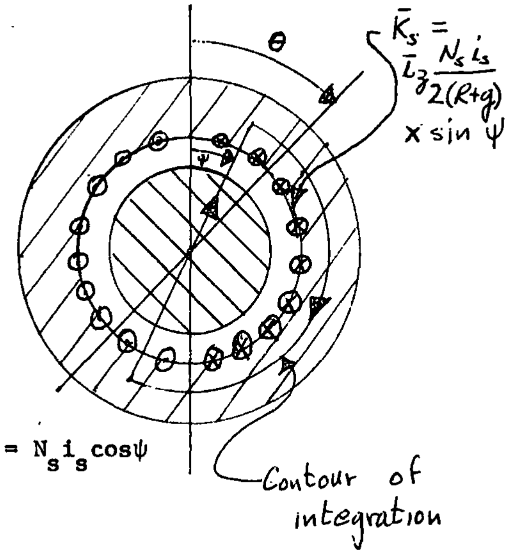


PROBLEM 4.1

Part a

With stator current acting alone the situation is as depicted at the right. Recognizing by symmetry that $H_{rs}(\psi+\pi) = -H_{rs}(\psi)$ we use the contour shown and Ampere's law to get

$$2H_{rs}(\psi)g = \int_{\psi}^{\psi+\pi} [\frac{N_s i_s}{2(R+g)} \sin \psi'](R+g)d\psi' = N_s i_s \cos\psi$$

from which

$$H_{rs}(\psi) = \frac{N_s i_s \cos\psi}{2g}$$

and

$$B_{rs}(\psi) = \frac{\mu_o N_s i_s \cos\psi}{2g}$$

Part b

Following the same procedure for rotor excitation alone we obtain

$$B_{rr}(\psi) = \frac{\mu_o N_r i_r \cos(\psi-\theta)}{2g}$$

Note that this result is obtained from part (a) by making the replacements

$$N_s \rightarrow N_r$$

$$i_s \rightarrow i_r$$

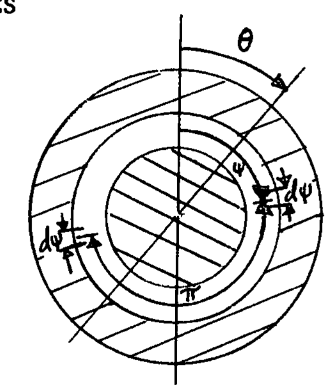
$$\psi \rightarrow (\psi-\theta)$$

Part c

The flux density varies around the periphery and the windings are distributed, thus a double integration is required to find inductances, whether they are found from stored energy or from flux linkages. We will use flux linkages.

The total radial flux density is

$$B_r = B_{rs} + B_{rr} = \frac{\mu_o}{2g} [N_s i_s \cos\psi + N_r i_r \cos(\psi-\theta)]$$



PROBLEM 4.1 (Continued)

Taking first the elemental coil on the stator having sides of angular span $d\psi$ at positions ψ and $\psi+\pi$ as illustrated. This coil links an amount of flux

$$d\lambda_s=\underbrace{\left[\frac{N_s}{2(R+g)}\sin\psi\right](R+g)d\psi}_{\text{number of turns in elemental coil}}\ \underbrace{\left[-\int_{\psi}^{\psi+\pi} B_r(\psi')(R+g)\ell d\psi'\right]}_{\text{flux linking one turn of elemental coil}}$$

$$d\lambda_s=-\frac{\mu_o N_s (R+g)\ell}{4g}\sin\psi d\psi\int_{\psi}^{\psi+\pi}[N_s i_s\cos\psi' + N_r i_r\cos(\psi'-\theta)]d\psi'$$

$$d\lambda_s=\frac{\mu_o N_s (R+g)\ell}{2g}\sin\psi[N_s i_s\sin\psi + N_r i_r\sin(\psi-\theta)]d\psi$$

To find the total flux linkage with the stator coil we add up all of the contributions

$$\lambda_s=\frac{\mu_o N_s (R+g)\ell}{2g}\int_0^{\pi}\sin\psi[N_s i_s\sin\psi + N_r i_r\sin(\psi-\theta)]d\psi$$

$$\lambda_s=\frac{\mu_o N_s (R+g)\ell}{2g}\left[\frac{\pi}{2}N_s i_s + \frac{\pi}{2}N_r i_r\cos\theta\right]$$

This can be written as

$$\lambda_s = L_s i_s + M i_r\cos\theta$$

where

$$L_s=\frac{\pi\mu_o N_s^2 R\ell}{4g}$$

$$M=\frac{\pi\mu_o N_s N_r R\ell}{4g}$$

and we have written $R+g \approx R$ because $g \ll R$.

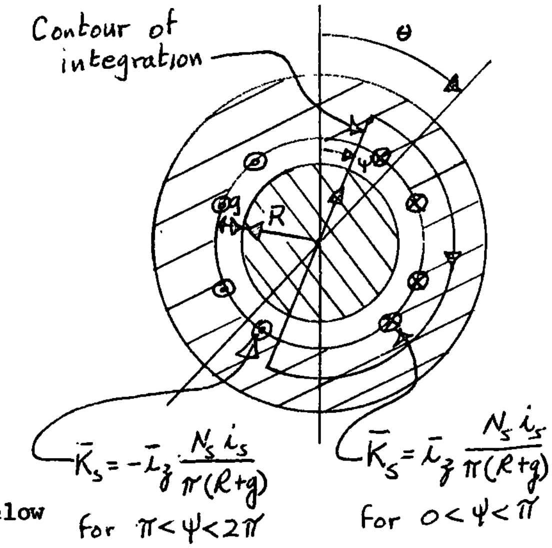
When a similar process is carried out for the rotor winding, it yields

$$\lambda_r = L_r i_r + M i_s\cos\theta$$

where

$$L_r=\frac{\pi\mu_o N_r^2 R\ell}{4g}$$

and M is the same as calculated before.



PROBLEM 4.2

Part a

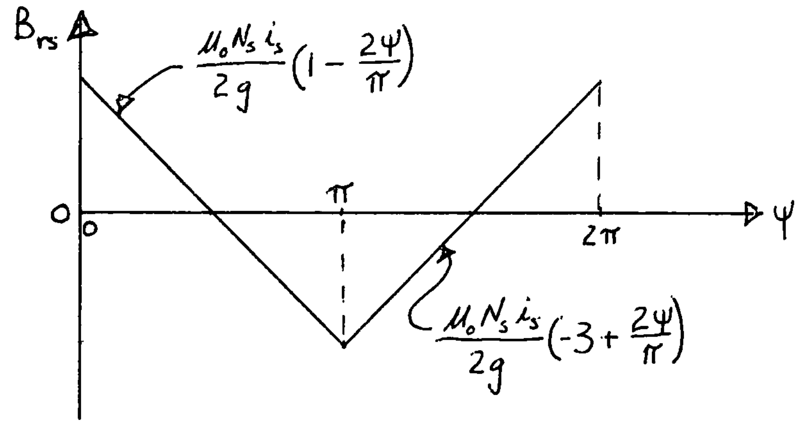
Application of Ampere's law with the contour shown and use of the symmetry condition

$H_{rs}(\psi+\pi) = -H_{rs}(\psi)$ yields

$2H_{rs}(\psi)g = N_s i_s(1 - \frac{2\psi}{\pi})$; for $0 < \psi < \pi$

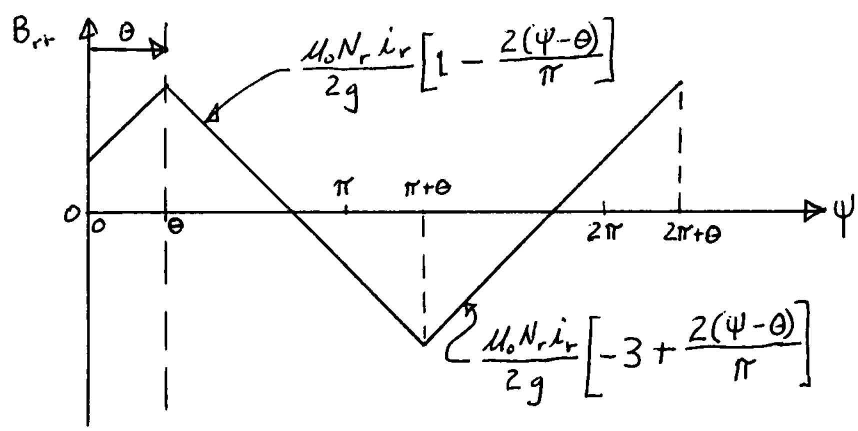
$2H_{rs}(\psi)g = N_s i_s(-3 + \frac{2\psi}{\pi})$; for $\pi < \psi < 2\pi$

The resulting flux density is sketched below



Part b

The same process applied to excitation of the rotor winding yields



PROBLEM 4.2 (Continued)

Part c

For calculating inductances it will be helpful to have both flux densities and turn densities in terms of Fourier series. The turn density on the stator is expressible as

$$n_s = \frac{4N_s}{\pi^2(R+g)} \sum_{n\,odd} \frac{1}{n} \sin n\psi$$

and the turn density on the rotor is

$$n_r = \frac{4N_r}{\pi^2 R} \sum_{n\,odd} \frac{1}{n} \sin(\psi-\theta)$$

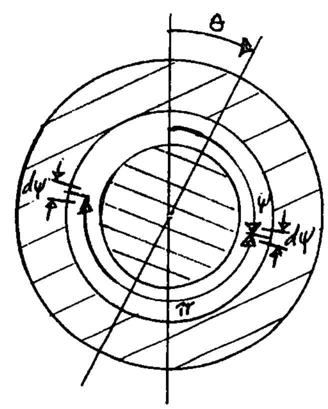
and the flux densities are expressible as

$$B_{rs} = \sum_{n\,odd} \frac{4\mu_o N_s i_s}{\pi^2 g n^2} \cos n\psi$$

$$B_{rr} = \sum_{n\,odd} \frac{4\mu_o N_r i_r}{\pi^2 g n^2} \cos n(\psi-\theta)$$

The total radial flux density is

$$B_r = B_{rs} + B_{rr}$$



First calculating stator flux linkages, we first consider the elemental coil having sides dψ long and π radians apart

$$d\lambda_s = \underbrace{n_s(R+g)\,d\psi}_{\text{number of turns}} \underbrace{\left[-\int_{\psi}^{\psi+\pi} B_r(\psi')(R+g)\ell\, d\psi'\right]}_{\text{flux linking one turn of elemental coil}}$$

Substitution of series for B_r yields

$$d\lambda_s = n_s(R+g)^2\ell\, d\psi \left[\sum_{n\,odd} \frac{8\mu_o N_s i_s}{\pi^2 g n^3} \sin n\psi + \sum_{n\,odd} \frac{8\mu_o N_r i_r}{\pi^2 g n^3} \sin n(\psi-\theta)\right]$$

The total flux linkage with the stator coil is

$$\lambda_s = \frac{32\mu_o N_s(R+g)\ell}{\pi^4 g} \int_0^{\pi} \left[\sum_{n\,odd} \frac{1}{n} \sin n\psi\right]\left[\sum_{n\,odd} \frac{N_s i_s}{n^3} \sin n\psi + \sum_{n\,odd} \frac{N_r i_r}{n^3} \sin n(\psi-\theta)\right] d\psi$$

PROBLEM 4.2 (Continued)

Recognition that

$$\int_0^{\pi} \sin n\psi \sin m(\psi-\theta)d\psi = 0 \text{ when } m \neq n$$

simplifies the work in finding the solution

$$\lambda_s = \frac{32\mu_o N_s (R+g)\ell}{\pi^4 g} \sum_{nodd} \left(\frac{\pi N_s i_s}{2n^4} + \frac{\pi N_r i_r}{2n^4} \cos n\theta\right)$$

This can be written in the form

$$\lambda_s = L_s i_s + \sum_{nodd} M_n \cos n\theta \, i_r$$

where

$$L_s = \frac{16\mu_o N_s^2 R\ell}{\pi^3 g} \sum_{nodd} \frac{1}{n^4}$$

$$M_n = \frac{16\mu_o N_s N_r R\ell}{\pi^3 g n^4}$$

In these expressions we have used the fact that $g \ll R$ to write $R+g \approx R$.

A similar process with the rotor winding yields

$$\lambda_r = L_r i_r + \sum_{nodd} M_n \cos n\theta \, i_s$$

where

$$L_r = \frac{16\mu_o N_s^2 R\ell}{\pi^3 g} \sum_{nodd} \frac{1}{n^4}$$

and M_n is as given above.

PROBLEM 4.3

With reference to the solution of Prob. 4.2, if the stator winding is sinusoidally distributed, λ_s becomes

$$\lambda_s = \frac{32\mu_o N_s (R+g)\ell}{\pi^4 g} \int_o^{\pi} \sin\psi \left[N_s i_s \sin\psi + \sum_{nodd} \frac{N_r i_r}{n^3} \sin n(\psi-\theta)\right] d\psi$$

Because $\int_o^{\pi} \sin\psi \sin n(\psi-\theta) = 0$ when $n \neq 1$

$$\lambda_s = \frac{32\mu_o N_s (R+g)\ell}{\pi^4 g} \int_o^{\pi} \sin\psi \left[N_s i_s \sin\psi + N_r i_r \sin(\psi-\theta)\right] d\psi$$

and the mutual inductance will contain no harmonic terms.

Similarly, if the rotor winding is sinusoidally distributed,

PROBLEM 4.3 (Continued)

$$\lambda_s = \frac{32\mu_o N_s (R+g)\ell}{\pi^4 g} \int_o^\pi \left[\sum_{nodd} \frac{1}{n} \sin n\psi\right]\left[\sum_{nodd} \frac{N_s i_s}{n^3} \sin n\psi + N_r i_r \sin(\psi-\theta)\right] d\psi$$

Using the orthogonality condition

$$\int_o^\pi \sin n\psi \sin(\psi-\theta) d\psi = 0 \text{ when } n \neq 1$$

$$\lambda_s = \frac{32\mu_o N_s (R+g)\ell}{\pi^4 g} \int_o^\pi \left[\sum_{nodd} \frac{N_s i_s}{n^4} \sin^2 n\psi + N_r i_r \sin\psi \sin(\psi-\theta)\right] d\psi$$

and the mutual inductance once again contains only a space fundamental term.

