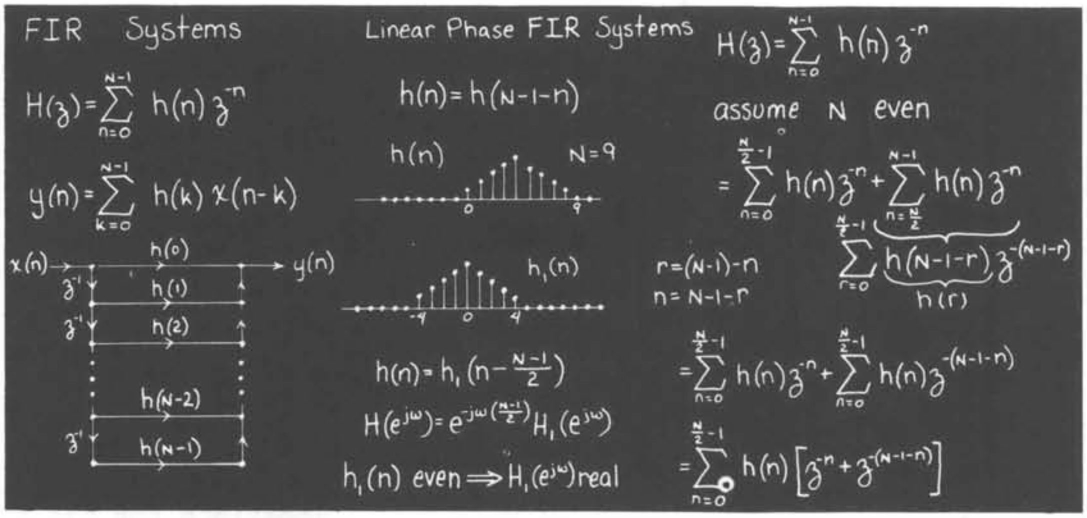
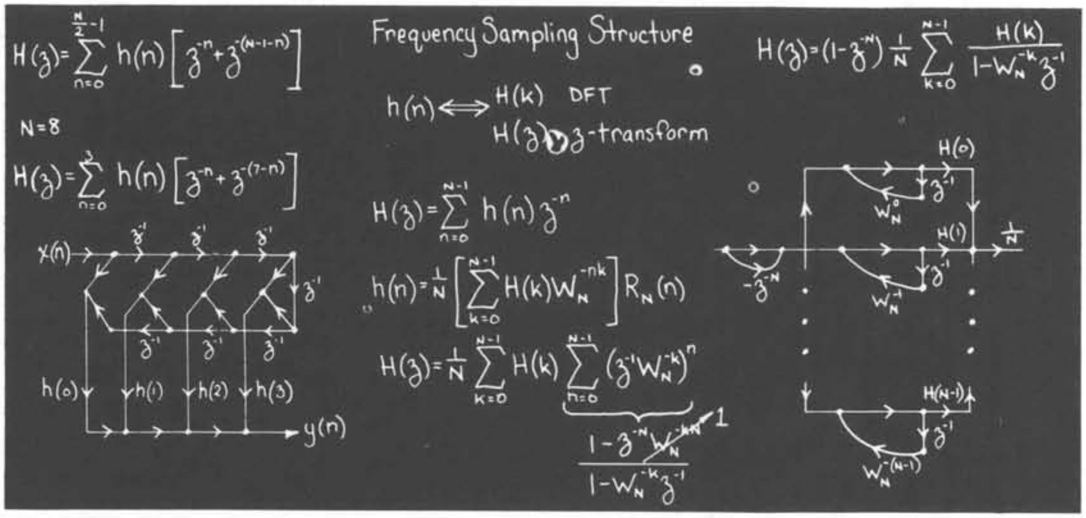
NETWORK STRUCTURES FOR FIR SYSTEMS AND PARAMETER QUANTIZATION EFFECTS IN DIGITAL FILTER STRUCTURES

