PROBLEMS

10.1. The current-carrying wire described in Section 10.1.2 is attached to a pair of dashpots with damping coefficients B and driven at $x = -l$, as shown in Fig. 10P.1.

(a) What is the boundary condition at $x = 0$?

(b) Compute the power absorbed in the dashpots for $\omega < \omega_c$, given the amplitude ξ_0 and other system parameters.

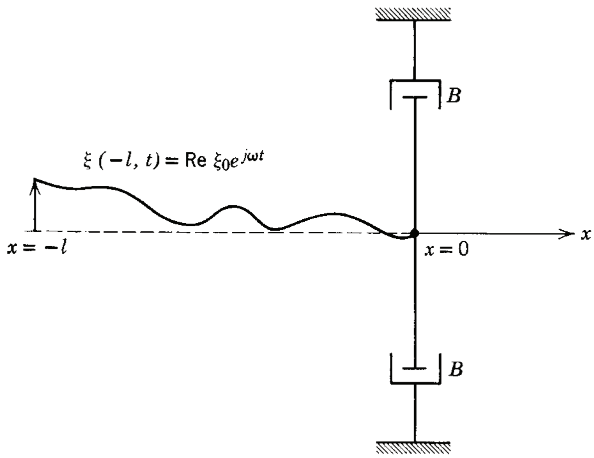


Fig. 10P.1

* Film Cartridge, produced by the National Committee for Fluid Mechanics Films, *Current-Induced Instability of a Mercury Jet*, may be obtained from Education Development Center, Inc., Newton, Mass. The instability seen in this film is convective, as would be the case here if the string were in motion with $U > v_s$.

† W. H. Louisell, *Coupled Modes and Parametric Electronics*, Wiley, 1960, p. 51.

10.2. Consider the same physical situation as that described in Section 10.1.2, except with the current-carrying wire constrained at $x = 0$, so that $(\partial\xi/\partial x)(0, t) = 0$, and driven at $x = -l$ such that $\xi(-l, t) = \xi_d \cos \omega_d t$.

(a) Find analytical expressions for $\xi(x, t)$ with $\omega_d > \omega_c$ and $\omega_d < \omega_c$.

(b) Sketch the results of (a) at an instant in time for cases in which $\omega_d = 0$, $\omega_d < \omega_c$, $\omega_d > \omega_c$.

(c) How could the boundary condition at $x = 0$ be realized physically?

10.3. The ends of the spring shown in Fig. 10.1.2 and discussed in Sections 10.1.2 and 10.1.3 are constrained such that

$$\frac{\partial \xi}{\partial x}(0, t) = 0,$$

$$\frac{\partial \xi}{\partial x}(-l, t) = 0.$$

(a) What are the eigenfrequencies of the spring with the current as shown in Fig. 10.1.2?

(b) What are these frequencies with I as shown in Fig. 10.1.9?

(c) What current I is required to make the equilibrium with $\xi = 0$ unstable? Give a physical argument in support of your answer.

10.4. In Section 10.1.2 a current-carrying wire in a magnetic field was described by the equation of motion

$$m\frac{\partial^2 \xi}{\partial t^2} = f\frac{\partial^2 \xi}{\partial x^2} - Ib\xi + F(x, t), \qquad \text{(a)}$$

where F is an externally applied force/unit length. We wish to consider the flow of power on the string. Because $F\,\partial\xi/\partial t$ is the power input/unit length to the string, we can find a conservation of power equation by multiplying (a) by $\partial\xi/\partial t$. Show that

$$P_{\text{in}} = \frac{\partial W}{\partial t} + \frac{\partial P}{\partial x}, \qquad \text{(b)}$$

where $P_{\text{in}} = F\,\partial\xi/\partial t$,

W = energy stored/unit length

$\frac{1}{2}m\left(\frac{\partial \xi}{\partial t}\right)^2 + \frac{1}{2}f\left(\frac{\partial \xi}{\partial x}\right)^2 + \frac{1}{2}Ib\xi^2,$

P = power flux

$= -f\frac{\partial \xi}{\partial x}\frac{\partial \xi}{\partial t}.$

10.5. Waves on the string in Problem 10.4 have the form

$$\xi(x, t) = \text{Re}[\hat{\xi}_+ e^{j(\omega t - kx)} + \hat{\xi}_- e^{j(\omega t + kx)}].$$

This problem makes a fundamental point of the way in which power is carried by ordinary waves in contrast to evanescent waves. The instantaneous power P carried by the string is given in Problem 10.4. Sinusoidal steady-state conditions prevail.

(a) Compute the time average power carried by the waves under the assumption that k is real. Your answer should show that the powers carried by the forward and backward waves are independent; that is,

$$\langle P\rangle = \frac{\omega k f}{2}(\hat{\xi}_+^*\hat{\xi}_+ - \hat{\xi}_-^*\hat{\xi}_-),$$

where $\hat{\xi}^*$ is the complex conjugate of $\hat{\xi}$.

(b) Show that if $k = j\beta$, β real we obtain by contrast

$$\langle P\rangle = \frac{-j\omega\beta f}{2}(\hat{\xi}_+^*\hat{\xi}_- - \hat{\xi}_-^*\hat{\xi}_+).$$

A single evanescent wave cannot carry power.

(c) Physically, how could it be argued that (b) must be the case rather than (a) for an evanescent wave?

10.6. Use the results of Problem 10.4 to show that the group velocity $v_g = d\omega/dk$ is given by the ratio of the time average power to the time average energy/unit length: $v_g = \langle P\rangle/\langle W\rangle$. Attention should be confined to the particular case of Problem 10.4 with $F = 0$.