PROBLEM 4.4

Part a

The open-circuit stator voltage is

$$v_s = \frac{d\lambda_s}{dt} = \frac{d}{dt}\left[I \sum_{nodd} \frac{M_o}{n^4} \cos n\omega t\right]$$

$$v_s(t) = - \sum_{nodd} \frac{\omega M_o I}{n^3} \sin n\omega t$$

Part b

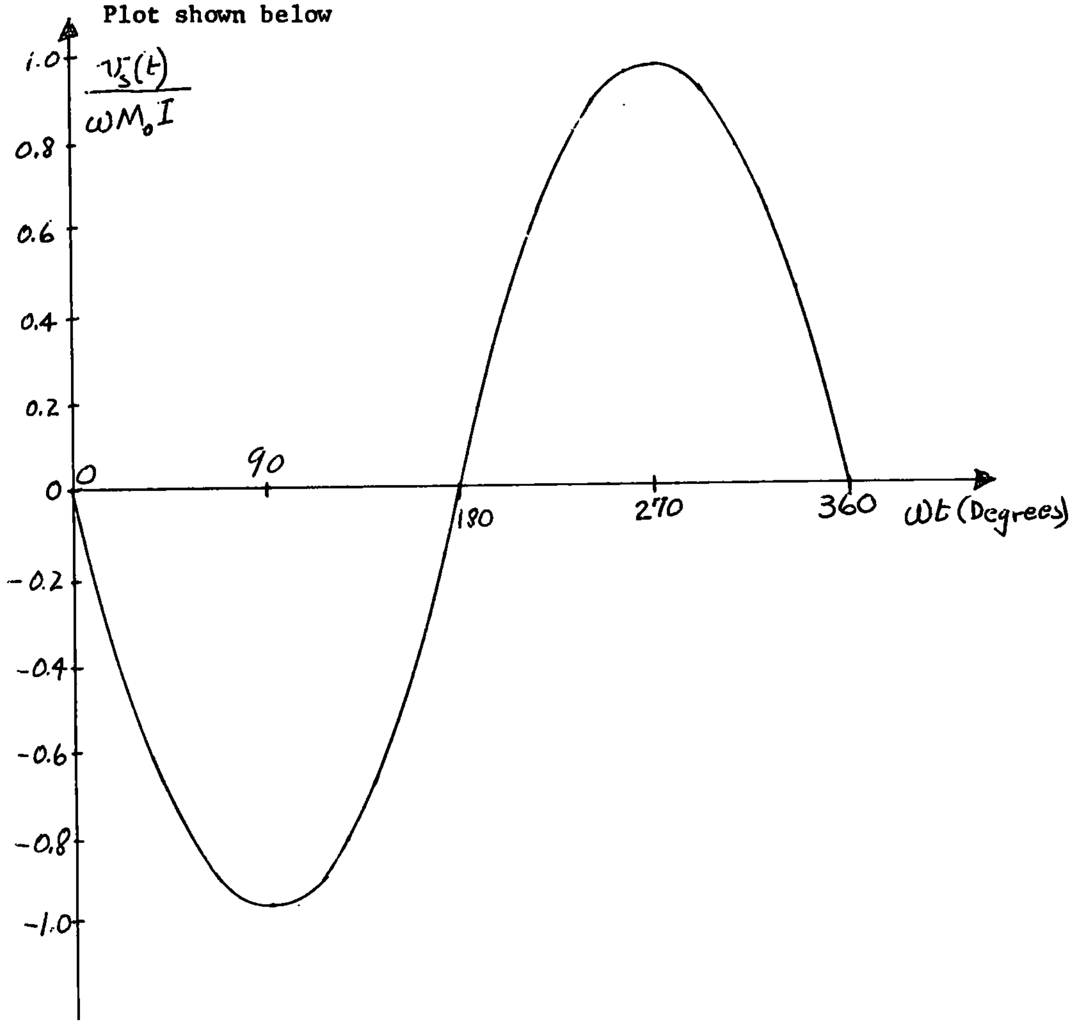
$$\frac{V_{sn}}{V_{s1}} = \frac{1}{n^3} \; ; \; \frac{V_{s3}}{V_{s1}} = \frac{1}{27} \approx 4 \text{ percent}$$

This indicates that uniform turn density does not yield unreasonably high values of harmonics.

PROBLEM 4.4 (Continued)

Part c

Plot shown below



PROBLEM 4.5

Given electrical terminal relations are

$$\lambda_s = L_s i_s + M i_r \cos\theta$$

$$\lambda_r = M i_s \cos\theta + L_r i_r$$

System is conservative so energy or coenergy is independent of path. Select currents and θ as independent variables and use coenergy (see Table 3.1). Assemble system first mechanically, then electrically so torque is not needed in calculation of coenergy. Selecting one of many possible paths of integration for i_s and i_r we have

PROBLEM 4.5 (Continued)

$$W_m'(i_s,i_r,\theta) = \int_0^{i_s} \lambda_s(i_s',0,\theta)di_s' + \int_0^{i_r} \lambda_r(i_s,i_r',\theta)di_r'$$

$$W_m'(i_s,i_r,\theta) = \frac{1}{2} L_s i_s^2 + M i_r i_s \cos\theta + \frac{1}{2} L_r i_r^2$$

$$T^e = \frac{\partial W_m'(i_s,i_r,\theta)}{\partial\theta} = - M i_r i_s \cos\theta$$

PROBLEM 4.6

The conditions existing at the time the rotor winding terminals are short-circuited lead to the constant rotor winding flux linkages

$$\lambda_r = M I_o$$

This constraint leads to a relation between i_r and $i_s = i(t)$

$$M I_o = M i_s \cos\theta + L_r i_r$$

$$i_r = \frac{M}{L_r} [I_o - i(t)\cos\theta]$$

The torque equation (4.1.8) is valid for any terminal constraint, thus

$$T^e = -M i_r i_s \cos\theta = - \frac{M^2}{L_r} i(t)[I_o - i(t)\cos\theta]\sin\theta$$

The equation of motion for the shaft is then

$$J \frac{d^2\theta}{dt^2} = - \frac{M^2}{L_r} i(t)[I_o - i(t)\cos\theta]\sin\theta$$

PROBLEM 4.7

Part a

Coenergy is

$$W_m'(i_s,i_r,\theta) = \frac{1}{2} L_s i_s^2 + \frac{1}{2} L_r i_r^2 + L_{sr}(\theta) i_s i_r$$

$$T^e = \frac{\partial W_m'(i_s,i_r,\theta)}{\partial\theta} = i_s i_r \frac{dL_{sr}(\theta)}{d\theta}$$

$$T^e = -i_s i_r [M_1 \sin\theta + 3M_3 \sin 3\theta]$$

Part b

With the given constraints

$$T^e = -I_s I_r \sin\omega_s t \sin\omega_r t [M_1 \sin(\omega_m t+\gamma) + 3M_3 \sin 3(\omega_m t+\gamma)]$$

PROBLEM 4.7 (Continued)

Repeated application of trigonometric identities leads to:

$$T^e = -\frac{M_1 I_s I_r}{4}\Big\{\sin[(\omega_m+\omega_s-\omega_r)t+\gamma]+\sin[(\omega_m-\omega_s+\omega_r)t+\gamma]$$
$$-\sin[(\omega_m+\omega_s+\omega_r)t+\gamma]-\sin[(\omega_m-\omega_s-\omega_r)t+\gamma]\Big\}$$
$$-\frac{3M_3 I_s I_r}{4}\Big\{\sin[(3\omega_m+\omega_s-\omega_r)t+3\gamma]+\sin[(3\omega_m-\omega_s+\omega_r)t+3\gamma]$$
$$-\sin[(3\omega_m+\omega_s+\omega_r)t+3\gamma]-\sin[(3\omega_m-\omega_s-\omega_r)t+3\gamma]\Big\}$$

To have a time-average torque, one of the coefficients of time must equal zero. This leads to the eight possible mechanical speeds

$$\omega_m = \pm\omega_s \pm \omega_r \text{ and } \omega_m = \pm\frac{\omega_s \pm \omega_r}{3}$$

For

$$\omega_m = \pm(\omega_s - \omega_r)$$
$$T^e_{avg} = -\frac{M_1 I_s I_r}{4}\sin\gamma$$

For

$$\omega_m = \pm(\omega_s + \omega_r)$$
$$T^e_{avg} = \frac{M_1 I_s I_r}{4}\sin\gamma$$

For

$$\omega_m = \pm\frac{(\omega_s-\omega_r)}{3}$$
$$T^e = -\frac{3M_3 I_s I_r}{4}\sin 3\gamma$$

For

$$\omega_m = \pm\frac{(\omega_s+\omega_r)}{3}$$
$$T^e_{avg} = \frac{3M_3 I_s I_r}{4}\sin 3\gamma$$

PROBLEM 4.8

From 4.1.8 and the given constraints the instantaneous torque is

$$T^e = -I_r M \sin\omega_r t\,\cos(\omega_m t+\gamma)(I_{s1}\sin\omega_s t + I_{s3}\sin 3\omega_s t)$$

Repeated use of trigonometric identities leads to:

PROBLEM 4.8 (Continued)

$$T^e = -\frac{I_r I_{s1} M}{4}\Big\{\cos[(\omega_r+\omega_m-\omega_s)t+\gamma]-\cos[(\omega_r+\omega_m+\omega_s)t+\gamma]$$
$$+\cos[(\omega_r-\omega_m-\omega_s)t-\gamma]-\cos[(\omega_r-\omega_m+\omega_s)t-\gamma]\Big\}$$
$$-\frac{I_r I_{s3} M}{4}\Big\{\cos[(\omega_r+\omega_m-3\omega_s)t+\gamma]-\cos[(\omega_r+\omega_m+3\omega_s)t+\gamma]$$
$$+\cos[(\omega_r-\omega_m-3\omega_s)t-\gamma]-\cos[(\omega_r-\omega_m+3\omega_s)t-\gamma]\Big\}$$

For a time-average torque one of the coefficients of t must be zero. This leads to eight values of ω_m:

$$\omega_m = \pm\,\omega_r \pm \omega_s \quad \text{and} \quad \omega_m = \pm\,\omega_r \pm 3\omega_s$$

For

$$\omega_m = \pm(\omega_r-\omega_s)$$

$$T^e_{avg} = -\frac{I_r I_{s1} M}{4}\cos\gamma$$

For

$$\omega_m = \pm(\omega_r+\omega_s)$$

$$T^e_{avg} = \frac{I_r I_{s1} M}{4}\cos\gamma$$

For

$$\omega_m = \pm(\omega_r-3\omega_s)$$

$$T^e_{avg} = -\frac{I_r I_{s3} M}{4}\cos\gamma$$

For

$$\omega_m = \pm(\omega_r+3\omega_s)$$

$$T^e_{avg} = \frac{I_r I_{s3} M}{4}\cos\gamma$$

PROBLEM 4.9

Electrical terminal relations are 4.1.19-4.1.22. For conservative system, coenergy is independent of path and if we bring system to its final mechanical configuration before exciting it electrically there is no contribution to the coenergy from the torque term. Thus, of the many possible paths of integration we choose one

PROBLEM 4.9 (Continued)

$$W_m'(i_{as},i_{bs},i_{ar},i_{br},\theta) = \int_o^{i_{as}} \lambda_{as}(i_{as}',0,0,0,\theta)di_{as}'$$

$$+ \int_o^{i_{bs}} \lambda_{bs}(i_{as},i_{bs}',0,0,\theta)di_{bs}'$$

$$+ \int_o^{i_{ar}} \lambda_{ar}(i_{as},i_{bs},i_{ar}',0,\theta)di_{ar}'$$

$$+ \int_o^{i_{br}} \lambda_{br}(i_{as},i_{bs},i_{ar},i_{br}',\theta)di_{br}'$$

The use of 4.1.19-4.1.22 in this expression yields

$$W_m' = \int_o^{i_{as}} L_s i_{as}' \; i_{as}' + \int_o^{i_{bs}} L_s i_{bs}' di_{bs}'$$

$$+ \int_o^{i_{ar}} (L_r i_{ar}' + M i_{as} \cos\theta + M i_{bs} \sin\theta) di_{ar}'$$

$$+ \int_o^{i_{br}} (L_r i_{br}' - M i_{as} \sin\theta + M i_{bs} \cos\theta) di_{br}'$$

Evaluation of these integrals yields

$$W_m' = \frac{1}{2} L_s i_{as}^2 + \frac{1}{2} L_s i_{bs}^2 + \frac{1}{2} L_r i_{ar}^2 + \frac{1}{2} L_r i_{br}^2$$

$$+ M i_{as} i_{ar} \cos\theta + M i_{bs} i_{ar} \sin\theta$$

$$- M i_{as} i_{br} \sin\theta + M i_{bs} i_{br} \cos\theta$$

The torque of electric origin is then (see Table 3.1)

$$T^e = \frac{\partial W_m'(i_{as},i_{bs},i_{ar},i_{br},\theta)}{\partial\theta}$$

$$T^e = -M[i_{as} i_{ar} \sin\theta - i_{bs} i_{ar} \cos\theta + i_{as} i_{br} \cos\theta + i_{bs} i_{br} \sin\theta]$$

PROBLEM 4.10

Part a

Substitution of currents into given expressions for flux density

$$B_r = B_{ra} + B_{rb}$$

$$B_r = \frac{\mu_o N}{2g} [I_a \cos \omega t \cos \psi + I_b \sin \omega t \sin \psi]$$

PROBLEM 4.10 (Continued)

Part b

Application of trigonometric identities and simplification yield.

$$B_r = \frac{\mu_o N}{2g}\left[\frac{I_a}{2}\cos(\omega t-\psi) + \frac{I_a}{2}\cos(\omega t + \psi)\right.$$

$$\left.+ \frac{I_b}{2}\cos(\omega t-\psi) - \frac{I_b}{2}\cos(\omega t + \psi)\right]$$

$$B_r = \frac{\mu_o N}{4g}\left[(I_a+I_b)\cos(\omega t-\psi)+(I_a-I_b)\cos(\omega t+\psi)\right]$$

The forward wave is

$$B_{rf} = \frac{\mu_o N(I_a + I_b)}{4g}\cos(\omega t-\psi)$$

For constant phase on the forward wave

$$\omega t - \psi = \text{constant}$$

$$\omega_f = \frac{d\psi}{dt} = \omega$$

The backward wave is

$$B_{rb} = \frac{\mu_o N(I_a - I_b)}{4g}\cos(\omega t + \psi)$$

For

$$\omega t + \psi = \text{constant}$$

$$\omega_b = \frac{d\psi}{dt} = -\omega$$

Part c

The ratio of amplitudes is

$$\frac{B_{rbm}}{B_{rfm}} = \frac{I_a-I_b}{I_a+I_b}$$

$$\frac{B_{rbm}}{B_{rfm}} \to 0 \quad \text{as} \quad I_a \to I_b$$

Part d

When $I_b = -I_a$

$$B_{rf} = 0$$

This has simply reversed the phase sequence.

PROBLEM 4.11

Part a

$$B_r = B_{ra} + B_{rb}$$

$$B_r = \frac{\mu_o NI}{2g}[\cos \omega t \cos \psi + \sin(\omega t + \beta)\sin\psi]$$

Part b

Using trigonometric identities

$$B_r = \frac{\mu_o NI}{2g}[\cos \omega t \cos \psi + \cos \beta \sin \omega t \sin \psi + \sin \beta \cos \omega t \sin \psi]$$

$$B_r = \frac{\mu_o NI}{2g}\ [\frac{1}{2} \cos(\omega t-\psi)+ \frac{1}{2} \cos(\omega t+\psi)$$

$$+ \frac{\cos\beta}{2} \cos(\omega t-\psi)- \frac{\cos\beta}{2} \cos(\omega t+\beta)$$

$$+ \frac{\sin\beta}{2} \sin(\omega t+\psi)- \frac{\sin\beta}{2} \sin(\omega t-\psi)]$$

$$B_r = \frac{\mu_o NI}{4g}\ [(1+\cos\beta)\cos(\omega t-\psi)-\sin\beta\sin(\omega t-\psi)$$

$$+ (1-\cos\beta)\cos(\omega t+\psi)+\sin\beta\sin(\omega t+\psi)]$$

Forward wave is

$$B_{rf} = \frac{\mu_o NI}{4g}\ [(1+\cos\beta)\cos(\omega t-\psi)-\sin\beta\sin(\omega t-\psi)]$$

For constant phase

$$\omega t - \psi = \text{constant}$$

and

$$\omega_f = \frac{d\psi}{dt} = \omega$$

Backward wave is

$$B_{rb} = \frac{\mu_o NI}{4g}[(1-\cos\beta)\cos(\omega t+\psi)+\sin\beta\sin(\omega t+\psi)]$$

For constant phase

$$\omega t + \psi = \text{constant}$$

and

$$\omega_b = \frac{d\psi}{dt} = -\ \omega$$

PROBLEM 4.11 (Continued)

Part c

The ratio of amplitudes is

$$\frac{B_{rbm}}{B_{rfm}} = \frac{\sqrt{(1-\cos\beta)^2 + \sin^2\beta}}{\sqrt{(1+\cos\beta)^2 + \sin^2\beta}} = \sqrt{\frac{1-\cos\beta}{1+\cos\beta}}$$

as $\beta \to 0$, $\frac{B_{rbm}}{B_{rfm}} \to 0$.