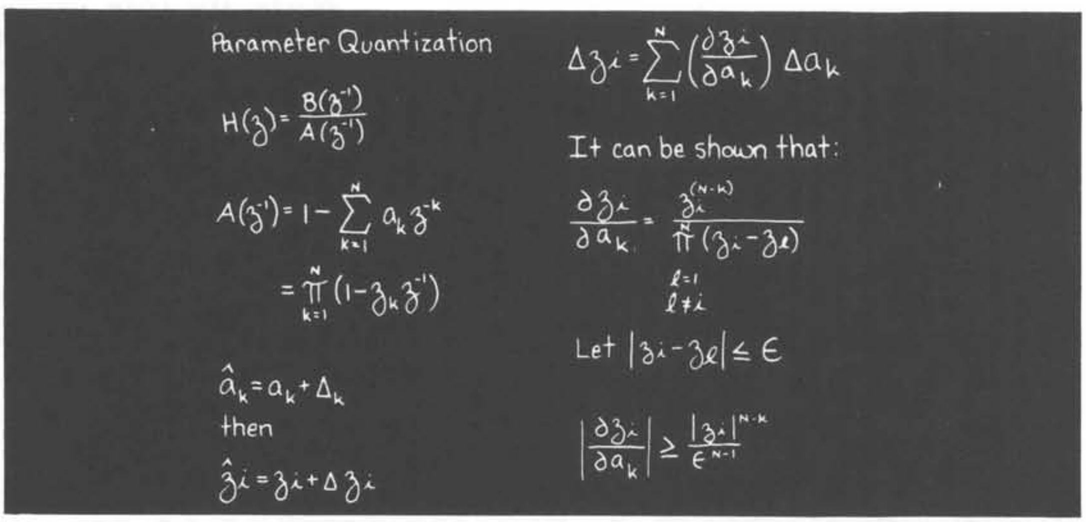
1. Lesson 13 - 51 minutes



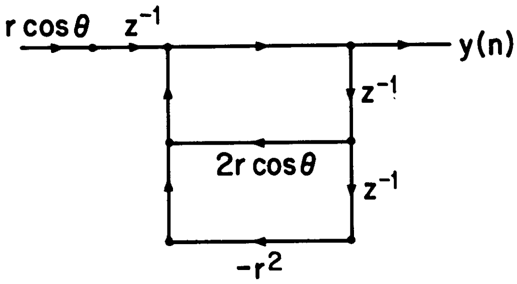
a.



b.



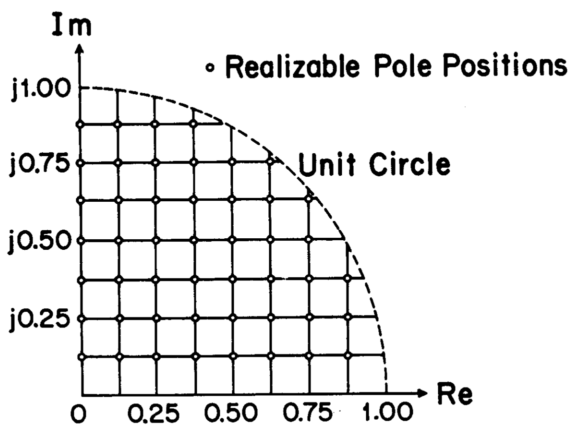
c.



Direct-form implementation of a complex conjugate pole pair.

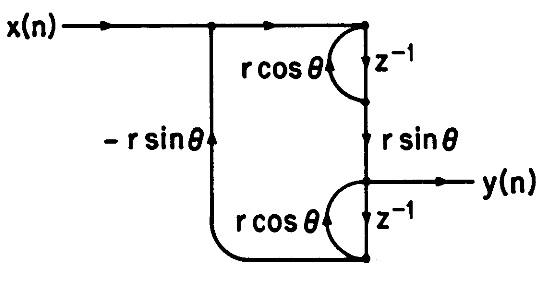
$$H(z) = \frac{r \cos \theta \, z^{-1}}{(1 - re^{j\theta} z^{-1})(1 - re^{-j\theta} z^{-1})}$$

d.



Grid of possible pole locations for the network of viewgraph d when the coefficients are quantized to three bits.

