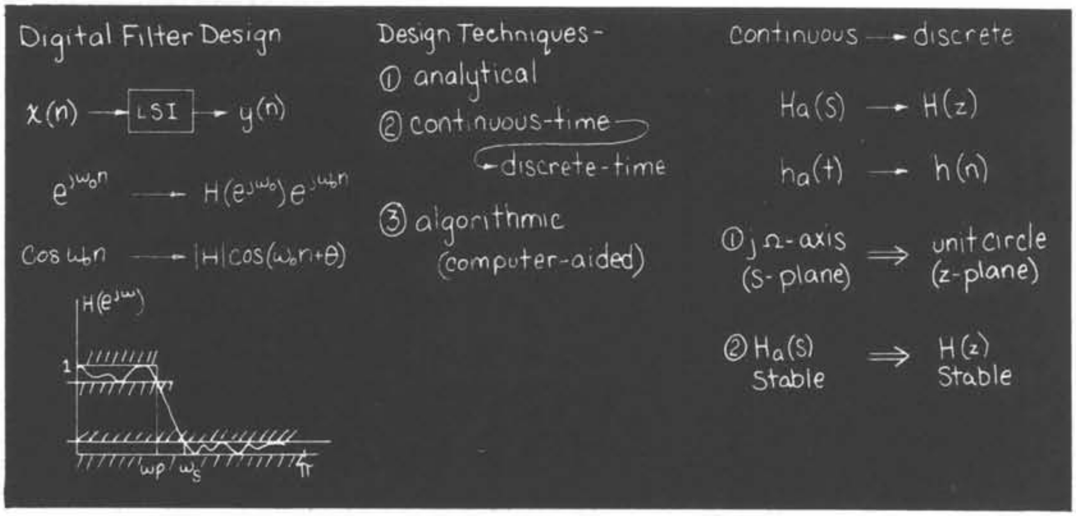
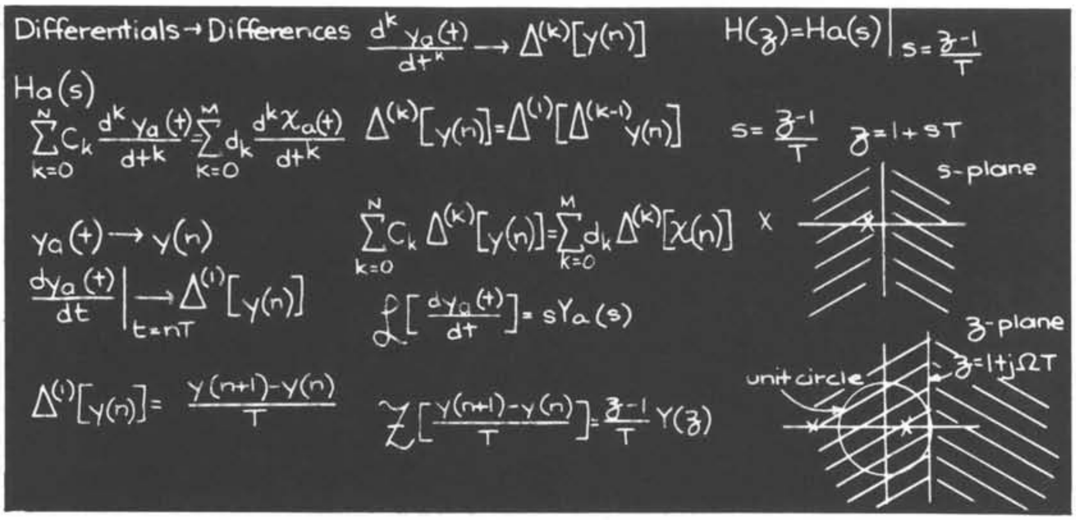
DESIGN OF IIR DIGITAL FILTERS - PART 1

1. Lecture 14 - 47 minutes



a.



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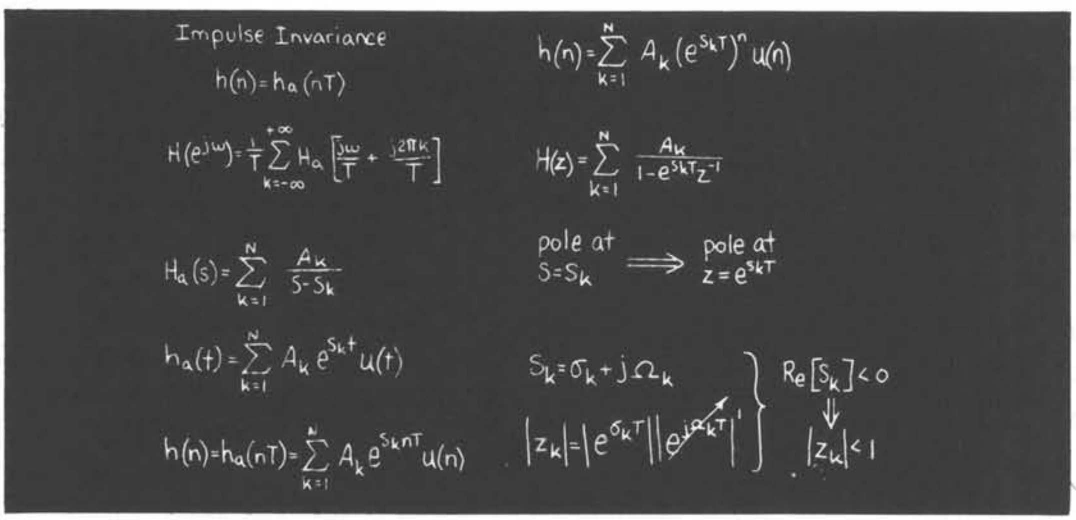
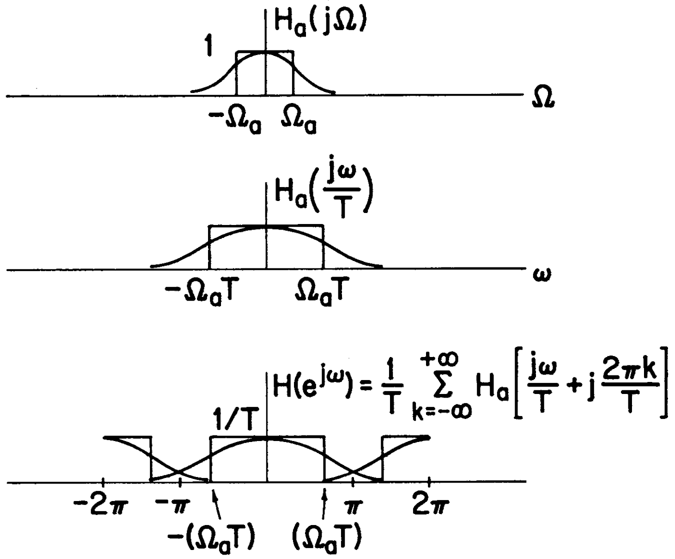