Part d

The forward wave amplitude will go to zero when $\beta = \pi$. The phase sequence has been reversed by reversing the phase of the current in the b-winding.

PROBLEM 4.12

Equation 4.1.53 is

$$p_e = v_{as}i_{as} + v_{bs}i_{bs}$$

For steady state balanced conditions we can write

$$i_{as} = I\cos\omega t; \quad i_{bs} = I\sin\omega t$$

$$v_{as} = V\cos(\omega t+\phi); \quad v_{bs} = V\sin(\omega t+\phi)$$

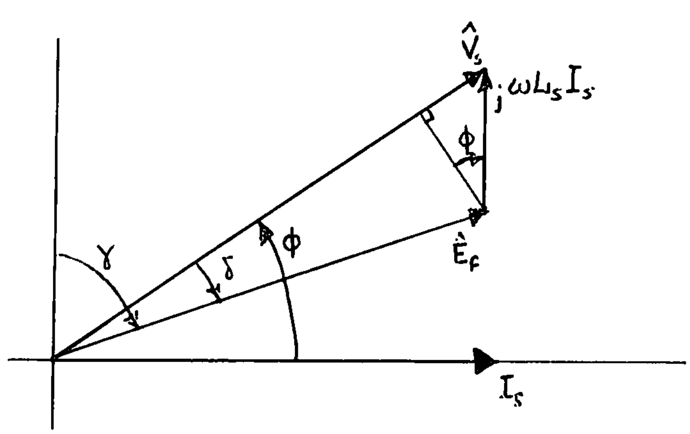
then

$$p_e = VI[\cos\omega t\cos(\omega t+\phi)+\sin\omega t\,\sin(\omega t+\phi)]$$

Using trigonometric identities

$$p_e = VI\cos\phi$$

Referring to Fig. 4.1.13(b) we have the vector diagram



PROBLEM 4.12 (Continued)

From this figure it is clear that

$$\omega L_s I_s \cos\phi = -E_f \sin\delta$$

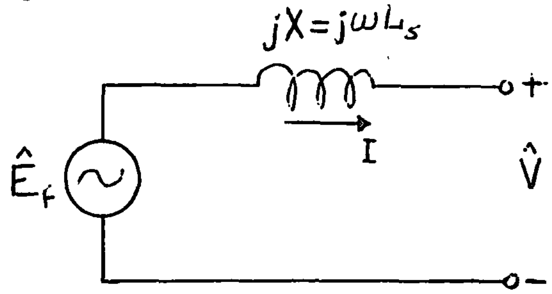
(remember that $\delta < 0$)

Then $p_e = -\frac{VE_f}{\omega L_s}\sin\delta$

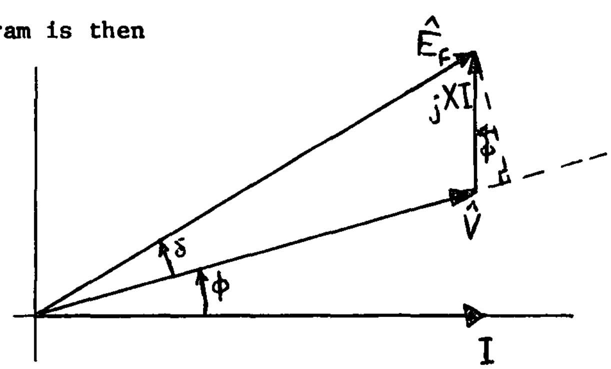
which was to be shown.

PROBLEM 4.13

For the generator we adopt the notation for one phase of the armature circuit (see Fig. 4.1.12 with current convention reversed)



The vector diagram is then



From the vector diagram

$$XI\sin\phi = E_f\cos\delta - V$$

$$XI\cos\phi = E_f\sin\delta$$

Also, the mechanical power input is

$$P = \frac{E_f V}{X}\sin\delta$$

Eliminating ϕ and δ from these equations and solving for I yields

PROBLEM 4.13 (Continued)

$$I = \frac{V}{X}\sqrt{\left(\frac{E_f}{V}\right)^2 - 2\sqrt{\left(\frac{E_f}{V}\right)^2 - \left(\frac{PX}{V^2}\right)^2} + 1}$$

Normalizing as indicated in the problem statement we define

I_o = rated armature current

I_{fo} = field current to give rated voltage on open circuit.

P_o = rated power

$$\frac{I}{I_o} = \frac{V}{I_o X}\sqrt{\left(\frac{I_f}{I_{fo}}\right)^2 + 1 - 2\sqrt{\left(\frac{I_f}{I_{fo}}\right)^2 - \left(\frac{P}{P_o}\right)^2\left(\frac{P_o X}{V^2}\right)^2}}$$

Injecting given numbers and being careful about rms and peak quantities we have

$$\frac{I}{I_o} = 0.431\sqrt{\left(\frac{I_f}{I_{fo}}\right)^2 + 1 - 2\sqrt{\left(\frac{I_f}{I_{fo}}\right)^2 - 3.92\left(\frac{P}{P_o}\right)^2}}$$

I_{fo} = 2,030 amps

and

$$\left(\frac{I_f}{I_{fo}}\right)_{max} = 3.00$$

The condition that $\delta = \frac{\pi}{2}$ is

$$E_f = \frac{PX}{V}$$

$$\left(\frac{I_f}{I_{fo}}\right)_{min} = \frac{PX}{\omega M I_{fo} V} = \frac{PX}{V^2} = 1.98\,\frac{P}{P_o}$$

For unity p.f., $\cos\phi = 1$, $\sin\phi = 0$

$$E_f \cos\delta = V \quad \text{and} \quad E_f \sin\delta = IX$$

eliminating δ we have

$$\frac{I}{I_o} = \frac{V}{XI_o}\sqrt{\left(\frac{E_f}{V}\right)^2 - 1}$$

$$\frac{I}{I_o} = 0.431\sqrt{\left(\frac{I_f}{I_{fo}}\right)^2 - 1}$$

PROBLEM 4.13 (Continued)

for 0.85 p.f.

$$E_f \sin \delta = 0.85\ IX$$

$$E_f \cos \delta - V = \sqrt{1-(0.85)^2}\ \ IX$$

eliminating δ, solving for I, and normalizing yields

$$\frac{I}{I_o} = 0.431 \left| [-0.527 \pm \sqrt{(\frac{I_f}{I_{fo}})^2 - 0.722}] \right|$$

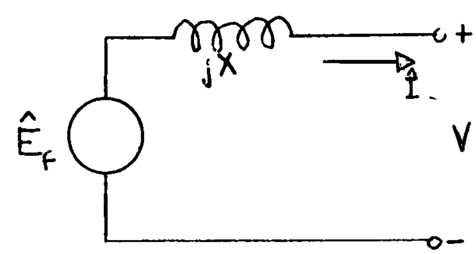
This is double-valued and the magnitude of the bracketed term is used.

The required curves are shown on the next page.

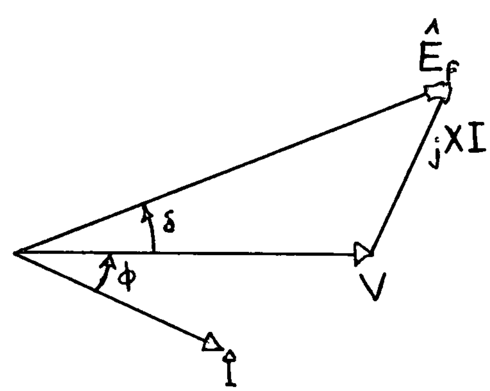
PROBLEM 4.14

The armature current limit is defined by a circle of radius VI_o, where I_o is the amplitude of rated armature current.

To find the effect of the field current limit we must express the complex power in terms of field current. Defining quantities in terms of this circuit

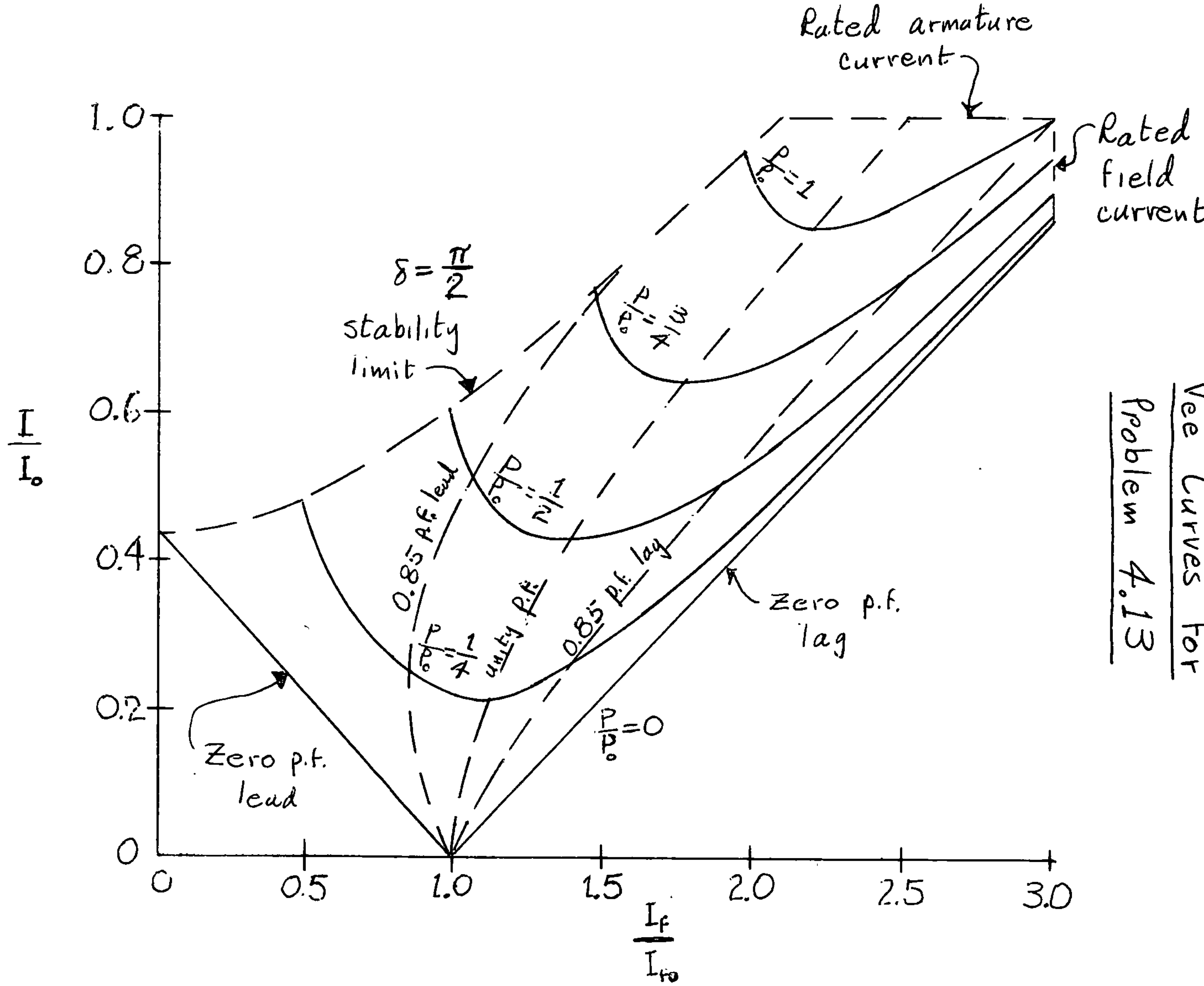


The vector diagram is



$$\hat{I} = \frac{\hat{E}_f - V}{jX}$$

$$P + jQ = V\hat{I}^* = \frac{V\hat{E}_f^* - V^2}{-jX} = j\,\frac{VE_f e^{-j\delta}}{X} - j\,\frac{V^2}{X}$$



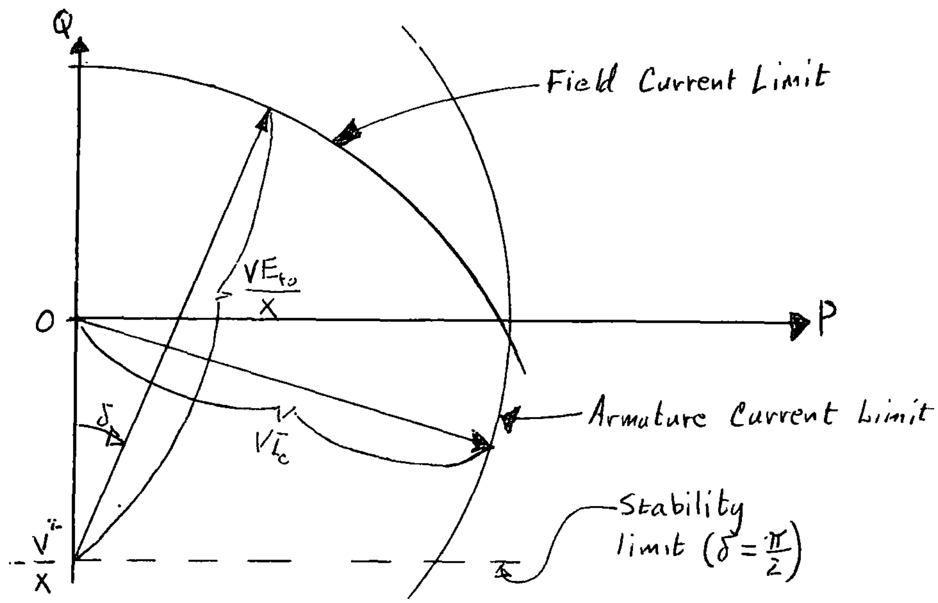
Vee Curves for Problem 4.13

PROBLEM 4.14 (Continued)

If we denote the voltage for maximum field current as E_{fo}, this expression becomes

$$P+jQ = -j\frac{V^2}{X} + \frac{VE_{fo}}{X}\sin\delta + j\frac{VE_{fo}}{X}\cos\delta$$

On a P+jQ plane this trajectory is as sketched below



The stability limit ($\delta = \frac{\pi}{2}$) is also shown in the sketch, along with the armature current limit.

The capability curve for the generator of Prob. 4.13 is shown on the next page.

P and Q are normalized to 724 MVA.