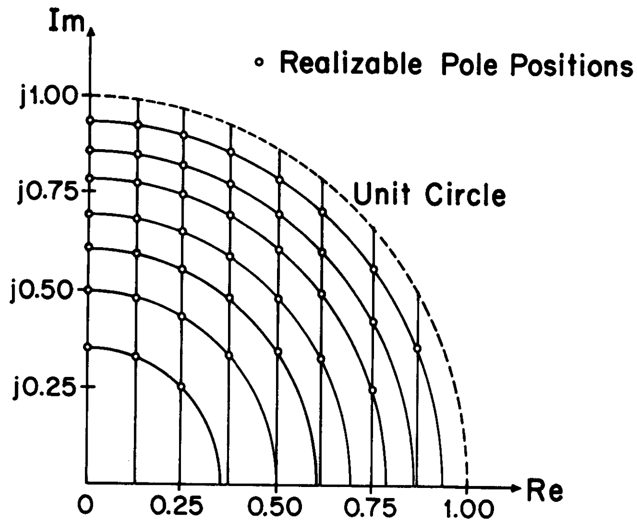
e.



Coupled form implementation of a complex conjugate pole pair. (Note that the transfer function has been corrected. The numerator factor is r sin θ not r cos θ as indicated in the lecture.)

$$H(z) = \frac{r \sin \theta \, z^{-1}}{(1 - re^{j\theta} z^{-1})(1 - re^{-j\theta} z^{-1})}$$

f.



Grid of possible pole locations for the network of viewgraph f when the coefficients are quantized to three bits.

g.

2. Comments

In the previous lecture we discussed several basic network structures for IIR filters. These structures apply also, of course, to the implementation of FIR filters. There are, however, several additional structures which apply specifically to FIR systems, two of which are discussed in this lecture.

The first of these structures applies specifically to linear phase FIR systems. FIR systems can be designed to have exactly linear phase by constraining the unit sample response to be symmetrical. With this symmetry there are only $\frac{N}{2}$ (for N even) or $\frac{N+1}{2}$ (for N odd) independent coefficients and such filters can be implemented with a structure requiring only $\frac{N}{2}$ or $\frac{N+1}{2}$ multiplications. The second FIR structure discussed in this lecture is referred to as the frequency-sampling structure because the coefficients of this structure are samples of the frequency response of the system.

The importance of different structures for digital filters is tied very closely to the considerations involved in a hardware implementation. One such consideration is the effect of finite register length. As an introduction to this issue we discuss briefly in this lecture the relation between structures with regard to parameter quantization effects. The primary result stressed is that the sensitivity of pole and zero locations to parameter quantization tends to be higher for a direct form structure than for a cascade form structure. Even within the cascade structure there is flexibility with regard to how the pole-zero pairs are implemented, resulting in different parameter quantization effects. In particular, coefficient quantization constrains the poles (and zeros) to lie on a grid in the z-plane. The location and density of the grid points is determined by the amount of parameter quantization and the form of the network structure.

3. Reading

Text: Sections 6.5 (page 313) and 6.8.

4. Problems

Problem 13.1

H(z) represents the system function for an FIR linear system, and is given by

$$H(z) = (1+ \tfrac{1}{2}z^{-1})\ (1 + 2z^{-1})\ (1 - \tfrac{1}{4}z^{-1})\ (1 - 4z^{-1})$$

Draw a flow-graph implementation of the system in each of the following forms:

(i) cascade form
(ii) direct form
(iii) linear-phase form
(iv) frequency-sampling form

Problem 13.2

We wish to implement a digital oscillator, i.e., a digital filter for which the unit-sample response is of the form

$$h(n) = A \cos(\omega_0 n + \phi)\ u(n) \qquad (13.2\text{-}1)$$

Shown in Figure P13.2-1 are a direct form and a coupled form second order filter.