10.7. A pair of perfectly conducting membranes has equilibrium spacing d from each other and from parallel rigid walls (Fig. 10P.7). The membranes and walls support currents such

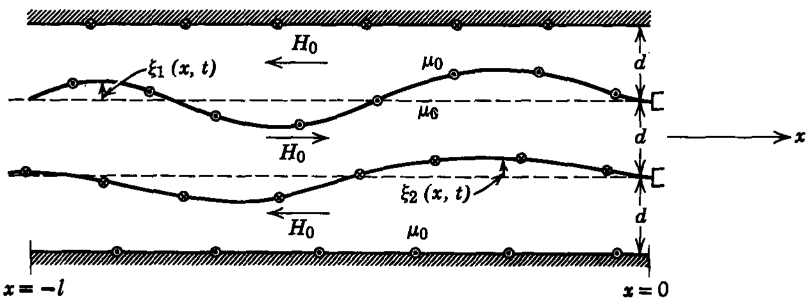


Fig. 10P.7

that with $\xi_1 = 0$ and $\xi_2 = 0$ the static uniform magnetic field intensities H_0 are as shown. As the membranes deform, the flux through each of the three regions between conductors is conserved.

(a) Assume that both membranes have tension S and mass/unit area σ_m. Write two equations of motion for ξ_1 and ξ_2.

(b) Assume that $\xi_1 = \text{Re}\,\hat{\xi}_1 \exp j(\omega t - kx)$ and $\xi_2 = \text{Re}\,\hat{\xi}_2 \exp j(\omega t - kx)$ and find the dispersion equation.

(c) Make an ω-k plot to show complex values of k for real values of ω. Show which branch of this plot goes with $\xi_1 = \xi_2$ and which with $\xi_1 = -\xi_2$. What are the respective cutoff frequencies for these odd and even modes?

(d) The membranes are fixed at $x = 0$ and given the displacements $\xi_1(-l, t) = -\xi_2(-l, t) = \text{Re}\,\xi_0 \exp j\omega t$. Find ξ_1 and ξ_2 and sketch for $\omega = 0$.

10.8. An electromagnetic wave can be transmitted through or reflected by a plasma, depending on the frequency of the wave relative to the plasma frequency ω_p. This phenomenon, which is fundamental to the propagation of radio signals in the ionosphere, is illustrated by the following simple example of a cutoff wave. In dealing with electromagnetic waves, we require that both the electric displacement current in Ampère's law and the

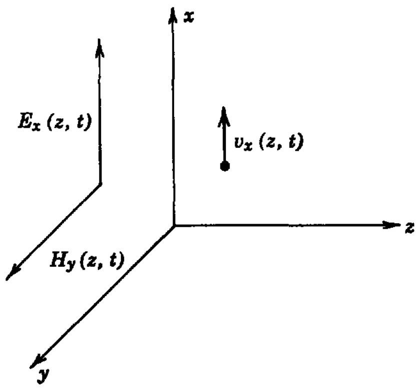


Fig. 10P.8

magnetic induction in Faraday's law (see Section B.2.1) be accounted for. We consider one-dimensional plane waves in which $\mathbf{E} = \mathbf{i}_x E_x(z, t)$ and $\mathbf{H} = \mathbf{i}_y H_y(z, t)$.

(a) Show that Maxwell's equations require that

$$\frac{\partial E_x}{\partial z} = \frac{-\partial \mu_0 H_y}{\partial t}, \qquad -\frac{\partial H_y}{\partial z} = \frac{\partial \epsilon_0 E_x}{\partial t} + J_x.$$

(b) The space is filled with plasma composed of equal numbers of ions and electrons. Assume that the more massive ions remain fixed and that n_e is the electron number density, whereas e and m are the electronic charge and mass. Use a linearized force equation to relate E_x and v_x, where v_x is the average electron velocity in the x-direction.

(c) Relate v_x and J_x to linear terms.

(d) Use the equations of (a)–(c) to find the dispersion equation for waves in the form of $\exp j(\omega t - kz)$.

(e) Define the plasma frequency as $\omega_p = \sqrt{n_e e^2/\epsilon_0 m}$ and describe the dynamics of a wave with $\omega < \omega_p$.

(f) Suppose that a wave in free space were to be normally incident on a layer of plasma (such as the ionosphere). What would you expect to happen? (See Problem 10.9 for a similar situation.)

10.9. A current-carrying string extends from $x = -\infty$ to $x = +\infty$. The section $-l < x < 0$ is subjected to a magnetic field with the distribution shown in Fig. 10.1.2. Hence the sections of string to the left of $x = -l$ and to the right of $x = 0$ support ordinary waves, whereas the section in between can support cutoff waves. Sinusoidal steady-state conditions

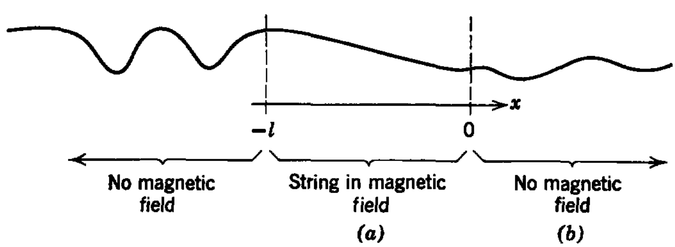


Fig. 10P.9

prevail, with waves incident from the left producing a deflection $\xi(-l, t) = \text{Re}\,\hat{\xi}_0 e^{j\omega t}$, $\omega < \omega_c$. Assume that waves propagating to the right are completely absorbed at $x \to \infty$ so that in interval (b) $\xi_b = \text{Re}\,\hat{\xi}_b e^{j(\omega t - k_b x)}$, $k_b = \omega/v_s$.