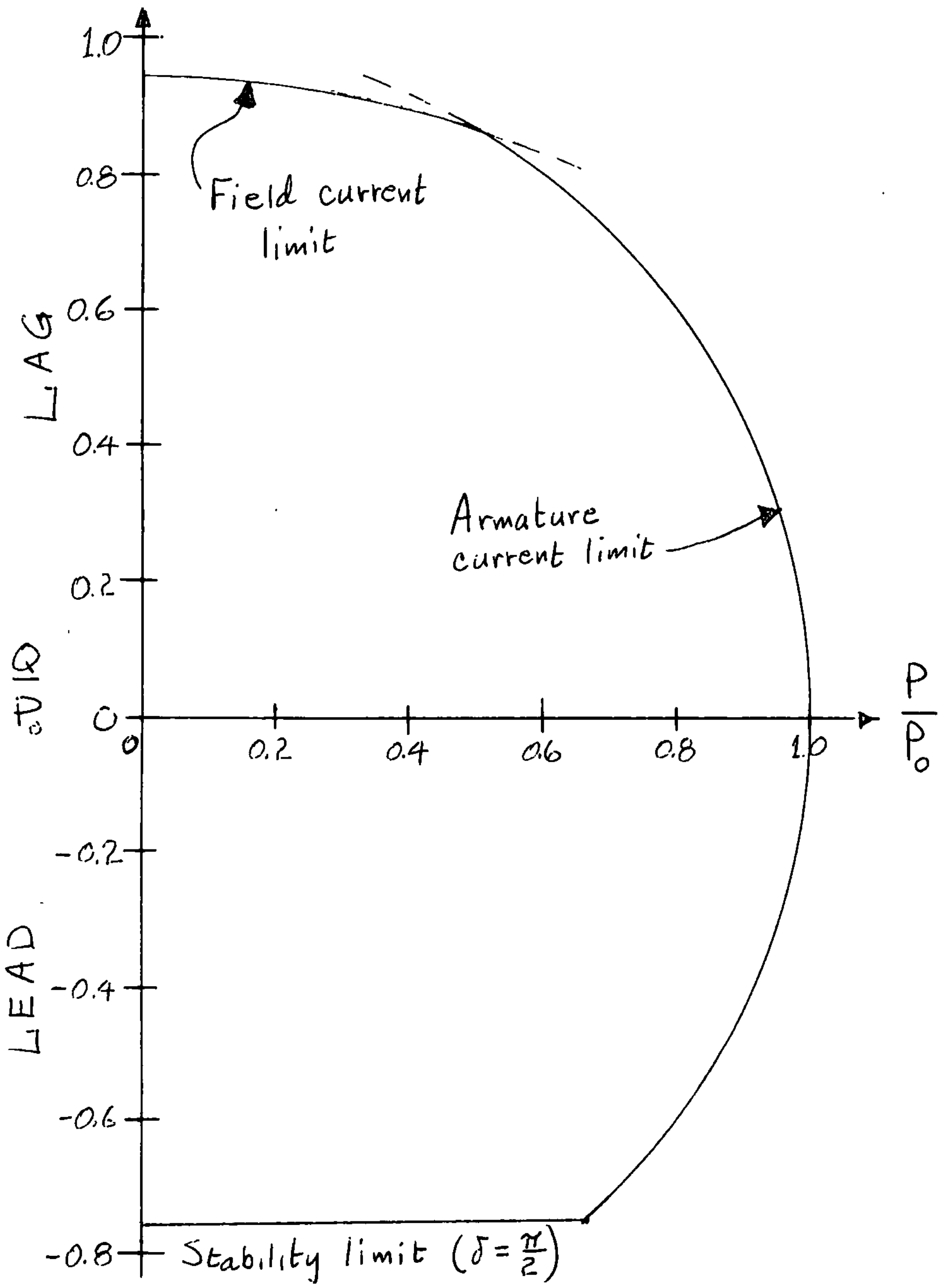
PROBLEM 4.15

The steady state deflection ψ of the rotatable frame is found by setting sum of torques to zero

$$T^e + T_s = 0 = T^e - K\psi \qquad (1)$$

where T^e is electromagnetic torque. This equation is solved for ψ.

Torque T^e is found from



Capability curve of Problem 4.14

PROBLEM 4.15 (Continued)

$$T^e = \frac{\partial W'_m(i_1,i_2,i_3,\phi,\psi)}{\partial\psi}$$

and the magnetic coenergy for this electrically linear system is

$$W'_m = \frac{1}{2}Li_1^2 + \frac{1}{2}Li_2^2 + \frac{1}{2}L_3i_3^2 + Mi_1i_3\cos(\phi-\psi)+Mi_2i_3\sin(\phi-\psi)$$

from which

$$T^e = Mi_1i_3\sin(\phi-\psi) - Mi_2i_3\cos(\phi-\psi)$$

For constant shaft speed ω, the shaft position is

$$\phi = \omega t.$$

Then, with $i_3 = I_o$ as given

$$\frac{d\lambda_1}{dt} = -\omega MI_o\sin(\omega t-\psi)+L\frac{di_1}{dt} = -i_1R$$

and

$$\frac{d\lambda_2}{dt} = \omega MI_o\cos(\omega t-\psi)+L\frac{di_2}{dt} = -i_2R$$

Using the given assumptions that

$$\left|L\frac{di_1}{dt}\right| << \left|Ri_1\right| \quad \text{and} \quad \left|L\frac{di_2}{dt}\right| << \left|Ri_2\right|$$

we have

$$i_1 = \frac{\omega MI_o}{R}\sin(\omega t-\psi)$$

$$i_2 = -\frac{\omega MI_o}{R}\cos(\omega t-\psi)$$

and the torque T^e is

$$T^e = MI_o\left(\frac{\omega MI_o}{R}\right)[\sin^2(\omega t-\psi)+\cos^2(\omega t-\psi)]$$

Hence, from (1)

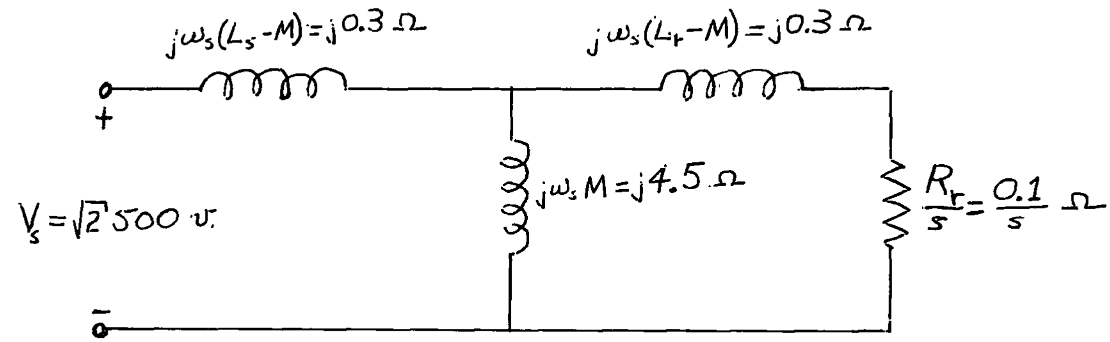
$$\psi = \frac{(MI_o)^2}{KR}\omega$$

which shows that pointer displacement ψ is a linear function of shaft speed ω which is in turn proportional to car speed.

Suppose we had not neglected the voltage drops due to self inductance. Would the final result still be the same?

PROBLEM 4.16

The equivalent circuit with parameter values as given is



From (4.1.82) the torque is

$$T^e = \frac{\left(\frac{k^2}{\omega_s}\right)\left(\frac{L_r}{L_s}\right)\left(\frac{R_r}{s}\right)V_s^2}{[\omega_s(1-k^2)L_r]^2+(R_r/s)^2}$$

where $k^2 = \frac{M^2}{L_r L_s}$ and $s = \frac{\omega_s-\omega_m}{\omega_s}$

Solution of (4.1.81) for I_s yields

$$I_s = \sqrt{\frac{\left(\frac{R_r}{s}\right)^2 + (\omega_s L_r)^2}{\left(\frac{R_r}{s}\right)^2 + [\omega_s L_r(1-k^2)]^2}}\quad\left(\frac{V_s}{\omega_s L_s}\right)$$

volt-ampere input is simply (for two phases)

$$(VA)_{in} = V_s I_s$$

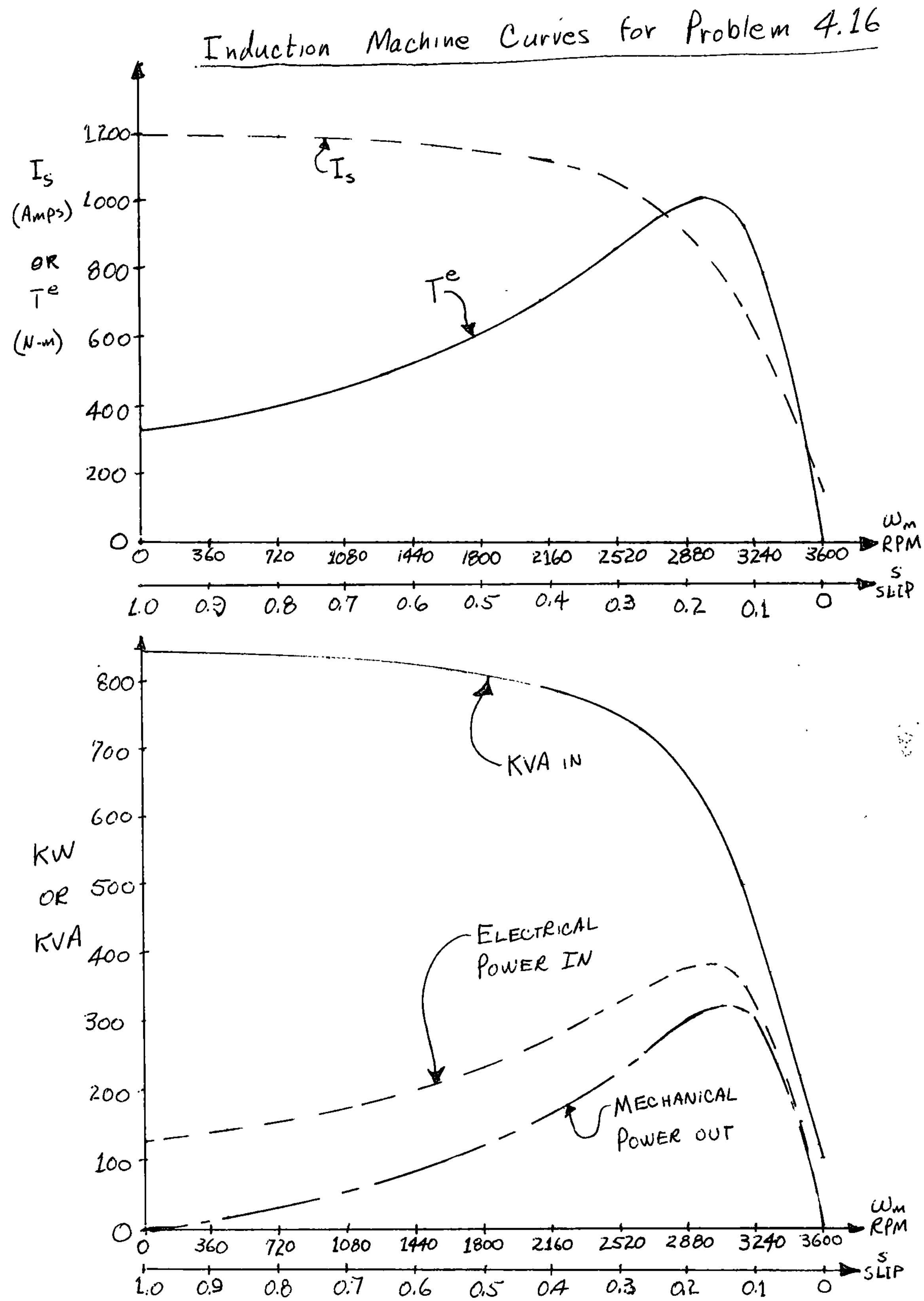
The electrical input power can be calculated in a variety of ways, the simplest being to recognize that in the equivalent circuit the power dissipated in R_r/s (for two phases) is just ω_s times the electromagnetic torque, hence

$$P_{in} = T^e\omega_s$$

Finally, the mechanical power output is

$$P_{mech} = T^e\omega_m$$

These five quantities are shown plotted in the attached graphs. Numerical constants used in the computations are


Induction Machine Curves for Problem 4.16
I_s (Amps) OR T^e (N-m)
1200
1000
800
600
400
200
0
I_s
T^e
ω_m RPM
0 360 720 1080 1440 1800 2160 2520 2880 3240 3600
s SLIP
1.0 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0
KW OR KVA
800
700
600
500
400
300
200
100
0
KVA IN
ELECTRICAL POWER IN
MECHANICAL POWER OUT
ω_m RPM
0 360 720 1080 1440 1800 2160 2520 2880 3240 3600
s SLIP
1.0 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0

PROBLEM 4.16 (Continued)

$$\omega_s L_s = \omega_s L_r = \omega_s M + 0.3 = 4.8\Omega$$

$$k^2 = (\frac{4.5}{4.8})^2 = 0.878$$

$$T^e = \frac{\frac{117}{s}}{0.342 + \frac{0.01}{s^2}} \text{ newton-meters}$$

$$I_s = \sqrt{\frac{23.0 + \frac{0.01}{s^2}}{0.342 + \frac{0.01}{s^2}}}\ 147 \text{ amps pk.}$$

$$s_{mT} = 0.188$$

PROBLEM 4.17

Part a

For ease in calculation it is useful to write the mechanical speed as

$$\omega_m = (1-s)\omega_s$$

and the fan characteristic as

$$T_m = -B\omega_s^3(1-s)^3$$

With $\omega_s = 120\pi$ rad/sec

$$B\omega_s^3 = 400 \text{ newton-meters}$$

The results of Prob. 4.16 for torque yields

$$400(1-s)^3 = \frac{\frac{117}{s}}{0.342 + \frac{0.01}{s^2}}$$

Solution of this equation by cut-and-try for s yields:

$$s = 0.032$$

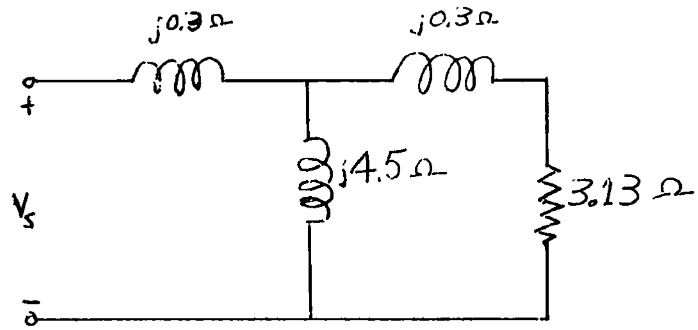
Then $P_{mech} = (400)\ (1-s)^3\omega_m = (400)(\omega_s)(1-s)^4$

$$P_{mech} = 133 \text{ kilowatts into fan}$$

$$P_{input} = \frac{P_{mech}}{1-s} = 138 \text{ kilowatts}$$

Circuit seen by electrical source is

PROBLEM 4.17 (Continued)



Input impedance is

$$Z_{in} = j0.3 + \frac{(j4.5)(3.13+j0.3)}{3.13 + j4.8} = \frac{-2.79+j15.0}{3.13+j4.8}$$

$$\angle Z_{in} = 100.6° - 56.8° = 43.8°$$

Hence,

$$\text{p.f.} = \cos\angle Z_{in} = 0.72 \text{ lagging}$$

Part b

Electromagnetic torque scales as the square of the terminal voltage, thus

$$T^e = \frac{\frac{117}{s}}{0.342 + \frac{0.01}{s^2}} \left(\frac{V_s}{V_{so}}\right)^2$$

where $V_{so} = \sqrt{2}\ 500$ volts peak. The slip for any terminal voltage is now found from

$$400(1-s)^3 = \frac{\frac{117}{s}}{0.342 + \frac{0.01}{s^2}} \left(\frac{V_s}{V_{so}}\right)^2$$

The mechanical power into the fan is

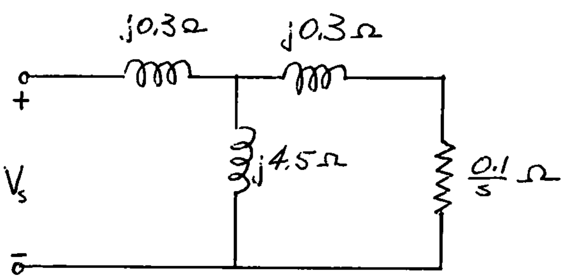
$$P_{mech} = 400\ \omega_s^4(1-s)^4$$

electrical power input is

$$P_{in} = \frac{P_{mech}}{1-s}$$

PROBLEM 4.17 (Continued)

and the power factor is found as the cosine of the angle of the input impedance of the circuit



These quantities are plotted as required on the attached graph.