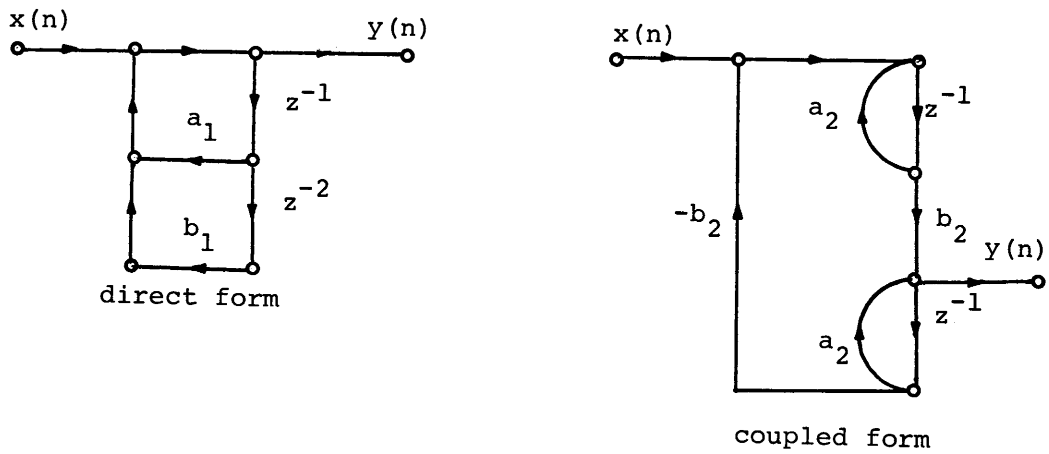


Figure P13.2-1

(a) Assuming that there is no parameter quantization, determine the coefficients a_1 and b_1 so that the unit-sample response of the direct form filter will be of the form of Eq. (13.2-1) with $\omega_0 = \frac{\pi}{4}$.

(b) Again, assuming that there is no parameter quantization determine the coefficients a_2 and b_2 so that the unit-sample response of the coupled form filter will be of the form of Eq. (13.2-1) with $\omega_0 = \frac{\pi}{4}$. Although the amplitude A and phase shift ϕ will be different for the direct form and coupled form, this difference is not important for this problem.

(c) Now, let us quantize the coefficients for the two filters. In particular, the fractional part of the absolute value of each coefficient will be represented by, at most three bits. The value of the quantized coefficient can be obtained as follows:

(i) multiply the coefficient by 8
(ii) discard the fractional part of the product
(iii) divide the result in (ii) by 8.

Determine whether, after quantization the unit-sample response of each filter will still be of the form of Eq. (13.2-1). If yes, determine the new value of ω_0.

Problem 13.3

A causal linear-phase FIR system has the property that $h(n) = h(N-1-n)$ for $n = 0, 1..., N-1$. This symmetry constraint was used in Sec. 6.5.3 of the text to show that systems satisfying this constraint have linear phase corresponding to a delay of $(N-1)/2$ samples. This constraint results in a significant simplication of the frequency-sampling realization of Eqs. (4.48) and (4.50) of the text.

(a) Using the above linear-phase constraint, show that, for N even, $\tilde{H}(N/2) = 0$.

(b) Consider H(k) expressed in the form $H(k) = \tilde{H}_a(k)e^{j\theta(k)}$ where $\tilde{H}_a(k)$ is real. Determine an expression for $\theta(k)$ for $k = 0, 1, ..., N-1$ that is valid for N even. You may find it helpful to refer to the results in Sec. 6.5.3 of the text.

(c) Assume that $\tilde{H}_a(k)$ defined in (b) is position i.e. that it corresponds to the magnitude of $\tilde{H}(k)$. Using the results of part (a) and (b), show that for h(n) linear phase and with N even, Eqs. (4.49) and (4.50) of text can be simplified to

$$H(z) = \frac{1 - z^{-N}}{N}\left[\sum_{k=1}^{(N/2)-1} \frac{(-1)^k |\tilde{H}(k)| 2\cos(\pi k/N)(1-z^{-1})}{1 - 2z^{-1}\cos(2\pi k/N) + z^{-2}} + \frac{\tilde{H}(0)}{1 - z^{-1}}\right].$$

(We have assumed for convenience that $r = 1$.)

(d) Draw a flow-graph representation of the system function derived in part (c).