(a) Find the attenuation factor $\hat{\xi}_b/\hat{\xi}_0$ for a wave passing through the cutoff section.
(b) What is $\hat{\xi}_b/\hat{\xi}_0$ as $l \to 0$? As $l \to \infty$?

10.10. A rigid straight rod supports a charge Q coulombs per unit length and is fixed. Just below this rod an insulating string is stretched between fixed supports, as shown in Fig.

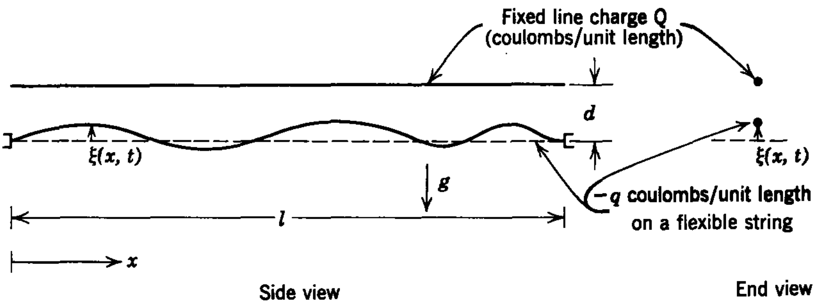


Fig. 10P.10

10P.10. This string, which has a tension f and mass per unit length m, supports a charge per unit length $-q$, where $q \ll Q$ and both Q and q are positive.

(a) What should qQ be in order that the string have the static equilibrium $\xi = 0$ in spite of the gravitational acceleration g?
(b) What is the largest value of m that is consistent with the equilibrium of part (a) being stable?
(c) How would you alter this physical situation to make the static equilibrium stable even with m larger than given by (b)?

10.11. A wire with the mass/unit length m and tension f is stretched between fixed supports, as shown in Fig. 10P.11. The wire carries a current I and is subject to the gravitational

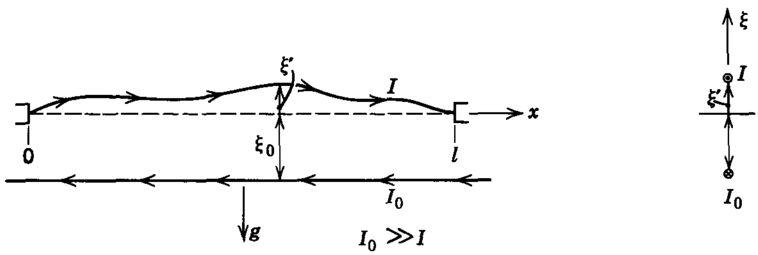


Fig. 10P.11

acceleration g. An adjacent wire carries the much larger current I_0. Because $I_0 \gg I$, the magnetic field produced by I can be ignored.

(a) Given all other system parameters, what value of I is required to hold the wire in static equilibrium with $\xi' = 0$?
(b) Write the differential equation of motion for vertical displacements $\xi'(x, t)$ of the wire from a horizontal equilibrium at $\xi = \xi_0$.
(c) Under what conditions will the equilibrium be stable?

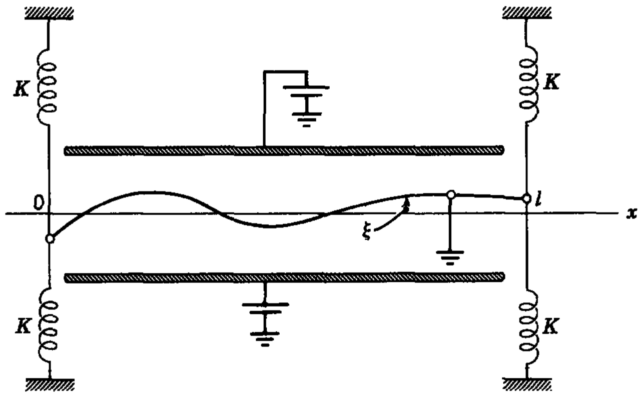


Fig. 10P.12

10.12. The conducting wire shown in Fig. 10P.12 is stressed by a transverse electric field and hence has transverse displacements that satisfy the force equation:

$$m\frac{\partial^2 \xi}{\partial t^2} = f\frac{\partial^2 \xi}{\partial x^2} + P\xi,$$

where m, f, and P are known constants. (P arises from the electric field.) The ends of the string are constrained by springs, but are otherwise free to move in the transverse direction.

(a) Write the boundary conditions in terms of $\xi(x, t)$ at $x = 0$ and $x = l$.

(b) Find an expression for the natural frequencies and illustrate its solution graphically. What effect does raising P have on the lowest eigenfrequency?

(c) What is the largest value of P consistent with stability in the limit where $K \to 0$?

10.13. A pair of perfectly conducting membranes are stretched between rigid supports at $x = 0$ and $x = L$, as shown in Fig. 10P.13. The membranes have the applied voltage V_0 with respect to each other and with respect to plane-parallel electrodes.