PROBLEM 4.18

Part a

The solution to Prob. 4.1 can be used to find the flux densities here. For the stator a-winding, the solution of Prob. 4.1 applies directly, thus, the radial component of flux density due to current in stator winding a is

$$B_{ra}(\psi) = \frac{\mu_o N_s i_a}{2g} \cos\psi$$

Windings b and c on the stator are identical with the a winding except for the indicated angular displacements, thus,

$$B_{rb}(\psi) = \frac{\mu_o N_s i_b}{2g} \cos(\psi - \frac{2\pi}{3})$$

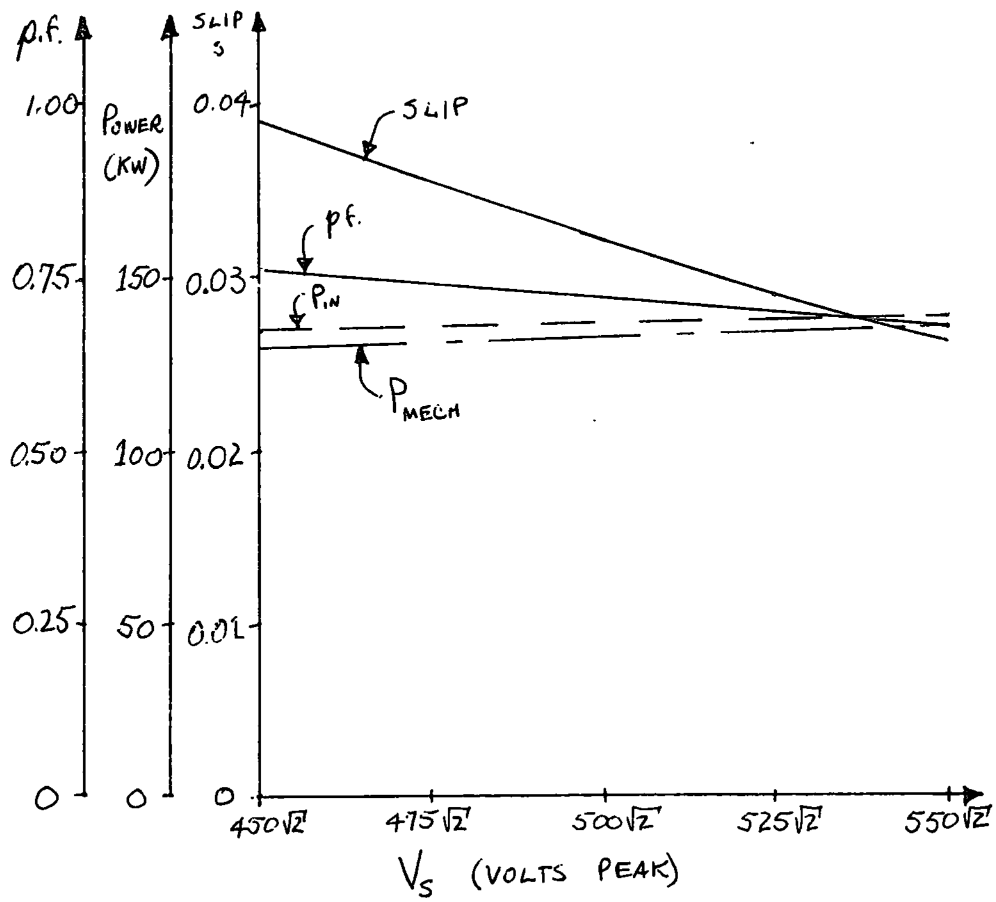
$$B_{rc}(\psi) = \frac{\mu_o N_s i_c}{2g} \cos(\psi - \frac{4\pi}{3})$$

The solution in Prob. 4.1 for the flux density due to rotor winding current applies directly here, thus,

$$B_{rr}(\psi) = \frac{\mu_o N_r i_r}{2g} \cos(\psi - \theta)$$

Part b

The method of part (c) of Prob. 4.1 can be used and the results of that analysis applied directly by replacing rotor quantities by stator b-winding quantities and θ by $2\pi/3$. The resulting mutual inductance is (assuming $g \ll R$)



Induction Machine Curves for Problem 4.17

PROBLEM 4.18 (Continued)

$$L_{ab} = \frac{\pi\mu_o N_s^2 R\ell}{2g} \cos\frac{2\pi}{3}.$$

$$L_{ab} = -\frac{\mu_o N_s^2 R\ell}{4g} = -\frac{L_s}{2}$$

where L_s is the self inductance of one stator winding alone. Note that $L_{ac} = L_{ab}$ because of relative geometry.

Part c

The λ-i relations are thus

$$\lambda_a = L_s i_a - \frac{L_s}{2} i_b - \frac{L_s}{2} i_c + M\cos\theta i_r$$

$$\lambda_b = -\frac{L_s}{2} i_a + L_s i_b - \frac{L_s}{2} i_c + M\cos(\theta - \frac{2\pi}{3}) i_r$$

$$\lambda_c = -\frac{L_s}{2} i_a - \frac{L_s}{2} i_b + L_s i_c + M\cos(\theta - \frac{4\pi}{3}) i_r$$

$$\lambda_r = M\cos\theta i_a + M\cos(\theta - \frac{2\pi}{3}) i_b$$

$$+ M\cos(\theta - \frac{4\pi}{3}) i_c + L_r i_r$$

where from Prob. 4.1,

$$L_s = \frac{\pi\mu_o N_s^2 R\ell}{2g}$$

$$M = \frac{\pi\mu_o N_s N_r R\ell}{2g}$$

$$L_r = \frac{\pi\mu_o N_r^2 R\ell}{2g}$$

Part d

The torque of electric origin is found most easily by using magnetic coenergy which for this electrically linear system is

$$W_m'(i_a, i_b, i_c, i_r, \theta) = \frac{1}{2} L_s(i_a^2 + i_b^2 + i_c^2)$$

$$+ \frac{1}{2} L_s(i_a i_b + i_a i_c + i_b i_c) + M\cos\theta i_r i_a$$

$$+ M\cos(\theta - \frac{2\pi}{3}) i_r i_b + M\cos(\theta - \frac{4\pi}{3}) i_r i_c$$

The torque of electric origin is

PROBLEM 4.18 (Continued)

$$T^e = \frac{\partial W'_m(i_a, i_b, i_c, i_r, \theta)}{\partial \theta}$$

$$T^e = -Mi_r[i_a \sin\theta + i_b \sin(\theta - \frac{2\pi}{3}) + i_c \sin(\theta - \frac{4\pi}{3})]$$

PROBLEM 4.19

Part a

Superimposing the three component stator flux densities from Part a of Prob. 4.18, we have

$$B_{rs} = \frac{\mu_o N_s}{2g}[i_a \cos\psi + i_b \cos(\psi - \frac{2\pi}{3}) + i_c \cos(\psi - \frac{4\pi}{3})]$$

Substituting the given currents

$$B_{rs} = \frac{\mu_o N_s}{2g}[I_a \cos\omega t \cos\psi + I_b \cos(\omega t - \frac{2\pi}{3})\cos(\psi - \frac{2\pi}{3})$$

$$+ I_c \cos(\omega t - \frac{4\pi}{3})\cos(\psi - \frac{4\pi}{3})$$

Using trigonometric identities and simplifying yields

$$B_{rs} = \frac{\mu_o N_s}{2g}\Bigg[\left(\frac{I_a + I_b + I_c}{2}\right)\cos(\omega t - \psi)$$

$$+ \frac{1}{2}(I_a + I_b \cos\frac{4\pi}{3} + I_c \cos\frac{2\pi}{3})\cos(\omega t + \psi)$$

$$+ \frac{1}{2}(I_b \sin\frac{4\pi}{3} + I_c \sin\frac{2\pi}{3})\sin(\omega t + \psi)\Bigg]$$

Positive traveling wave has point of constant phase defined by

$$\omega t - \psi = \text{constant}$$

from which

$$\frac{d\psi}{dt} = \omega$$

This is positive traveling wave with amplitude

$$B_{rfm} = \frac{\mu_o N_s}{4g}(I_a + I_b + I_c)$$

Negative traveling wave has point of constant phase

$$\omega t + \psi = \text{constant}$$

from which

$$\frac{d\psi}{dt} = -\omega$$

This defines negative traveling wave with amplitude

PROBLEM 4.19 (Continued)

$$B_{rbm} = \frac{\mu_o N_s}{4g} \sqrt{\left(I_a - \frac{I_b}{2} - \frac{I_c}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2} I_b + \frac{\sqrt{3}}{2} I_c\right)^2}$$

Part b

When three phase currents are balanced

$$I_a = I_b = I_c$$

and $B_{rbm} = 0$ leaving only a forward (positive) traveling wave.

PROBLEM 4.20

Part a

Total radial flux density due to stator excitation is

$$B_{rs} = \frac{\mu_o N}{2g} (i_a \cos 2\psi + i_b \sin 2\psi)$$

Substituting given values for currents

$$B_{rs} = \frac{\mu_o N}{2g} (I_a \cos \omega t \cos 2\psi + I_b \sin \omega t \sin 2\psi)$$

Part b

$$B_{rs} = \frac{\mu_o N}{2g} \left| \left(\frac{I_a + I_b}{2}\right) \cos(\omega t - 2\psi) + \left(\frac{I_a - I_b}{2}\right) \cos(\omega t + 2\psi) \right|$$

The forward (positive-traveling) component has constant phase defined by

$$\omega t - 2\psi = \text{constant}$$

from which

$$\frac{d\psi}{dt} = \frac{\omega}{2}$$

The backward (negative-traveling) component has constant phase defined by

$$\omega t + 2\psi = \text{constant}$$

from which

$$\frac{d\psi}{dt} = -\frac{\omega}{2}$$

Part c

From part b, when $I_a = I_b$, $I_a - I_b = 0$ and the backward-wave amplitude goes to zero. When $I_b = -I_a$, $I_a + I_b = 0$ and the forward-wave amplitude goes to zero.

PROBLEM 4.21

Referring to the solution for Prob. 4.20,

Part a

$$B_{rs} = \frac{\mu_o N}{2g}(i_a \cos p\psi + i_b \sin p\psi)$$

$$B_{rs} = \frac{\mu_o N}{2g}(I_a \cos \omega t \cos p\psi + I_b \sin \omega t \sin p\psi)$$

Part b

Using trigonometric identities yields

$$B_{rs} = \frac{\mu_o N}{2g}\left[\left(\frac{I_a + I_b}{2}\right)\cos(\omega t - p\psi) + \left(\frac{I_a - I_b}{2}\right)\cos(\omega t + p\psi)\right]$$

Forward wave has constant phase

$$\omega t - p\psi = \text{constant}$$

from which

$$\frac{d\psi}{dt} = \frac{\omega}{p}$$

Backward wave has constant phase

$$\omega t + p\psi = \text{constant}$$

from which

$$\frac{d\psi}{dt} = -\frac{\omega}{p}$$

Part c

From part b, when $I_b = I_a$, $I_a - I_b = 0$, and backward-wave amplitude goes to zero. When $I_b = -I_a$, $I_a + I_b = 0$, and forward-wave amplitude goes to zero.

PROBLEM 4.22

This is an electrically linear system, so the magnetic coenergy is

$$W'_m(i_s, i_r, \theta) = \frac{1}{2}(L_o + L_2 \cos 2\theta)i_s^2 + \frac{1}{2} L_r i_r^2 + M i_r i_s \cos \theta$$

Then the torque is

$$T^e = \frac{\partial W'_m(i_s, i_r, \theta)}{\partial \theta} = -M i_r i_s \sin \theta - L_2 i_s^2 \sin 2\theta$$

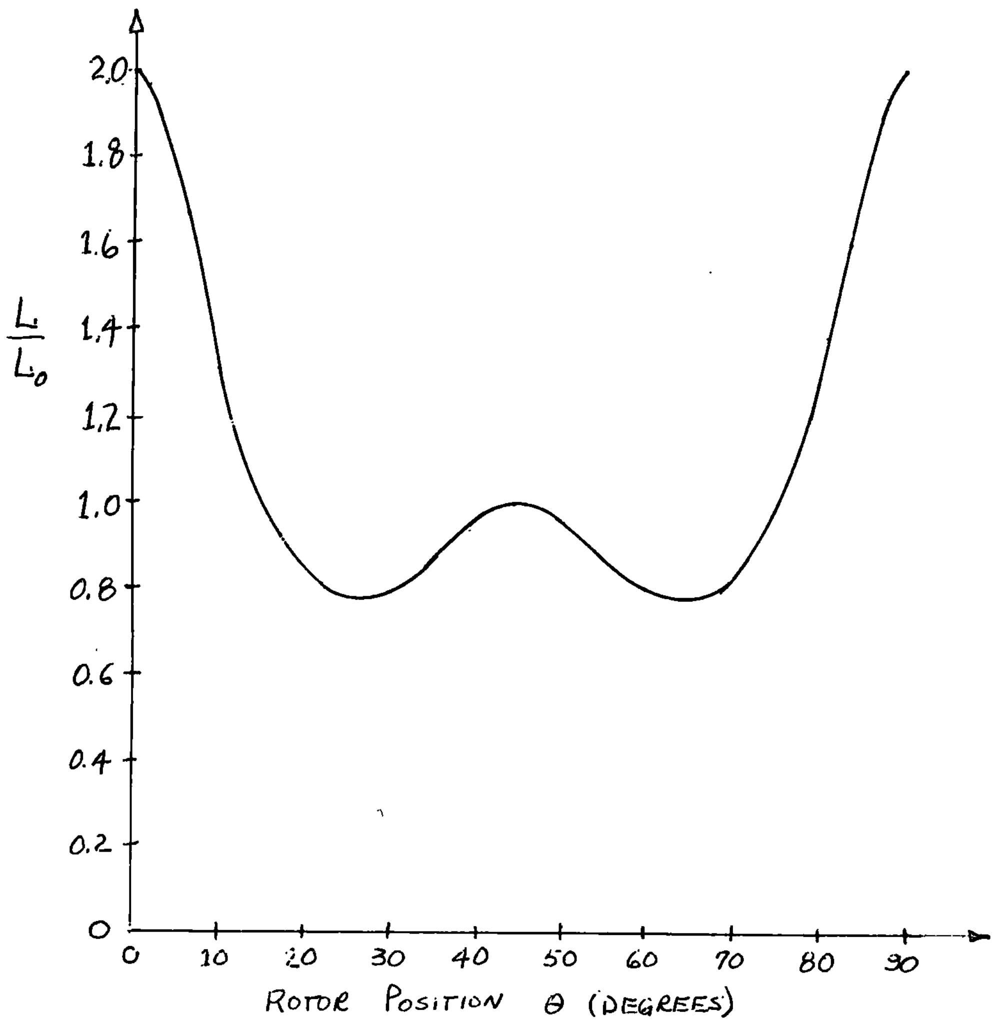
PROBLEM 4.23

Part a

$$L = \frac{L_o}{(1-0.25 \cos 4\theta - 0.25 \cos 8\theta)}$$

PROBLEM 4.23 (Continued)

The variation of this inductance with θ is shown plotted below.



