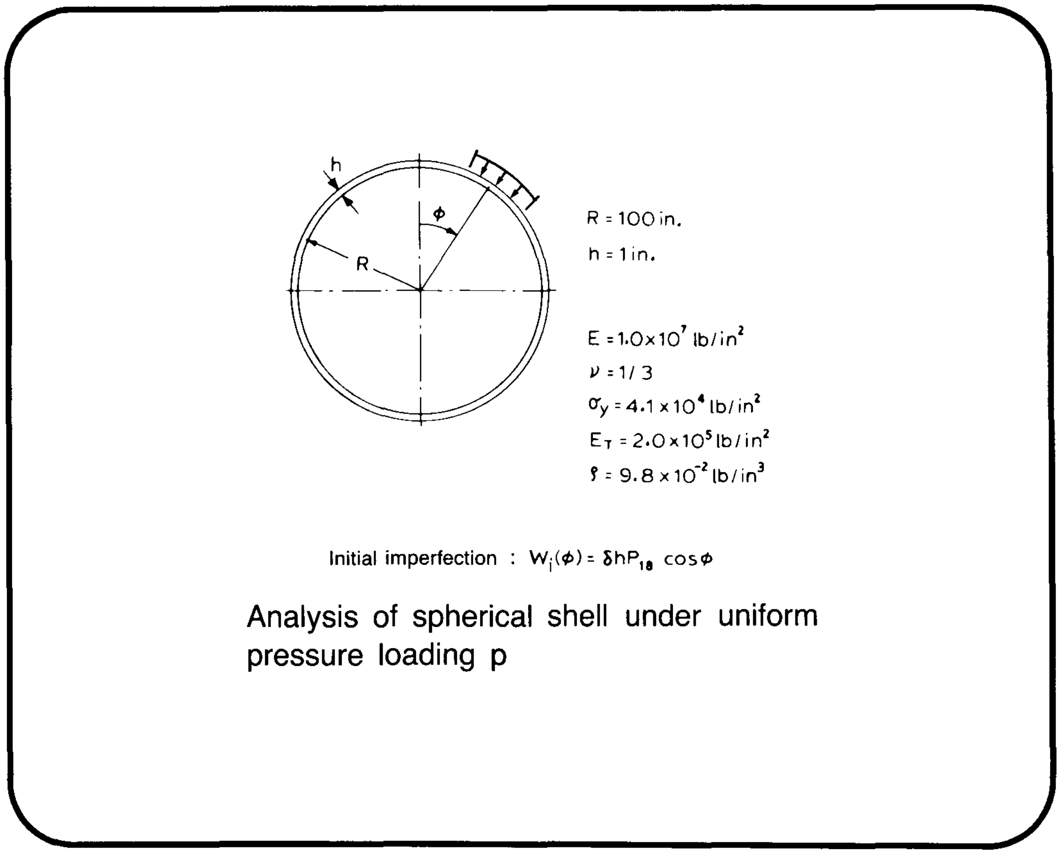
$$^{t}\underline{K}\,\Delta\underline{U}^{(i)} = {}^{t+\Delta t}\underline{R} - {}^{t+\Delta t}\underline{F}^{(i-1)}$$

$$^{t+\Delta t}\underline{U}^{(i)} = {}^{t+\Delta t}\underline{U}^{(i-1)} + \Delta\underline{U}^{(i)}$$

using

$$^{t+\Delta t}\underline{F}^{(0)} = {}^{t}\underline{F}\,, \qquad {}^{t+\Delta t}\underline{U}^{(0)} = {}^{t}\underline{U}$$