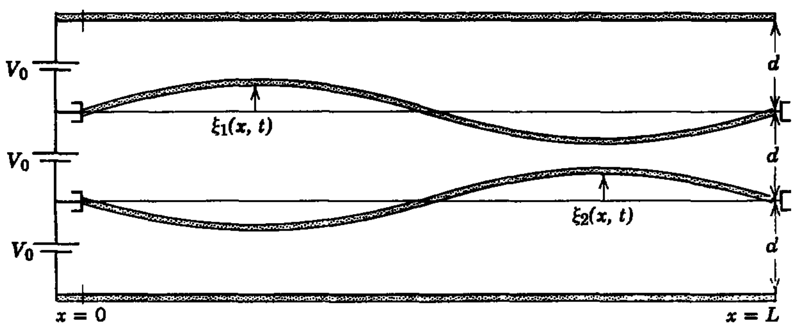


Fig. 10P.13

(a) Write a pair of differential equations in $\xi_1(x, t)$ and $\xi_2(x, t)$ which describe the membrane motions. Assume that ξ_1 and ξ_2 are small enough to warrant linearization and that the wavelengths are long enough that the membranes appear flat to the electric field at any one value of x.

(b) Assume that

$$\xi_1 = \text{Re}\,\hat{\xi}_1 e^{j(\omega t - kx)}$$
$$\xi_2 = \text{Re}\,\hat{\xi}_2 e^{j(\omega t - kx)}$$

and find a dispersion equation relating ω and k.

(c) Make an ω-k plot showing the results of part (b), including imaginary values of ω for real values of k. (This equation should be biquadratic in ω.)

(d) At what potential V_0 will the static equilibrium $\xi_1 = 0$ and $\xi_2 = 0$ first become unstable? Describe the mode of instability.

10.14. A spring is immersed in a viscous fluid so that damping forces of the type discussed in Section 10.1.4 are important. The spring is fixed at $x = 0$ and $x = -l$. When $t = 0$, it is static and released from the initial deflection shown.

(a) Find $\xi(x, t)$ in terms of the system normal modes and ξ_0.

(b) Compare this physical situation with that developed in Section 7.1.1.

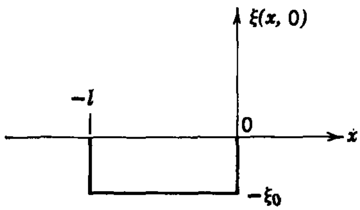


Fig. 10P.14

10.15. A string with tension f and mass/unit length m moves in the x-direction with the velocity $U > \sqrt{f/m}$. The string may be regarded as infinitely long. When $t = 0$, the string has no deflection: $[\xi(x, 0) = 0]$. It has, however, the transverse velocity $(\partial\xi/\partial t)(x, 0)$ given in Fig. 10P.15. Find an analytical expression for $\xi(x, t)$ and sketch it as a function of (x, t). (Your sketch should have an appearance similar to that of Figs. 10.2.4 and 10.2.5.)

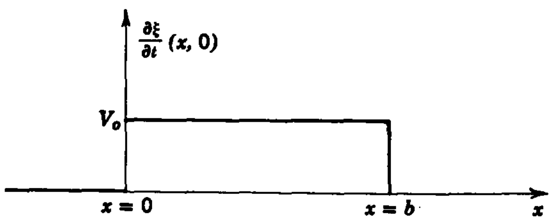


Fig. 10P.15

10.16. A string with the tension f and mass/unit length m has an equilibrium velocity U in the x-direction, where $U > \sqrt{f/m}$. At $x = 0$ it is constrained such that

$$\xi(0, t) = \xi_0 \cos \omega t,$$

$$\frac{\partial \xi}{\partial x}(0, t) = 0.$$

(a) Find the sinusoidal steady-state response $\xi(x, t)$.

(b) Sketch the results of (a) at an instant in time.

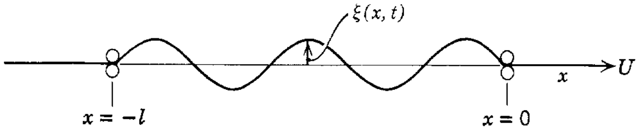


Fig. 10P.17

10.17. A string with the longitudinal velocity U is excited sinusoidally at $x = -l$, $\xi(-l, t) = \text{Re}\, \xi_0 e^{j\omega t}$, and constrained to zero deflection at $x = 0$ by pairs of rollers.

(a) Find the driven response $\xi(x, t)$ in the interval $-l < x < 0$.

(b) What are the natural frequencies of the system? How do they depend on U?

(c) For what values of U are the results of (a) and (b) physically meaningful?

10.18. A wire under the tension f is closed on itself as shown. The resulting loop rotates with the constant angular velocity Ω. We consider deflections ξ from a circular equilibrium which have short wavelengths compared with the radius R. Hence each section of the wire is essentially straight and effects of the curvature on the dynamics can be ignored.

(a) Show that the partial differential equation of motion is

$$m\left(\frac{\partial}{\partial t} + \Omega\frac{\partial}{\partial \theta}\right)^2 \xi = \frac{f}{R^2}\frac{\partial^2 \xi}{\partial \theta^2},$$

where m, f, and R are given constants.

(b) For $t < 0$ the pulse of deflection (Fig. 10P.18b) is imposed externally and is stationary when $t = 0$. At $t = 0$ this pulse is released. You are given that $\Omega R = 2\sqrt{f/m}$. Plot the deflection $\xi(0, t)$ for $0 \le t \le 2\pi R/\sqrt{f/m}$.