PROBLEM 4.23 (Continued)

From this plot and the configuration of Fig. 4P.23, it is evident that minimum reluctance and maximum inductance occur when $\theta = 0, \pi/2, \pi, \ldots \frac{n}{2}\pi, \ldots$. The inductance is symmetrical about $\theta = 0, \frac{\pi}{2}, \ldots$ and about $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \ldots \frac{\pi}{4} + \frac{n\pi}{2}, \ldots$ as it should be. Minimum inductance occurs on both sides of $\theta = \frac{\pi}{4}$ which ought to be maximum reluctance.

The general trend of the inductance is correct for the geometry of Fig. 4P.23 but the equation would probably be a better representation if the sign of the 8θ term were reversed.

Part b

For this electrically linear system, the magnetic stored energy is

$$W_m(\lambda,\theta) = \frac{1}{2}\frac{\lambda^2}{L}$$

$$W_m(\lambda,\theta) = \frac{\lambda^2(1-0.25\cos 4\theta - 0.25\cos 8\theta)}{2L_o}$$

The torque is then

$$T^e = -\frac{\partial W_m(\lambda,\theta)}{\partial\theta}$$

$$T^e = -\frac{\lambda^2}{2L_o}(\sin 4\theta + 2\sin 8\theta)$$

Part c

With $\lambda = \Lambda_o \cos\omega t$ and $\theta = \Omega t + \delta$

$$T^e = -\frac{\Lambda_o^2\cos^2\omega t}{2L_o}[\sin(4\Omega t+4\delta)+2\sin(8\Omega t+8\delta)]$$

Repeated use of trig identities yields for the instantaneous converted power

$$\Omega T^e = -\frac{\Omega\Lambda_o^2}{4L_o}[\sin(4\Omega t+4\delta) + 2\sin(8\Omega t+8\delta)$$

$$+\frac{1}{2}\sin(2\omega t + 4\Omega t + 4\delta) + \frac{1}{2}\sin(4\Omega t - 2\omega t + 4\delta)$$

$$+\sin(2\omega t + 8\Omega t + 8\delta) + \sin(8\Omega t - 2\omega t + 8\delta)]$$

This can only have a non-zero average value when $\Omega \neq 0$ and a coefficient of t in one argument is zero. This gives 4 conditions

$$\Omega = \pm\frac{\omega}{2}, \pm\frac{\omega}{4}$$

When $\Omega = \pm\frac{\omega}{2}$

$$[\Omega T^e]_{avg} = -\frac{\Omega\Lambda_o^2}{8L_o}\sin 4\delta$$

PROBLEM 4.23 (Continued)

and when $\Omega = \pm \frac{\omega}{4}$

$$[\Omega T^e]_{avg} = -\frac{\Omega \Lambda_o^2}{4L_o} \sin 8\delta$$

PROBLEM 4.24

It will be helpful to express the given ratings in alternative ways. Rated output power = 6000 HP = 4480 KW at 0.8 p.f. this is

$$\frac{4480}{0.8} = 5600 \text{ KVA total}$$

or

2800 KVA per phase

The rated phase current is then

$$I_s = \frac{2800 \times 10^3}{3 \times 10^3} = 933 \text{ amps rms} = 1320 \text{ amps pk.}$$

Given:

Direct axis reactance $\omega(L_o+L_2) = 4.0$ ohms

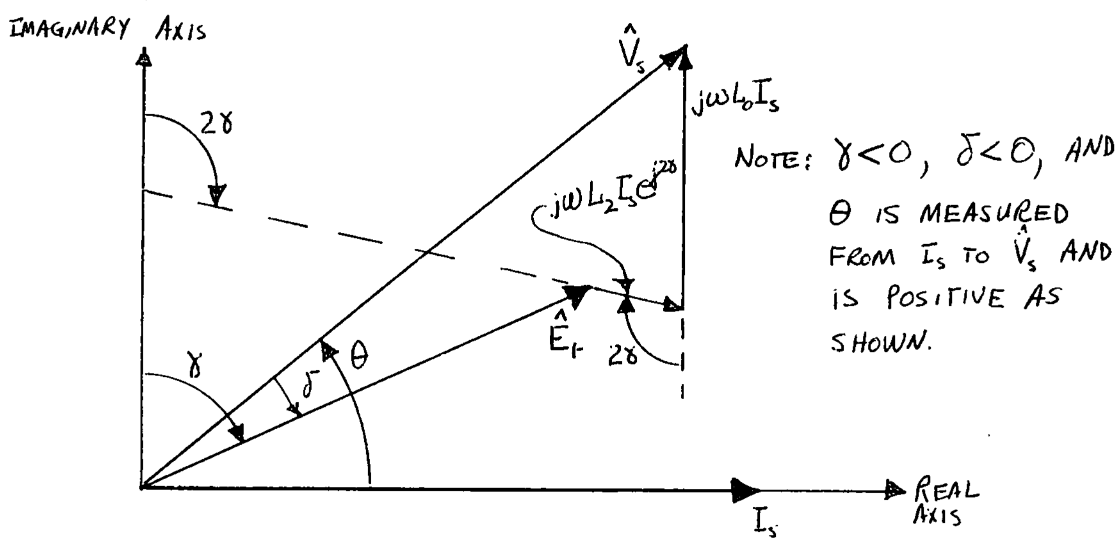
Quadrature axis reactance $\omega(L_o-L_2) = 2.2$ ohms

$\omega L_o = 3.1$ ohms $\qquad \omega L_2 = 0.9$ ohms

The number of poles is not given in the problem statement. We assume 2 poles.

Part a

Rated field current can be found in several ways, all involving cut-and-try procedures. Our method will be based on a vector diagram like that of Fig. 4.2.5(a), thus



PROBLEM 4.24 (Continued)

Evaluating the horizontal and vertical components of $\hat{V}_s$ we have (remember that $\gamma < 0$)

$$V_s \cos\theta = E_f \cos(\frac{\pi}{2} + \gamma) + \omega L_2 I_s \cos(\frac{\pi}{2} + 2\gamma)$$

$$V_s \sin\theta = E_f \sin(\frac{\pi}{2} + \gamma) + \omega L_2 I_s \sin(\frac{\pi}{2} + 2\gamma) + \omega L_o I_s$$

Using trigonometric identities we rewrite these as

$$V_s \cos\theta = -E_f \sin\gamma - \omega L_2 I_s \sin 2\gamma$$

$$V_s \sin\theta = E_f \cos\gamma + \omega L_2 I_s \cos 2\gamma + \omega L_o I_s$$

Next, it will be convenient to normalize these equations to V_s,

$$\cos\theta = -e_f \sin\gamma - \frac{\omega L_2 I_s}{V_s} \sin 2\gamma$$

$$\sin\theta = e_f \cos\gamma + \frac{\omega L_2 I_s}{V_s} \cos 2\gamma + \frac{\omega L_o I_s}{V_s}$$

where

$$e_f = \frac{E_f}{V_s}$$

Solution of these two equations for e_f yields

$$e_f = \frac{\sin\theta - \frac{\omega L_2 I_s}{V_s} \cos 2\gamma - \frac{\omega L_o I_s}{V_s}}{\cos\gamma}$$

$$e_f = \frac{-\cos\theta - \frac{\omega L_2 I_s}{V_s} \sin 2\gamma}{\sin\gamma}$$

For rated conditions as given the constants are:

$$\cos\theta = \text{p.f.} = 0.8$$

$$\sin\theta = -\ 0.6 \text{ (negative sign for leading p.f.)}$$

$$\frac{\omega L_2 I_s}{V_s} = 0.280; \quad \frac{\omega L_o I_s}{V_s} = 0.964$$

Solution by trial and error for a value of γ that satisfies both equations simultaneously yields

$$\gamma = -\ 148°$$

PROBLEM 4.24 (Continued)

and the resulting value for e_f is

$$e_f = 1.99$$

yielding for the rated field current

$$I_r = \frac{V_s e_f}{\omega M} = 24.1 \text{ amps.}$$

where V_s is in volts peak.

Part b

The V-curves can be calculated in several ways. Our choice here is to first relate power converted to terminal voltage and field generated voltage by multiplying (4.2.46) by ω, thus

$$P = \omega T^e = -\frac{E_f V_s}{X_d} \sin\delta - \frac{(X_d - X_q)V_s^2}{2X_d X_q} \sin 2\delta$$

where

$$X_d = \omega(L_o + L_2)$$

$$X_q = \omega(L_o - L_2)$$

We normalize this expression with respect to V_s^2/X_d, then

$$\frac{PX_d}{V_s^2} = -e_f \sin\delta - \frac{(X_d - X_q)}{2X_q} \sin 2\delta$$

Pull-out torque occurs when the derivative of this power with respect to δ goes to zero. Thus pull-out torque angle is defined by

$$\frac{\partial}{\partial\delta}\left(\frac{PX_d}{V_s^2}\right) = -e_f \cos\delta - \frac{(X_d - X_q)}{X_q} \cos 2\delta = 0$$

The use of (4.2.44) and (4.2.45) then yield the armature (stator) current amplitude as

$$I_s = \sqrt{\left(\frac{V_s}{X_q} \sin\delta\right)^2 + \left(\frac{V_s}{X_d} \cos\delta - \frac{E_f}{X_d}\right)^2}$$

A more useful form is

$$I_s = \frac{V_s}{X_d} \sqrt{\left(\frac{X_d}{X_q} \sin\delta\right)^2 + (\cos\delta - e_f)^2}$$

The computation procedure used here was to fix the power and assume values of δ over a range going from either rated armature current or rated field current to pull-out. For each value of δ, the necessary value of e_f is calculated

PROBLEM 4.24 (Continued)

from the expression for power as

$$e_f = \frac{\frac{PX_d}{V_s^2} + \frac{(X_d - X_q)}{2X_q}\sin 2\delta}{-\sin\delta}$$

and then the armature current magnitude is calculated from

$$I_s = \frac{V_s}{X_d}\sqrt{\left(\frac{X_d}{X_q}\sin\delta\right)^2 + (\cos\delta - e_f)^2}$$

For zero load power, $\gamma = 0$ and $\delta = 0$ and, from the vector diagram given earlier, the armature current amplitude is

$$I_s = \frac{|V_s - E_f|}{\omega(L_o + L_2)}$$

with pull-out still defined as before. The required V-curves are shown in the followinggraph. Note that pull-out conditions are never reached because range of operation is limited by rated field current and rated armature current.

PROBLEM 4.25

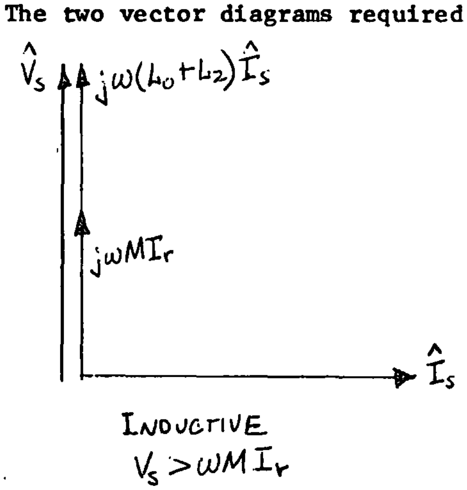
Equation (4.2.41) is (assuming arbitrary phase for I_s)

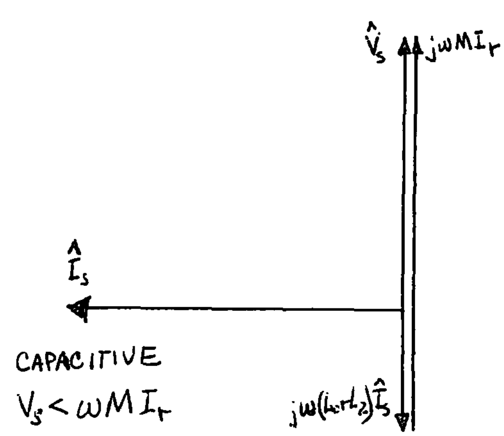
$$\hat{V}_s = j\omega L_o\hat{I}_s + j\omega L_2\hat{I}_s e^{j2\gamma} + j\omega MI_r e^{j2\gamma}$$

With $\gamma = 0$ as specified

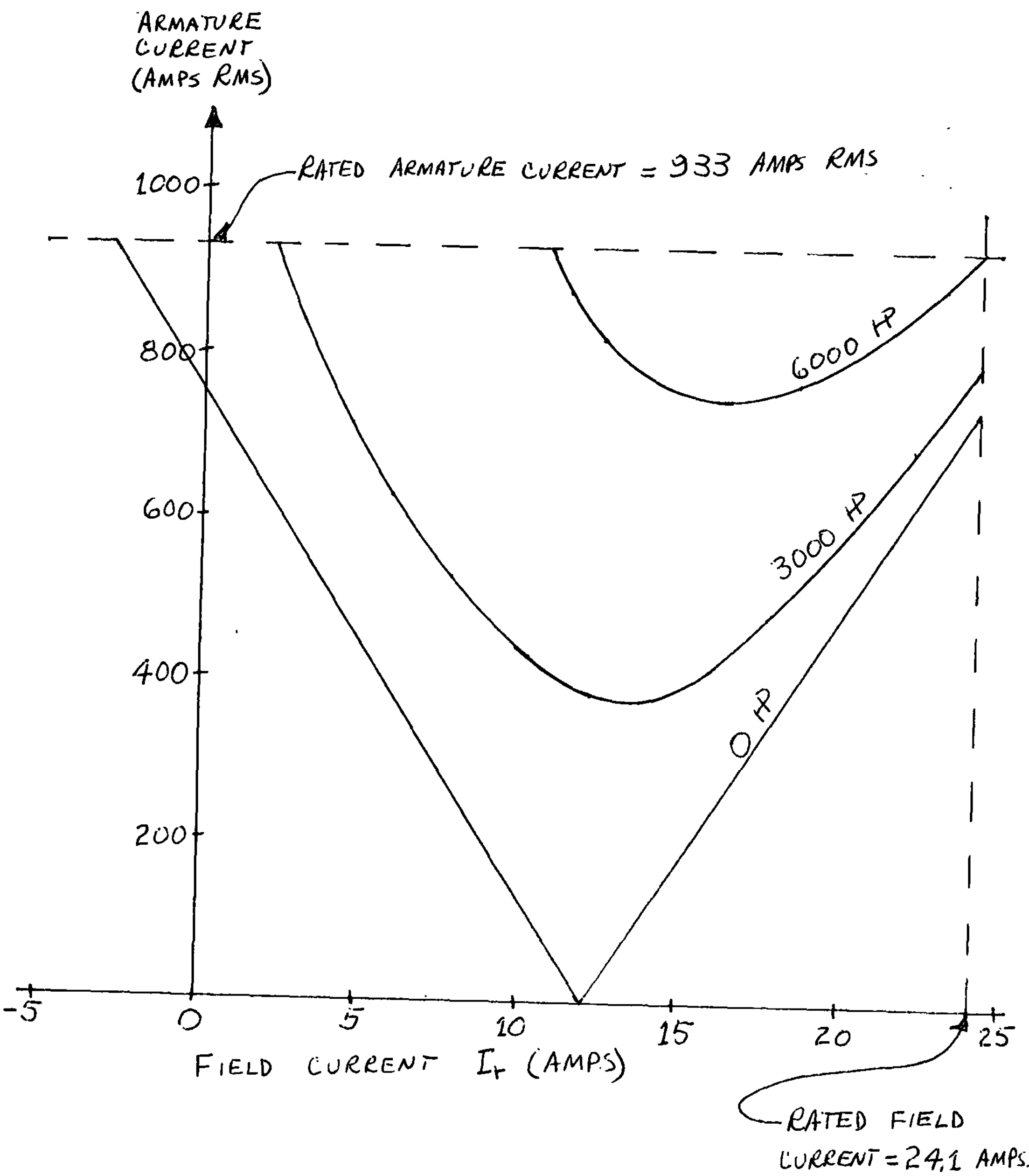
$$\hat{V}_s = j\omega(L_o + L_2)\hat{I}_s + j\omega MI_r$$