**Slide 1-1**

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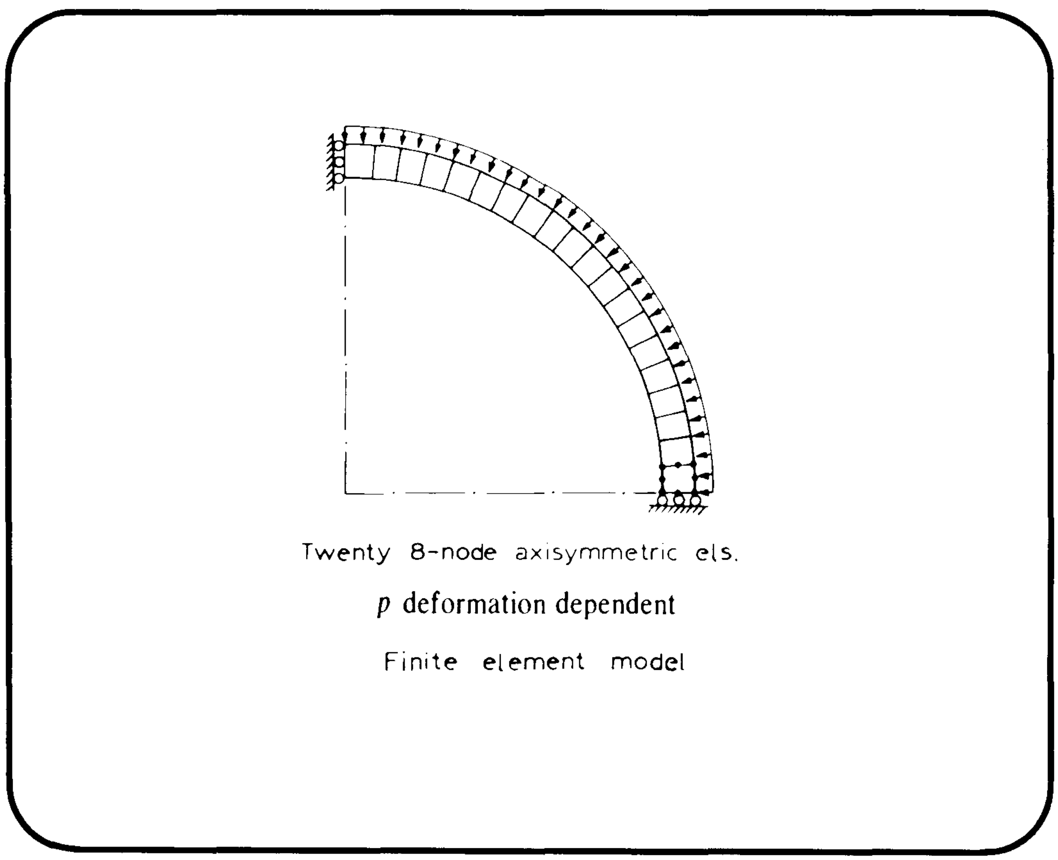
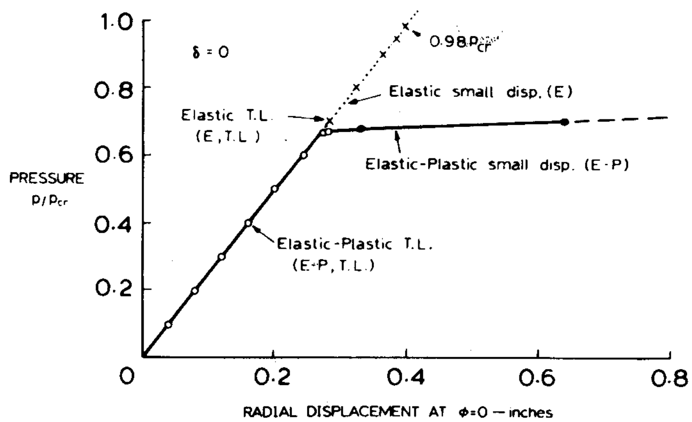
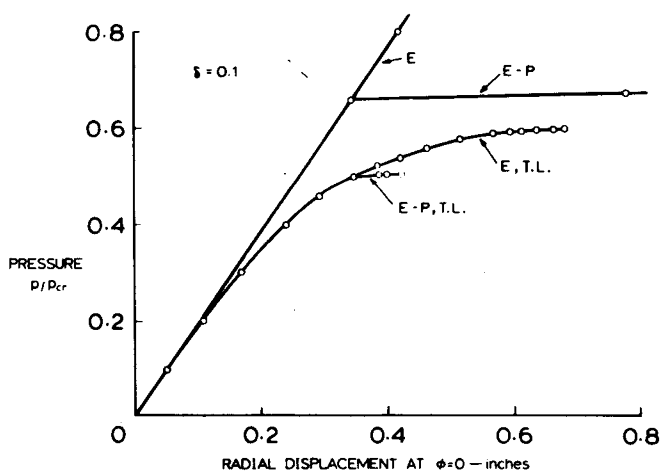