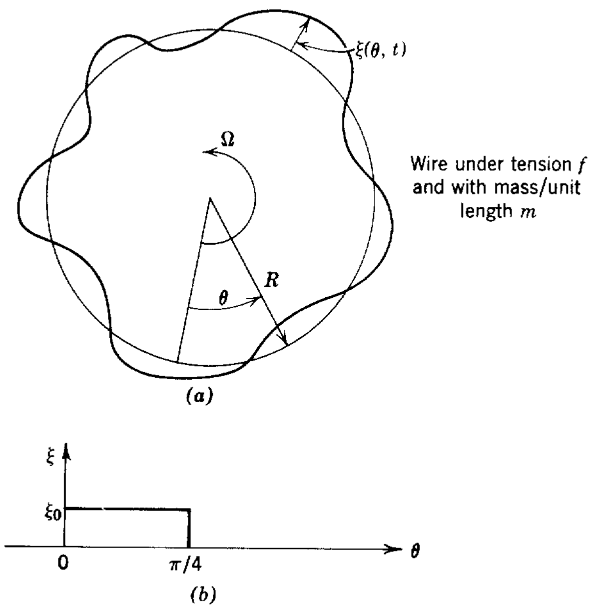


Fig. 10P.18

10.19. A string has the velocity U in the x-direction and is subject to arbitrary inputs of energy from a distributed force $F(x, t)$. Use the equation of motion to find a conservation of power equation in the form $P_{in} = \frac{\partial W}{\partial t} + \frac{\partial P}{\partial x}$. (See Problem 10.4.)

10.20. Find the sinusoidal steady-state response for the conditions outlined in Problem 10.16 with the additional effect of a destabilizing force included (see Section 10.2.3). Sketch the deflections at an instant in time under conditions in which the response takes the form of an amplifying wave.

10.21. A perfectly conducting membrane with the tension S and mass per unit area σ_m is ejected from a nozzle along the x-axis with a velocity U. Gravity acts as shown in Fig.

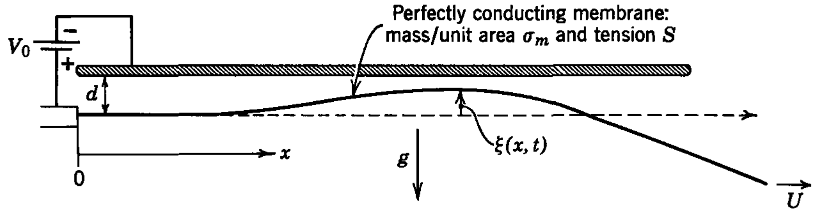


Fig. 10P.21

10P.21. A planar electrode above the membrane has the constant potential V_0 relative to the membrane.

(a) What value of V_0 is required to make the membrane assume an equilibrium parallel to the electrode?

(b) Now, under the conditions in (a), the membrane is excited at the frequency ω_d; what is the lowest frequency of excitation that will not lead to spatially growing deflections? Assume that $U > \sqrt{S/\sigma_m}$.

10.22. An elastic membrane with tension S and mass/unit area σ_m is closed on itself, as shown in Fig. 10P.22. When it is in a steady-state equilibrium, the membrane has radius R

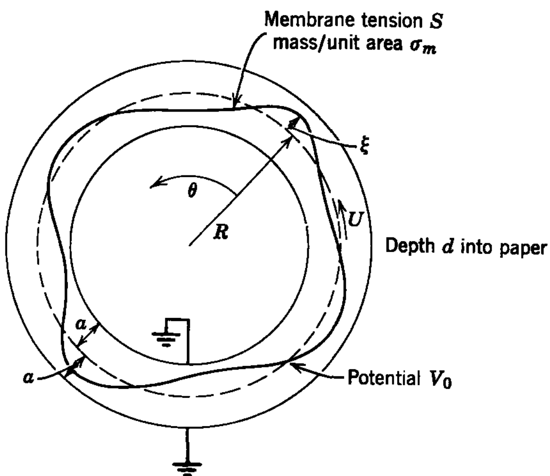


Fig. 10P.22

and rotates with angular velocity Ω. Any point on its surface has an azimuthal velocity $U = \Omega R$. At a distance a to either side of the membrane are coaxial electrodes which, like the membrane, are perfectly conducting. There is a constant potential difference V_0 between the membrane and each of the electrodes. The radius R is very large, so that effects of the membrane and electrode curvatures can be ignored. In addition, wavelengths on the membrane are much greater than a.

(a) Show that the equation of motion for membrane deflections takes the form

$$\left(\frac{\partial}{\partial t} + \Omega\frac{\partial}{\partial \theta}\right)^2 \xi = \Omega_s^2\left(\frac{\partial^2 \xi}{\partial \theta^2} + m_c^2 \xi\right); \qquad \Omega_s \text{ and } m_c = ?$$

(b) Assume that solutions have the form $\xi = \text{Re}\,\hat{\xi}\exp j(\omega t - m\theta)$ and find the ω-m relation. Plot complex ω for real m and complex m for real ω.

(c) Under what conditions is this system absolutely stable?

10.23. A pair of perfectly conducting membranes move in the x-direction with the velocity U. The membranes have the applied voltage V_0 with respect to one another and to plane-parallel electrodes. They enter the region between the plates from rollers at $x = 0$.