The two vector diagrams required are





V-CURVES FOR PROBLEM 4.24



PROBLEM 4.26

Part a

From Fig. 4P.26(a)

$$\frac{\hat{V}}{\hat{V}_s} = \frac{\frac{1}{Y} e^{j\phi}}{jX_s + \frac{1}{Y} e^{j\phi}}$$

from which the ratio of the magnitudes is

$$\frac{|\hat{V}|}{|\hat{V}_s|} = \frac{\frac{1}{Y}}{\sqrt{(\frac{1}{Y}\cos\phi)^2 + (\frac{1}{Y}\sin\phi + X_s)^2}}$$

For the values Y = 0.01 mho, X_s = 10 ohms

$$\frac{|\hat{V}|}{|\hat{V}_s|} = \frac{100}{\sqrt{(100\cos\phi)^2 + (100\sin\phi + 10)^2}}$$

Then, for $\phi = 0$

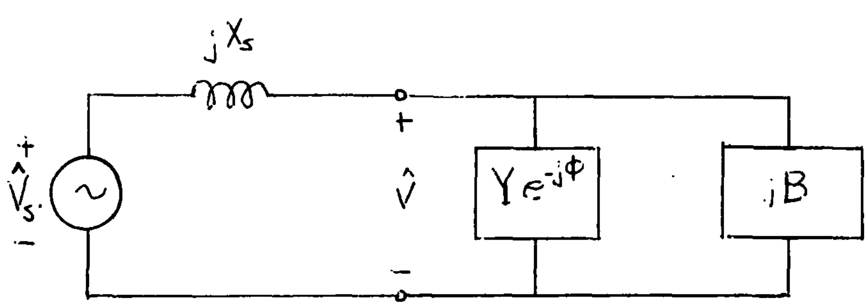
$$\frac{|\hat{V}|}{|\hat{V}_s|} = \frac{100}{\sqrt{10{,}000 + 100}} = 0.995$$

and, for $\phi = 45°$

$$\frac{|\hat{V}|}{|\hat{V}_s|} = \frac{100}{\sqrt{(\frac{100}{\sqrt{2}})^2 + (\frac{100}{\sqrt{2}} + 10)^2}} = 0.932$$

Part b

It is instructive to represent the synchronous condenser as a susceptance jB, then when B is positive the synchronous condenser appears capacitive. Now the circuit is



PROBLEM 4.26 (Continued)

Now the voltage ratio is

$$\frac{\hat{V}}{\hat{V}_s} = \frac{\frac{1}{Ye^{-j\phi} + jB}}{\frac{1}{Ye^{-j\phi} + jB} + jX_s} = \frac{1}{1 + jX_s Ye^{-j\phi} - BX_s}$$

$$\frac{\hat{V}}{\hat{V}_s} = \frac{1}{1 - BX_s + X_s Y \sin\phi + jX_s Y \cos\phi}$$

Then

$$\frac{|\hat{V}|}{|\hat{V}_s|} = \frac{1}{\sqrt{(1 - BX_s + X_s Y \sin\phi)^2 + (X_s Y \cos\phi)^2}}$$

For $\phi = 0$

$$\frac{|\hat{V}|}{|\hat{V}_s|} = \frac{1}{\sqrt{(1 - BX_s)^2 + (X_s Y)^2}}$$

If this is to be unity

$$(1 - BX_s)^2 + (X_s Y)^2 = 1$$

$$1 - BX_s = \sqrt{1 - (X_s Y)^2}$$

$$B = \frac{1 - \sqrt{1 - (X_s Y)^2}}{X_s}$$

for the constants given

$$B = \frac{1 - \sqrt{1 - 0.01}}{10} = \frac{0.005}{10} = \underline{0.0005 \text{ mho}}$$

Volt-amperes required from synchronous condenser

$$(VA)_{sc} = |\hat{V}|^2 B = (2)(10^{10})(5)(10^{-4}) = 10{,}000 \text{ KVA}$$

Real power supplied to load

$$P_L = |\hat{V}|^2 Y \cos\phi = |\hat{V}|^2 Y \quad \text{for } \phi = 0$$

Then

$$\frac{(VA)_{sc}}{P_L} = \frac{B}{Y} = \frac{0.0005}{0.01} = 0.05$$

For $\phi = 0$ the synchronous condenser needs to supply reactive volt amperes equal to 5 percent of the load power to regulate the voltage perfectly.

PROBLEM 4.26 (Continued)

For $\phi = 45°$

$$\frac{|\hat{V}|}{|\hat{V}_s|} = \frac{1}{\sqrt{\left(1-BX_s + \frac{X_sY}{\sqrt{2}}\right)^2 + \left(\frac{X_sY}{\sqrt{2}}\right)^2}}$$

In order for this to be unity

$$\left(1-BX_s + \frac{X_sY}{\sqrt{2}}\right)^2 + \left(\frac{X_sY}{\sqrt{2}}\right)^2 = 1$$

$$B = \frac{1+\frac{X_sY}{\sqrt{2}} - \sqrt{1-\left(\frac{X_sY}{\sqrt{2}}\right)^2}}{X_s}$$

For the constants given

$$B = \frac{1 + 0.0707 - \sqrt{1-0.005}}{10} = 0.00732 \text{ mho}$$

Volt-amperes required from synchronous condenser

$$(VA)_{sc} = |\hat{V}|^2B = (2)(10^{10})(7.32)(10^{-3}) = 146{,}400 \text{ KVA}$$

Real power supplied to load

$$P_L = |\hat{V}|^2Y \cos\phi = \frac{|\hat{V}|^2Y}{\sqrt{2}} \text{ for } \phi = 45°$$

Then

$$\frac{(VA)_{sc}}{P_L} = \frac{B\sqrt{2}}{Y} = \frac{(\sqrt{2})(0.00732)}{0.01} = 1.04$$

Thus for a load having power factor of 0.707 lagging a synchronous condenser needs to supply reactive volt-amperes equal to 1.04 times the power supplied to the load to regulate the voltage perfectly.

These results, of course, depend on the internal impedance of the source. That given is typical of large power systems.

PROBLEM 4.27

Part a

This part of this problem is very much like part a of Prob. 4.24. Using results from that problem we define

PROBLEM 4.27 (Continued)

$$e_f = \frac{E_f}{V_s} = \frac{\omega M I_r}{V_s}$$

where V_s is in volts peak. Then

$$e_f = \frac{\sin\theta - \frac{\omega L_2 I_s}{V_s}\cos 2\gamma - \frac{\omega L_o I_s}{V_s}}{\cos\gamma}$$

$$e_f = \frac{-\cos\theta - \frac{\omega L_2 I_s}{V_s}\sin 2\gamma}{\sin\gamma}$$

From the constants given

$$\cos\theta = 1.0; \quad \sin\theta = 0$$

$$\omega L_o = 2.5 \text{ ohms} \quad \omega L_2 = 0.5 \text{ ohm}$$

Rated power

$$P_L = 1000 \text{ HP} = 746 \text{ KW}$$

Armature current at rated load is

$$I_s = \frac{746{,}000}{\sqrt{2}\ 1000} = 527 \text{ amps peak} = 373 \text{ amps RMS}$$

Then

$$\frac{\omega L_2 I_s}{V_s} = 0.186; \quad \frac{\omega L_o I_s}{V_s} = 0.932$$

Using the constants

$$e_f = \frac{-0.186\cos 2\gamma - 0.932}{\cos\gamma}$$

$$e_f = \frac{-1 - 0.186\sin 2\gamma}{\sin\gamma}$$

The use of trial-and-error to find a value of γ that satisfies these two equations simultaneously yields

$$\gamma = -127° \text{ and } e_f = 1.48$$

Using the given constants we obtain

$$I_r = \frac{e_f V_s}{\omega M} = \frac{(1.48)(\sqrt{2})(1000)}{150} = 14.0 \text{ amps}$$

For L_f/R_f very large compared to a half period of the supply voltage the field

PROBLEM 4.27 (Continued)

current will essentially be equal to the peak of the supply voltage divided by the field current; thus, the required value of R_f is

$$R_f = \frac{V_s}{I_r} = \frac{\sqrt{2}\ (1000)}{14.0} \approx 100 \text{ ohms}$$

Part b

We can use (4.2.46) multiplied by the rotational speed ω to write the output power as

$$P_L = \omega T^e = -\frac{E_f V_s}{X_d} \sin\delta - \frac{(X_d - X_q)V_s^2}{2X_d X_q} \sin 2\delta$$

where

$$X_d = \omega(L_o + L_2) = \text{direct axis reactance}$$

$$X_q = \omega(L_o - L_2) = \text{quadrature axis reactance}$$

With the full-wave rectifier supplying the field winding we can express

$$E_f = \omega M I_r = \frac{\omega M}{R_f} V_s$$

Then

$$P_L = -\frac{\omega M\, V_s^2}{R_f X_d} \sin\delta - \frac{(X_d - X_q)V_s^2}{2X_d X_q} \sin 2\delta$$

Factoring out V_s^2 yields

$$P_L = V_s^2 \left[-\frac{\omega M}{R_f X_d} \sin\delta - \frac{(X_d - X_q)}{2X_d X_q} \sin 2\delta\right]$$

Substitution of given constants yields

$$746 \times 10^3 = V_s^2\ [-0.500 \sin\delta - 0.083 \sin 2\delta]$$

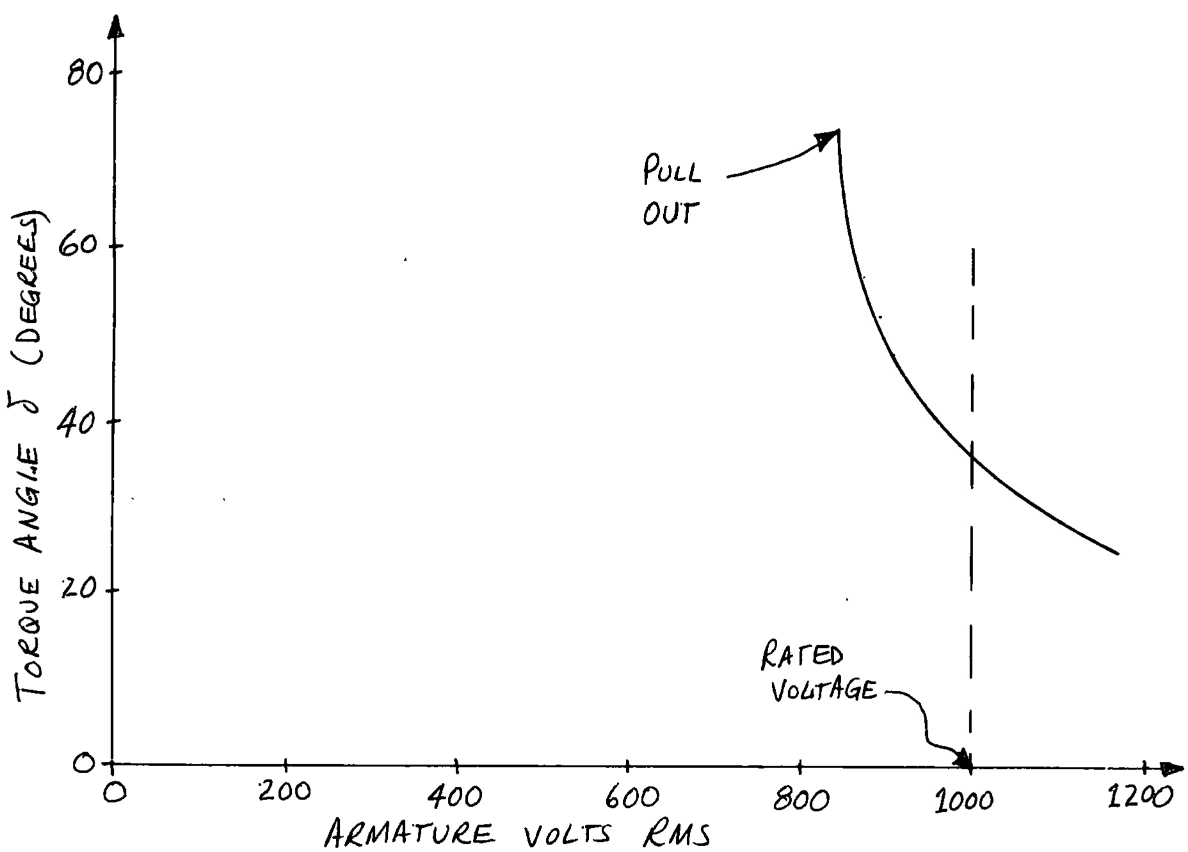
To find the required curve it is easiest to assume δ and calculate the required V_s, the range of δ being limited by pull-out which occurs when

$$\frac{\partial P_L}{\partial \delta} = 0 = -0.500 \cos\delta - 0.166 \cos 2\delta$$

The resulting curve of δ as a function of V_s is shown in the attached graph.

Note that the voltage can only drop 15.5% before the motor pulls out of step.

PROBLEM 4.27 (Continued)



Although it was not required for this problem, calculations will show that operation at reduced voltage will lead to excessive armature current, thus, operation in this range must be limited to transient conditions.