c.



An analog frequency response and the corresponding digital frequency response obtained through impulse invariance.

d.

2. Comments

With this lecture we begin the discussion of digital filter design techniques. The concept of frequency selective filtering for discrete-time signals is identical to that for continuous-time signals and stems from the fact that complex exponentials or sinusoids are eigenfunctions of linear shift-invariant systems. Just as with analog filters, ideal frequency response characteristics cannot be achieved exactly and must be approximated.

Design methods for analog filters have a long history and a variety of elegant design procedures have been developed. Many of the most useful digital filter design techniques are directed at transforming these analog filter designs to digital filter designs, thus taking advantage of a rich collection of available filter designs.

In this lecture two such transformation procedures are discussed. The first corresponds to approximating the linear constant coefficient differential equation for the analog filter by a linear constant coefficient difference equation by replacing derivative by differences. As we see, this transformation is not a useful one since it does not map the analog frequency response onto the unit circle and does not guarantee that a stable analog filter will yield a stable digital filter.

The second transformation discussed is the use of impulse invariance, corresponding to obtaining the discrete-time unit sample response by sampling the analog impulse response. Except for the effect of aliasing the digital frequency response obtained is a scaled replica of the analog frequency response.

3. Reading

Text: Sections 7.0 (page 403) and 7.1 up to example 7.3. (Example 7.3 will be covered in lecture 16.)

4. Problems

Problem 14.1

Consider an analog filter for which the input $x_a(t)$ and output $y_a(t)$ are related by the linear constant-coefficient differential equation

$$\frac{dy_a(t)}{dt} + 0.9\, y_a(t) = x_a(t)$$

A digital filter is obtained by replacing the first derivative by the first forward difference so that with x(n) and y(n) denoting the input and output of the digital filter,

$$\left[\frac{y(n+1) - y(n)}{T}\right] + 0.9\, y(n) = x(n)$$

Throughout this problem the digital filter is assumed to be causal.

(a) Determine and sketch the magnitude of the frequency response of the analog filter.

(b) Determine and sketch the magnitude of the frequency response of the digital filter for $T = 10/9$.

(c) Determine the range of values of T for which the digital filter is unstable. (Note that the analog filter is stable.)

Problem 14.2

$h_a(t)$ denotes the impulse response of an analog filter and is given by

$$h_a(t) = \begin{cases} e^{-0.9t} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Let h(n) denote the unit sample response and H(z) denote the system function for the digital filter designed from this analog filter by impulse invariance, i.e. with

$$h(n) = h_a(nT)$$

Determine H(z), including T as a parameter, and show that for any positive value of T, the digital filter is stable. Indicate also whether the digital filter approximates a lowpass filter or a highpass filter.

Problem 14.3

We now wish to design a digital filter from the analog filter of problem 14.2 using step invariance. Let $s_a(t)$ denote the step response of the analog filter of problem 4.2 and $s(n)$ the step response of the digital filter, so that

$$s(n) = s_a(nT)$$

(a) Determine $s_a(t)$

(b) Determine $s(n)$

(c) Determine $H(z)$, the system function of the digital filter. Note, in particular, that it is not the same as the system function obtained in problem 14.2 by using impulse invariance.

Problem 14.4

The system function $H_a(s)$ of an analog filter is

$$H_a(s) = \frac{s}{(s+1)(s+2)}$$

Determine the system function $H(z)$ of the digital filter obtained from this analog filter by impulse invariance.

Problem 14.5*

An ideal bandlimiting differentiator with delay τ is defined by the frequency response

$$H_a(j\Omega) = \begin{cases} j\Omega e^{-j\Omega\tau} & |\Omega| \le \Omega_c \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine the frequency response $H_d(e^{j\omega})$ of the digital filter obtained from this analog filter by impulse invariance. Assume that $\frac{\pi}{T} > \Omega_c$.

(b) Let $\hat{h}_d(n)$ denote the unit sample response of the filter determined (a) with $\tau = 0$. For certain values of τ, $h_d(n)$ can be expressed as $\hat{h}_d(n)$ delayed, i.e.

$$h_d(n) = \hat{h}_d(n - n_\tau)$$

where n_τ is an integer. Determine this set of values of τ and the resulting delay n_τ.