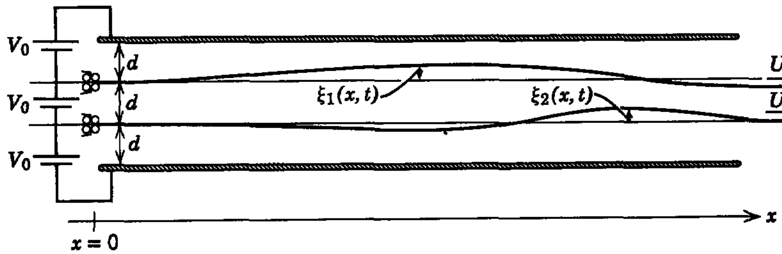


Fig. 10P.23

(a) Write a pair of differential equations in $\xi_1(x, t)$ and $\xi_2(x, t)$ to describe the membrane motions. Assume that ξ_1 and ξ_2 are small enough to warrant linearization and that the wavelengths are long enough for the membrane to appear flat to the electric field at any one value of x.

(b) Assume that

$$\xi_1 = \text{Re}\,\hat{\xi}_1 e^{j(\omega t - kx)},$$
$$\xi_2 = \text{Re}\,\hat{\xi}_2 e^{j(\omega t - kx)}$$

and find a dispersion equation relating ω and k.

(c) Make an ω-k plot to show the results of part (b), including complex values of k for real values of ω. This equation can be factored into two quadratic equations for k. Assume that $U > \sqrt{S/\sigma_m}$.

(d) One of the quadratic factors in part (c) describes motions in which $\xi_1(x, t) = \xi_2(x, t)$, whereas the other describes motions in which $\xi_1(x, t) = -\xi_2(x, t)$. Show that this is true by assuming first that $\xi_1 = \xi_2$ and then that $\xi_1 = -\xi_2$ in parts (a) and (b).

(e) Now suppose that the rollers at $x = 0$ are given the sinusoidal excitation $\xi_1(0, t) = \text{Re}\,\hat{\xi}e^{j\omega t} = -\xi_2(0, t)$, where $\hat{\xi}$ is the same real constant for each excitation. Also, $0 = \partial\xi_1/\partial x = \partial\xi_2/\partial x$ at $x = 0$. Find $\xi_1(x, t)$ and $\xi_2(x, t)$.

(f) What voltage V_0 is required to make the waves excited in part (e) amplify?

(g) Sketch the spatial dependence of ξ_1 and ξ_2 at an instant in time with $V_0 = 0$ and with V_0 large enough to produce amplifying waves.

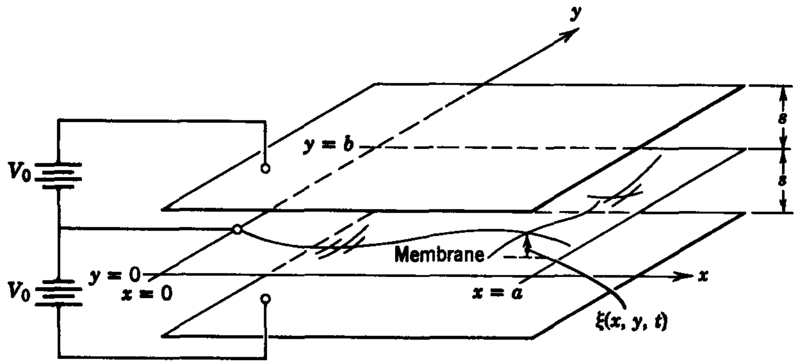


Fig. 10P.24

10.24. A perfectly conducting membrane with tension S and mass/unit area σ_m is fixed at $x = 0$ and $x = a$ and at $y = 0$ and $y = b$. Perfectly conducting plane-parallel electrodes have an equilibrium distance s from the membrane and a potential V_0 relative to the membrane.

(a) What is the largest value of V_0 that will still allow the membrane to be in a state of stable static equilibrium? You may assume that $a \gg s$ and $b \gg s$.

(b) What are the natural frequencies of the membrane?

(c) Given that the membrane is stationary when $t = 0$ and that

$$\xi(x, y, 0) = J_0 u_0\left(x - \frac{a}{2}\right) u_0\left(y - \frac{b}{2}\right),$$

where u_0 is the unit impulse and J_0 is an arbitrary constant, find the response $\xi(x, y, t)$.

10.25. A membrane with tension S and mass/unit area σ_m is fixed along its edges at $y = 0$ and $y = a$. It is also fixed along the edge $x = b$. At $x = 0$ it is driven and has the displacement shown in Fig. 10P.25*b*. Find the sinusoidal steady-state driven response $\xi(x, y, t)$.