PROBLEM 4.28

Part a

This is similar to part a of Prob. 4.24 except that now we are considering a number of pole pairs greater than two and we are treating a generator. Considering first the problem of pole pairs, reference to Sec. 4.1.8 and 4.2.4 shows that when we define electrical angles γ_e and δ_e as

$$\gamma_e = p\gamma \text{ and } \delta_e = p\delta$$

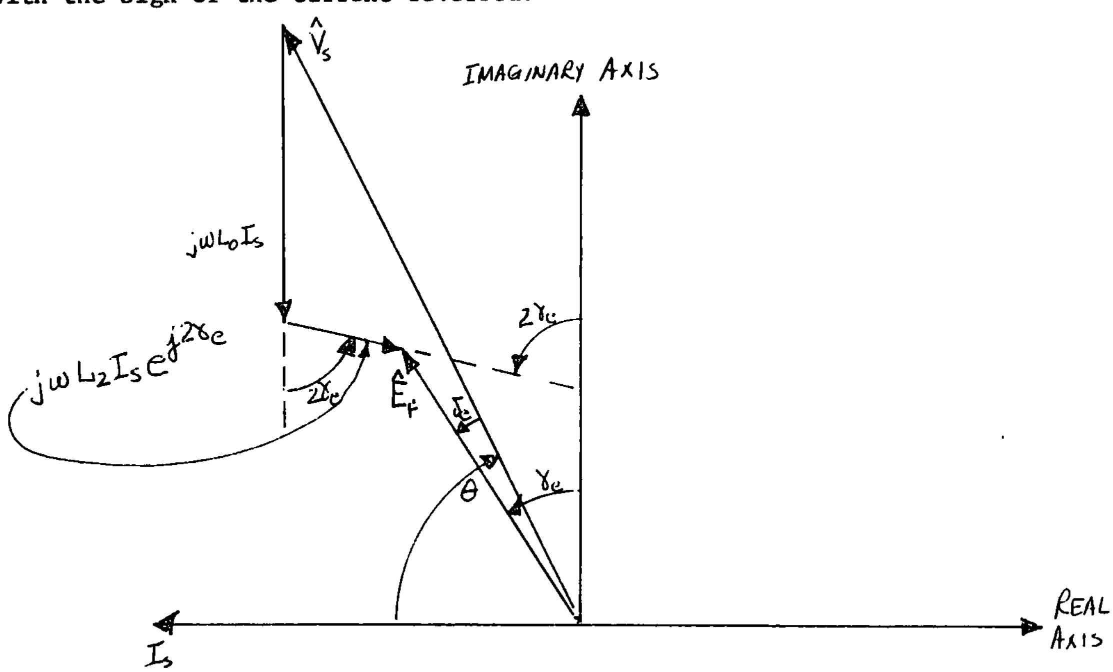
where p is number of pole pairs (36 in this problem) and when we realize that the electromagnetic torque was obtained as a derivative of inductances with respect to angle we get the results

$$T^e = -\frac{p}{\omega}\frac{V_sE_f}{X_d}\sin\delta_e - \frac{p(X_d-X_q)\;V_s^2}{\omega 2X_dX_q}\sin 2\delta_e$$

where $X_d = \omega(L_o+L_2)$ and $X_q = \omega(L_o-L_2)$, and, because the synchronous speed is ω/p (see 4.1.95) the electrical power output from the generator is

$$P = -\frac{\omega}{p}T^e = \frac{V_sE_f}{X_d}\sin\delta_e + \frac{(X_d-X_q)V_s^2}{2X_dX_q}\sin 2\delta_e$$

Next, we are dealing with a generator so it is convenient to replace I_s by $-I_s$ in the equations. To make clear what is involved we redraw Fig. 4.2.5(a) with the sign of the current reversed.



PROBLEM 4.28 (Continued)

Now, evaluating horizontal and vertical components of V_s we have

$$V_s \cos\theta - \omega L_2 I_s \sin 2\gamma_e = E_f \sin\gamma_e$$

$$-V_s \sin\theta = \omega L_o I_s + \omega L_2 I_s \cos 2\gamma_e + E_f \cos\gamma_e$$

From these equations we obtain

$$e_f = \frac{\cos\theta - \frac{\omega L_2 I_s}{V_s}\sin 2\gamma_e}{\sin\gamma_e}$$

$$e_f = \frac{-\sin\theta - \frac{\omega L_o I_s}{V_s} - \frac{\omega L_2 I_s}{V_s}\cos 2\gamma_e}{\cos\gamma_e}$$

where

$$e_f = \frac{E_f}{V_s} = \frac{\omega M I_r}{V_s}$$

with V_s in volts peak

I_s in amps peak

ω is the electrical frequency

For the given constants

$$\cos\theta = \text{p.f.} = 0.850 \qquad \sin\theta = 0.528$$

$$\frac{\omega L_2 I_s}{V_s} = 0.200 \qquad \frac{\omega L_o I_s}{V_s} = 1.00$$

and

$$e_f = \frac{0.850 - 0.200\sin 2\gamma_e}{\sin\gamma_e}$$

$$e_f = \frac{-1.528 - 0.200\cos 2\gamma_e}{\cos\gamma_e}$$

Trial-and-error solution of these two equations to find a positive value of γ_e that satisfies both equations simultaneously yields

$$\gamma_e = 147.5^\circ \quad \text{and} \quad e_f = 1.92$$

From the definition of e_f we have

$$I_r = \frac{e_f V_s}{\omega M} = \frac{(1.92)(\sqrt{2})(10{,}000)}{(120)(\pi)(0.125)} = 576 \text{ amps}$$

PROBLEM 4.28 (Continued)

Part b

From Prob. 4.14 the definition of complex power is

$$\hat{V}_s\hat{I}_s^* = P + jQ$$

where $\hat{V}_s$ and $\hat{I}_s$ are complex amplitudes.

The capability curve is not as easy to calculate for a salient-pole machine as it was for a smooth-air-gap machine in Prob. 4.14. It will be easiest to calculate the curve using the power output expression of part a

$$P = \frac{V_s E_f}{X_d} \sin \delta_e + \frac{(X_d - X_q)V_s^2}{2X_d X_q} \sin 2\delta_e$$

the facts that

$$P = V_s I_s \cos \theta$$

$$Q = V_s I_s \sin \theta$$

and that I_s is given from (4.2.44) and (4.2.45) as

$$I_s = \sqrt{\left(\frac{V_s}{X_q} \sin \delta_e\right)^2 + \left(\frac{V_s}{X_d} \cos \delta_e - \frac{E_f}{X_d}\right)^2}$$

First, assuming operation at rated field current the power is

$$P = 320 \times 10^6 \sin \delta_e + 41.7 \times 10^6 \sin 2\delta_e \text{ watts.}$$

We assume values of δ_e starting from zero and calculate P; then we calculate I_s for the same values of δ_e from

$$I_s = 11{,}800 \sqrt{(1.50 \sin \delta_e)^2 + (\cos \delta_e - 1.92)^2} \text{ amps peak}$$

Next, because we know P, V_s, and I_s we find θ from

$$\cos \theta = \frac{P}{V_s I_s}$$

From θ we then find Q from

$$Q = V_s I_s \sin \theta.$$

This process is continued until rated armature current

$$I_s = \sqrt{2}\ 10{,}000 \text{ amps peak}$$

is reached.

The next part of the capability curve is limited by rated armature current which defines the trajectory

PROBLEM 4.28 (Continued)

$$\sqrt{P^2 + Q^2} = V_s I_s$$

where V_s and I_s are rated values.

For $Q < 0$, the capability curve is limited by pull-out conditions defined by the condition

$$\frac{dP}{d\delta_e} = 0 = \frac{V_s E_f}{X_d} \cos \delta_e + \frac{(X_d - X_q) V_s^2}{X_d X_q} \cos 2\delta_e$$

To evaluate this part of the curve we evaluate e_f in terms of δ_e from the power and current expressions

$$e_f = \frac{\dfrac{PX_d}{V_s^2} - \dfrac{(X_d - X_q)}{2X_q} \sin 2\delta_e}{\sin \delta_e}$$

$$e_f = \cos \delta_e - \sqrt{\left(\frac{I_s X_d}{V_s}\right)^2 - \left(\frac{X_d}{X_q} \sin \delta_e\right)^2}$$

For each level of power at a given power factor we find the value of δ_e that simultaneously satisfies both equations. The resulting values of e_f and δ_e are used in the stability criterion

$$\frac{dP}{d\delta_e} = \frac{V_s^2 e_f}{X_d} \cos \delta_e + \frac{(X_d - X_q) V_s^2}{X_d\, X_q} \cos 2\delta_e \geq 0$$

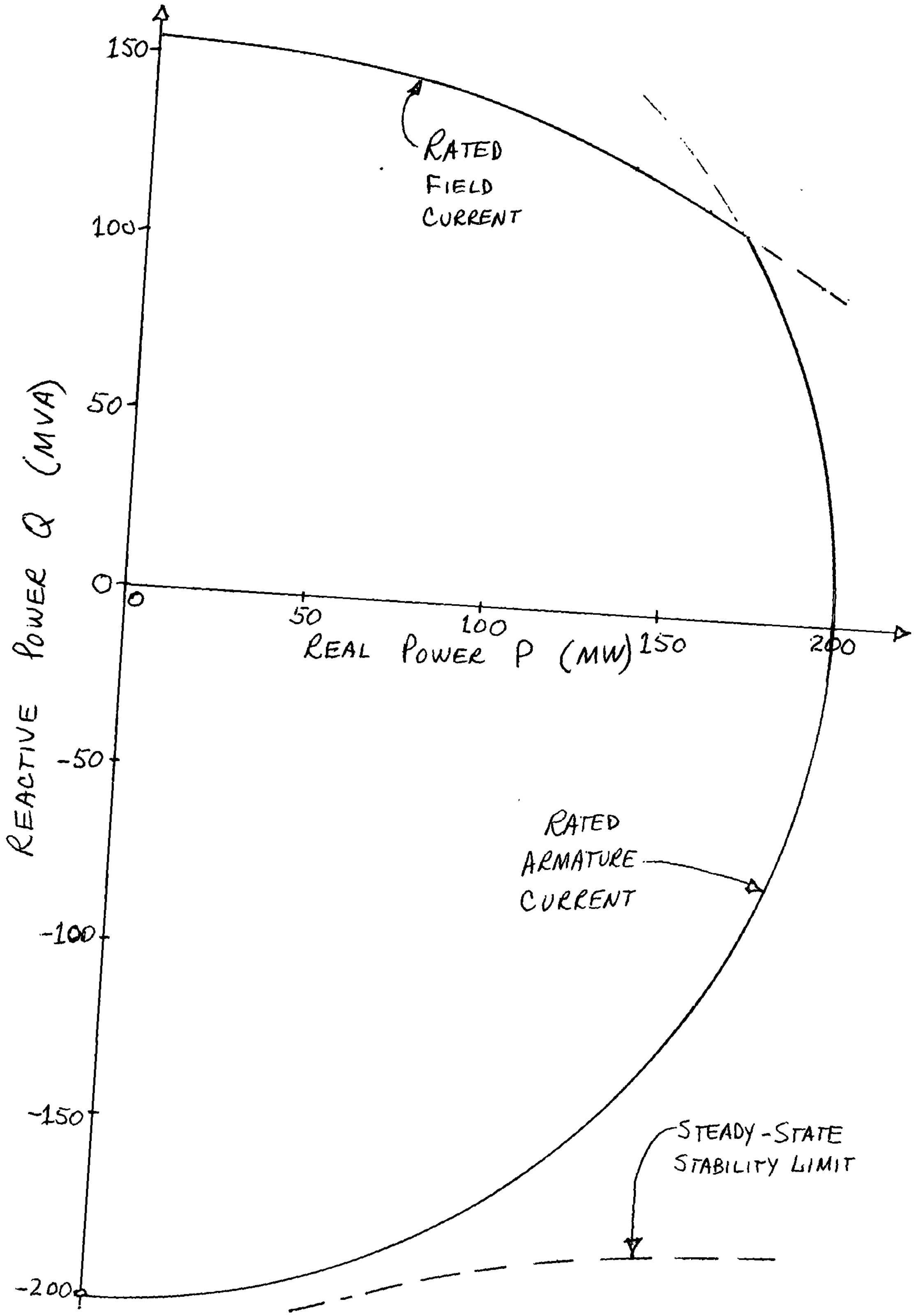
When this condition is no longer met (equal sign holds) the stability limit is reached. For the given constants

$$e_f = \frac{\dfrac{P}{167 \times 10^6} - 0.25 \sin 2\delta_e}{\sin \delta_e}$$

$$e_f = \cos \delta_e - \sqrt{\left(\frac{I_s}{11{,}800}\right)^2 - (1.5 \sin \delta_e)^2}$$

$$\frac{dP}{d\delta_e} = e_f \cos \delta_e + 0.5 \cos 2\delta_e \geq 0$$

The results of this calculation along with the preceding two are shown on the attached graph. Note that the steady-state stability never limits the capability. In practice, however, more margin of stability is required and the capability in the fourth quadrant is limited accordingly.


150
100
50
0
-50
-100
-150
-200
REACTIVE POWER Q (MVA)
0
50
100
150
200
REAL POWER P (MW)
RATED FIELD CURRENT
RATED ARMATURE CURRENT
STEADY-STATE STABILITY LIMIT

PROBLEM 4.29

Part a

For this electrically linear system the electric coenergy is

$$W_e'(v_1,v_2,\theta) = \frac{1}{2} C_o(1 + \cos 2\theta)v_1^2 + \frac{1}{2} C_o(1 + \sin 2\theta)v_2^2$$

The torque of electric origin is

$$T^e = \frac{\partial W_e'(v_1,v_2,\theta)}{\partial \theta} = C_o(v_2^2 \cos 2\theta - v_1^2 \sin 2\theta)$$

Part b

With $v_1 = V_o \cos \omega t$; $v_2 = V_o \sin \omega t$

$$T^e = C_o V_o^2(\sin^2 \omega t \cos 2\theta - \cos^2 \omega t \sin 2\theta)$$

Using trig identities

$$T^e = \frac{C_o V_o^2}{2}[\cos 2\theta - \cos 2\omega t \cos 2\theta - \sin 2\theta - \cos 2\omega t \cos 2\theta]$$

$$T^e = \frac{C_o V_o^2}{2} (\cos 2\theta - \sin 2\theta) - \frac{C_o V_o^2}{2} [\cos(2\omega t - 2\theta) + \cos(2\omega t + 2\theta)]$$

Three possibilities for time-average torque:

Case I:

Shaft sitting still at fixed angle θ

Case II:

Shaft turning in positive θ direction

$$\theta = \omega t + \gamma$$

where γ is a constant

Case III:

Shaft turning in negative θ direction

$$\theta = -\omega t + \delta$$

where δ is a constant.

Part c

The time average torques are:

Case I: θ = const.

$$\langle T^e \rangle = \frac{C_o V_o^2}{2}(\cos 2\theta - \sin 2\theta)$$

PROBLEM 4.29 (Continued)

Case II: $\theta = \omega t + \gamma$

$$\langle T^e \rangle = -\frac{C_o V_o^2}{2} \cos 2\gamma$$

Case III: $\theta = -\omega t + \delta$

$$\langle T^e \rangle = -\frac{C_o V_o^2}{2} \cos 2\delta$$

PROBLEM 4.30

For an applied voltage v(t) the electric coenergy for this electrically linear system is

$$W_e'(v,\theta) = \frac{1}{2}(C_o + C_1 \cos 2\theta)v^2$$

The torque of electric origin is then

$$T^e = \frac{\partial W_e'(v,\theta)}{\partial\theta} = -C_1 \sin 2\theta\, v^2$$

For $v = V_o \sin \omega t$

$$T^e = -C_1 V_o^2 \sin^2\omega t \sin 2\theta$$

$$T^e = -\frac{C_1 V_o^2}{2} (\sin 2\theta - \cos 2\omega t \cos 2\theta)$$

$$T^e = -\frac{C_1 V_o^2}{2} \sin 2\theta + \frac{C_1 V_o^2}{4} [\cos(2\omega t - 2\theta) + \cos(2\omega t + 2\theta)]$$

For rotational velocity ω_m we write

$$\theta = \omega_m t + \gamma$$

and then

$$T^e = -\frac{C_1 V_o^2}{2} \sin 2(\omega_m t + \gamma) + \frac{C_1 V_o^2}{4} \{\cos[2(\omega - \omega_m)t - 2\gamma] + \cos[2(\omega + \omega_m)t + 2\gamma]\}$$

This device can behave as a motor if it can produce a time-average torque for ω_m = constant. This can occur when

$$\omega_m = \pm\, \omega$$