Static response of perfect ($\delta = 0$) shell

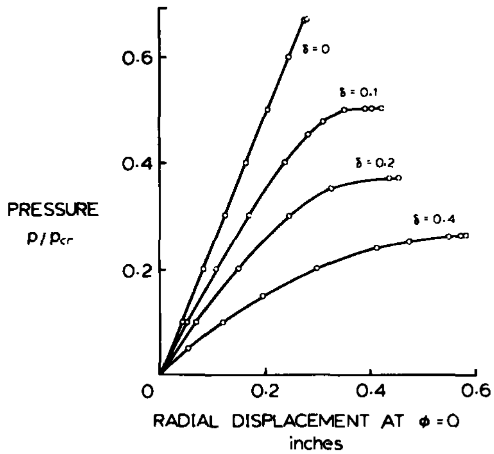
**Slide 1-3**




Static response of imperfect ($\delta = 0.1$) shell

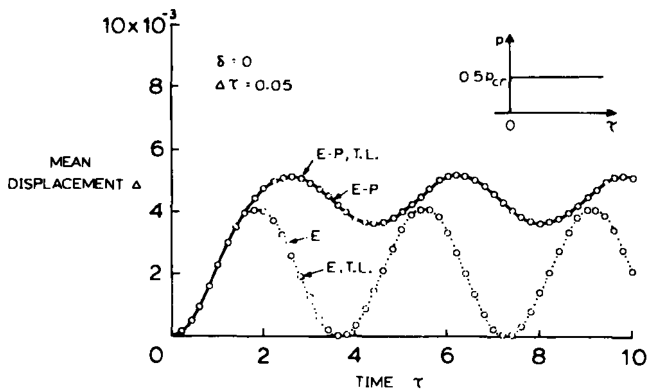
**Slide 1-4**

**Slide 1-5**

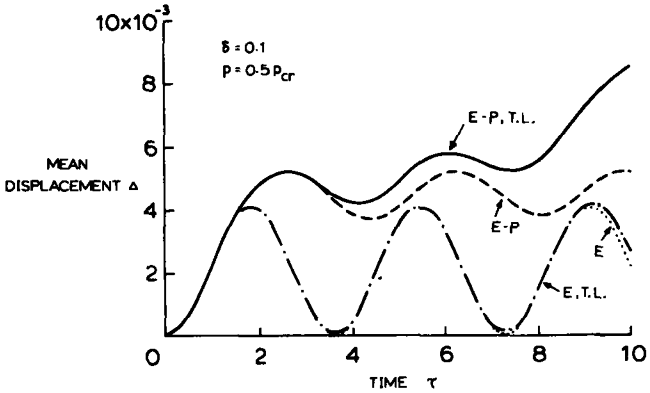



Elastic-plastic static buckling behavior of the shell with various levels of initial imperfection

**Slide 1-6**

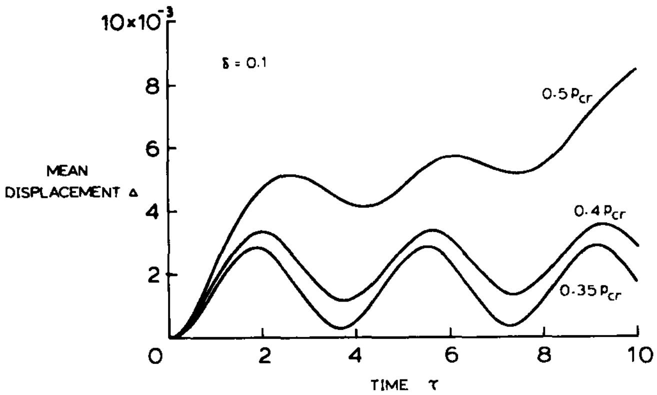



Dynamic response of perfect ($\delta = 0$) shell under step external pressure.




**Dynamic response of imperfect ($\delta = 0.1$) shell under step external pressure.**

**Slide 1-7**




Elastic–plastic dynamic response of imperfect ($\delta = 0.1$) shell

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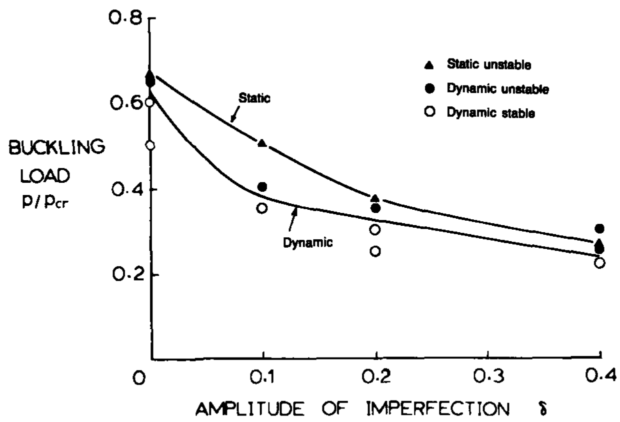
**Slide 1-9**




Effect of initial imperfections on the elastic–plastic buckling load of the shell