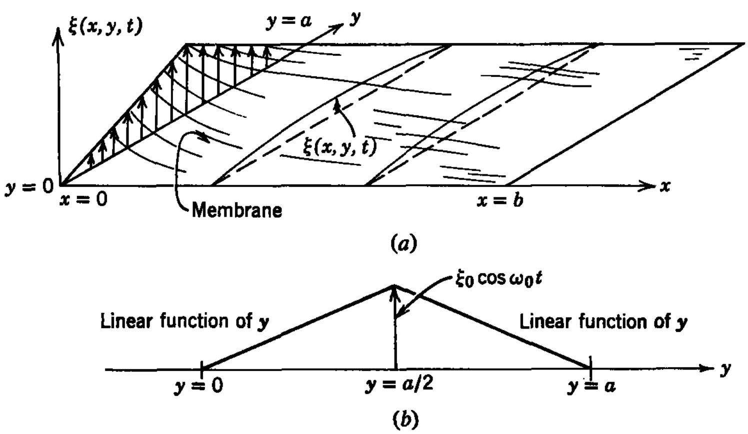


Fig. 10P.25

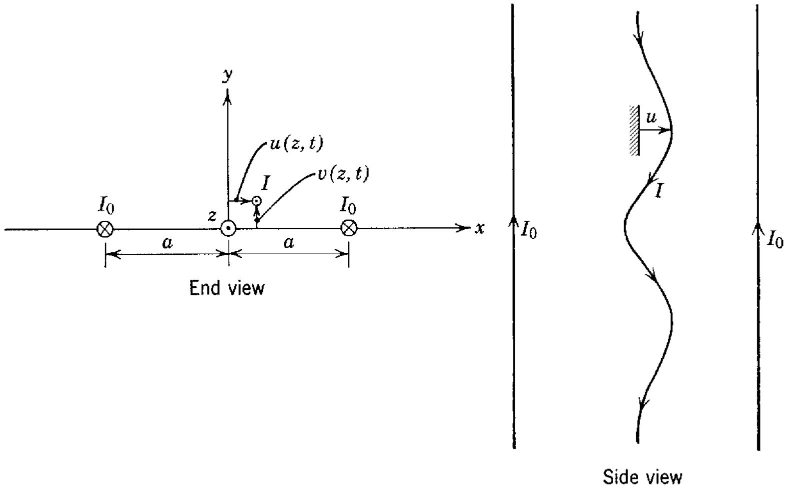


Fig. 10P.26

10.26. A pair of conductors, separated by a distance $2a$, carries currents I_0 (amperes) in the $-z$-direction, as shown in Fig. 10P.26. A conducting wire of mass density/unit length m is stretched along the z-axis and carries a current I $(I \ll I_0)$. Deflections of the wire from the z-axis are given by $u(z, t)\mathbf{i}_x + v(z, t)\mathbf{i}_y$.

(a) Show that the equations of motion for the wire in the magnetic field have the form

$$m \frac{\partial^2 u}{\partial t^2} = f \frac{\partial^2 u}{\partial z^2} - Ibu,$$

$$m \frac{\partial^2 v}{\partial t^2} = f \frac{\partial^2 v}{\partial z^2} + Ibv,$$

under the assumption that deflections u and v are small. What is the constant b in terms of I_0 and a? What fundamental law requires that if Ibu appears with a positive sign in the second equation it must appear with a negative sign in the first equation?

(b) Consider solutions that have the form $u = \text{Re}\, \hat{u}e^{j(\omega t - kz)}$ and $v = \text{Re}\, \hat{v}e^{j(\omega t - kz)}$ and find the relationship between ω-k for x and for y displacements. Make dimensioned plots in each case of real and imaginary values of k for real values of ω. Make dimensioned plots in each case of real and imaginary values of ω for real values of k. (Throughout this problem consider $I_0 > 0$, $I > 0$.)

(c) The wire is now fixed at $z = 0$ and, given the deflection

$$u(-l, t)\mathbf{i}_x + v(-l, t)\mathbf{i}_y = u_0 \cos \omega_0 t \mathbf{i}_x + v_0 \sin \omega_0 t \mathbf{i}_y \qquad (\omega_0 \text{ is real}).$$

Find $u(z, t)$ and $v(z, t)$.

(d) For what values of the currents (I, I_0) will it be possible to establish the sinusoidal steady-state solution of part (c)? For what values of ω_0, in terms of f, m, and l, will the wire support evanescent waves as x-deflections and remain stable?

(e) The frequency ω_0 is set $\omega_0 = (\pi/2l)\sqrt{f/m}$. Sketch the peak deflections u and v as functions of z for several values of I (starting with $I = 0$). Summarize in words how the deflections will change as the current I is raised.

10.27. This is a continuation of Problem 10.26. Now the wire has an equilibrium velocity U along the z-axis with $U > \sqrt{f/m}$.

(a) Write the differential equations for the deflections $u(z, t)$ and $v(z, t)$, including the effect of U.

(b) Consider solutions $u = \operatorname{Re} \hat{u}e^{j(\omega t - kz)}$ and $v = \operatorname{Re} \hat{v}e^{j(\omega t - kz)}$ and find the relationship between ω-k for x and y displacements. Make dimensioned plots in each case of real and imaginary values of k for real values of ω. Make dimensioned plots in each case of real and imaginary values of ω for real values of k ($I_0 > 0$, $I > 0$).

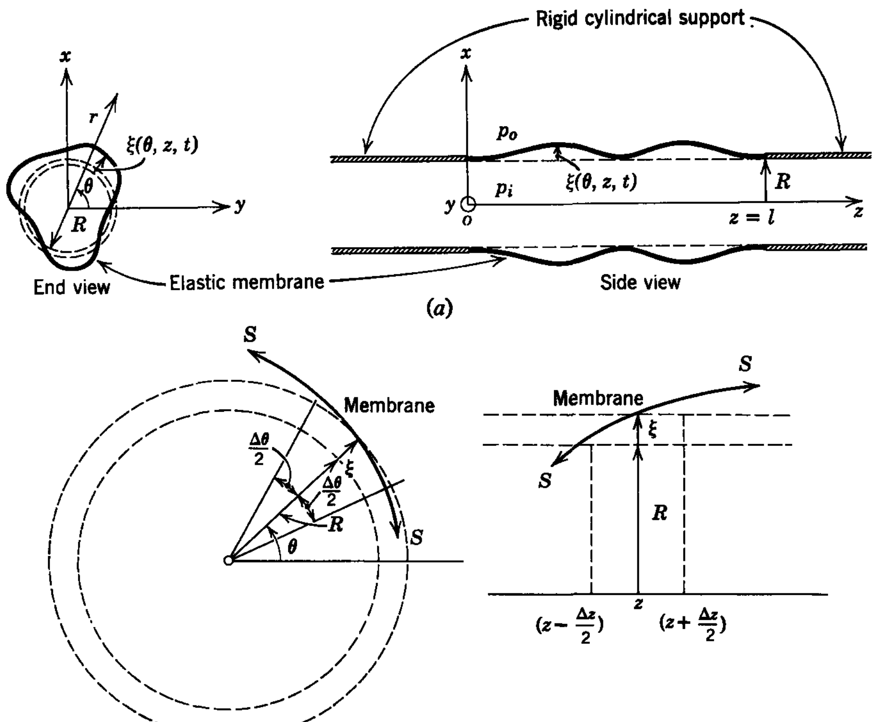
(c) Why would it not be possible to impose the boundary conditions of part (c) in Problem 10.26 to solve this problem? The wire is driven at $z = 0$ by the deflection $u(0, t)\mathbf{i}_x + v(0, t)\mathbf{i}_y = u_0 \cos \omega_0 t \mathbf{i}_x + v_0 \sin \omega_0 t \mathbf{i}_y$ with the slopes

$$\frac{\partial u}{\partial z}(0, t) = 0, \qquad \frac{\partial v}{\partial z}(0, t) = 0.$$

Find $u(z, t)$ and $v(z, t)$.

(d) For a given driving frequency ω_0 sketch the peak deflections u and v as functions of z for several values of I (starting with $I = 0$). Summarize in words how the deflections would change as the current I is raised.

(e) How would you devise an experiment to demonstrate the results of the preceding parts (i.e., what would you use as the moving "wire" and how would you excite it?).



(b)

Fig. 10P.28

10.28. An elastic membrane with constant tension S has a circular cylindrical equilibrium geometry, as shown in Fig. 10P28*a*. It is supported at $z = 0$ and at $z = l$ by circular rigid tubes. Intuitively, we expect that the membrane will collapse inward if the pressure inside the membrane (p_i) is not larger than that outside (p_o). We could imagine stopping up one of the supporting tubes and pushing a cork into the end of the other tube just far enough to maintain the required pressure difference, as might be done in extruding a hollow section of molten glass or plastic or a soap film.

(a) Show that for a static equilibrium to exist with $\xi = 0, p_i - p_o = S/R$.

(b) There is, of course, no guarantee that if we establish this pressure difference the equilibrium will be stable. To examine this question write a linearized equation of radial force equilibrium for a small section of the membrane. The sketches of surface deformation shown in Fig. 10P.28*b* should be helpful in writing the force per unit area due to the tension S. Your equation of motion should be

$$\sigma_m \frac{\partial^2 \xi}{\partial t^2} = S\left(\frac{\xi}{R^2} + \frac{1}{R^2}\frac{\partial^2 \xi}{\partial \theta^2} + \frac{\partial^2 \xi}{\partial z^2}\right)$$

(c) Under what conditions is the equilibrium stable?

(d) Describe the lowest modes of oscillation for the membrane.

(e) Reconsider Problem 10.22, taking into account the effect of the curvature.

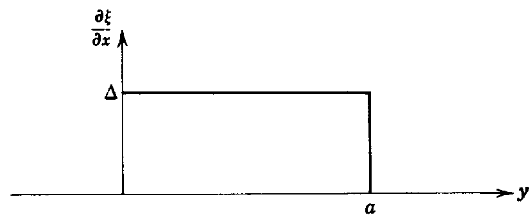


Fig. 10P.29

10.29. A membrane has the velocity $U > v_s$ in the x-direction, as described in Section 10.4.2. At $x = 0$, $\xi = 0$, and $\partial\xi/\partial x$ has the distribution shown. Assume that the membrane is infinitely wide in the y-direction.

(a) Find and sketch $\xi(x, y)$ for $x > 0$. Assume that $M^2 = 2$.

(b) Describe how you would physically produce the postulated excitation at $x = 0$.

10.30. A membrane moves with the velocity $U > v_s$ in the x-direction (see Section 10.4.2). Its edges are prevented from undergoing transverse motions along boundaries at $y = 0$ and

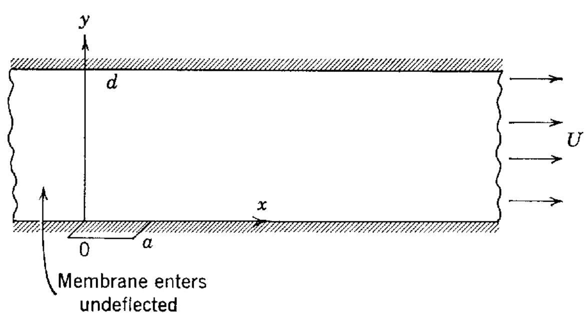


Fig. 10P.30

$y = d$, except for the segment $0 < x < a$, where the membrane is constrained to have the constant amplitude ξ_0. Assume that $M^2 = 2$ and find the resulting deflection $\xi(x, t)$. Your answer should be presented as a sketch similar to Fig. 10.4.3.

10.31. Plot the ω-k relation (10.4.30) for the example described in Section 10.4.3 to show complex ω for real k and complex k for real ω. Indicate the modes (fast wave or slow wave) represented by each branch of the curves.

10.32. Consider the example of Section 10.4.3, but with the wire having a longitudinal velocity $U > v_s$.

(a) Find the revised dispersion equation.
(b) Sketch the ω-k relation and show complex values of k for real values of ω.
(c) Describe the response of the wire to a sinusoidal excitation.